Strategic Voting in Plurality Elections

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* Duke University Manuscript, 2007.

ABSTRACT

This paper extends McKelvey and Ordeshook's (1972) *Calculus of Voting*, providing a direct derivation of the conditions under which voters will vote *strategically*: choose their second-most preferred candidate in order to prevent their least-preferred candidate from winning. Addressing this theoretical problem is important, as nearly all empirical research on strategic voting either implicitly or explicitly tests hypotheses which originate from this seminal model. The formal result allows us to *isolate the subset of voters to which strategic voting hypotheses properly apply*, and in turn motivates a critical reevaluation of past empirical work. In making this argument, we develop a unified and parsimonious framework for understanding competing models of tactical voter choice. The typology helps to elucidate the methodological difficulties in studying tactical behavior when faced with heterogeneous explanatory models, and suggests the need for both theoretical caution and more precise data instruments in future empirical work.

I. Introduction

Democratic institutions frequently provide electors the incentive to engage in *strategic voting*: choosing a candidate or policy alternative ranked lower than 1st in one's preference ordering, due to the fact that this most-preferred option has little to no chance of winning. Building on Duverger's famous 'wasted-vote' hypothesis (Duverger 1954), a stream of research tackled the issue of strategic voting in single-member district plurality elections. Such work generally comes in one of two guises. The first uses *game theory* to demonstrate that, under most circumstances, strategic voting will restrict the effective number of candidates receiving positive vote shares in an electoral district (Palfrey 1989, Cox 1994). Consider for example a single-member district election with three candidates. These authors demonstrate that, as long as voters have some shared information as to the identity of the two leading candidates, in equilibrium only these two 'viable' candidates will receive positive vote shares (Duvergerian Equilibrium). On the other hand, if expectations are such that it is impossible for voters to identify exactly the 1st and 2nd place candidates, in equilibrium all three candidates will receive votes (Non-Duvergerian Equilibrium). Such models thus generate aggregate level empirical predictions as to the situations under which Duvergerian and non-Duvergerian outcomes should obtain, and as to the relationship between an electoral district's magnitude (i.e. the number of seats which it contains) and its effective number of vote receiving parties.[1]

A second domain of research investigates strategic voting in mass elections from a *decision-theoretic* perspective. Since the 1970's this largely empirical literature has become one of the most prominent in behavioral political science. While its guiding

[1] These predictions have been tested with encouraging success in a variety of empirical settings (Ordeshook and Shvetsova 1994, Cox 1997).

intuition is similar to that in the above-reviewed game theoretic work, its predictions reside at the individual level. In particular, all such research either explicitly or implicitly tests hypotheses that originate in McKelvey and Ordeshook's seminal Calculus of Voting model, a decision-theoretic analysis of voter behavior in plurality elections (McKelvey and Ordeshook 1972). Interestingly, McKelvey and Ordeshook themselves only hinted that their model could be extended to study strategic voting, without ever presenting a full theoretical exposition. Thus, unlike the game theoretic research reviewed above, empirical studies in the decision-theoretic literature still suffer from the lack of a rigorous theoretical statement of their most basic hypotheses.

In part as a result of this theoretical lacuna, such empirical studies have at times employed mutually exclusive methodologies, such that the correctness of one approach entails the incorrectness of others. One prominent such example involves identifying the proper initial 'pool' of voters upon which to investigate strategic voting hypotheses. In this paper, we introduce a typology of voter choice in 3-candidate elections (Section II) which helps to elucidate different methods various authors have proposed for identifying this initial pool. We then present the first direct theoretical derivation of the conditions for strategic voting in the Calculus of Voting (Section III). The result allows us to isolate the *subset of voters to which strategic voting hypotheses properly apply*, and in turn identify which among the many proposed methodological options most accurately captures the model's comparative static implications. This same section presents empirical evidence in favor of our methodological recommendations. The concluding discussion demonstrates our typology's strength as a general framework for understanding the relationship between competing models of tactical voter behavior, and offers a number of additional cautionary notes regarding future empirical studies of strategic voting. Taken together, the paper's theoretical and methodological contributions represent an important step in identifying the explanatory power of instrumental rationality as a paradigm for the study of voter behavior, and more generally in our quest for rigorous empirical tests of formal theoretic models.

II. A Typology of Voter Choice

Downs (1957) presented the first informal specification of voting in *expected utility terms.* The expected utility model of voting in 2-candidate elections, dubbed the Calculus of Voting (henceforth COV), was formally specified and developed by Riker and Ordeshook (1968). This model yields unequivocal predictions as to voter choice in 2-candidate plurality elections: a rational voter will always choose her most-preferred candidate, denoted henceforth as '1' (a voter's second-most preferred candidate will be denoted '2', and so on). The following is the familiar condition for voter turnout in 2-candidate contests:

$$E_i^{\,1} - E_i^{\,0} \quad = \quad \tfrac{1}{2}\cdot\left(q_{12}^{\;1} + q_{12}^{\;0}\right)\cdot\left(U_1 - U_2\right) - C \quad > \quad 0, \qquad \textbf{(1)}$$

where $E_i^{\,1}$ is voter i's expected utility given she votes for 1, and $E_i^{\,0}$ is the utility associated with abstaining. Further, for $j \in \{1,2\}$, $q_{12}^{\;j}$ is the probability that 1 and the second-most-preferred candidate 2 tie given that i votes for j; U_j is voter i's utility payoff if j wins the election; and C represents the exogenous cost of voting.

Building on this framework, McKelvey and Ordeshook generalized the COV to elections with more than 2 candidates (1972). While, like Riker and Ordeshooks's original model, this generalization was undertaken first to study the question of voter turnout, the authors also illustrate the possibility of extending the framework to strategic voter choice, a task later taken up explicitly in papers by Black (1978), Cain (1978), and Gutkowski and Georges (1993). The gist of their theoretical results is intuitive: when a voter's most-preferred candidate 1 has little chance of winning, and the same voter perceives little utility difference between 1 and 2 but a large difference between 2 and her least-preferred candidate 3, this voter may under some circumstances maximize expected utility by *choosing 2 over 1 to prevent 3 from assuming office*.

This initial theoretical work stimulated a stream of empirical research. Though a number of empirical methodologies have been employed to study strategic voting, including those based on aggregate vote returns and those based on voters' self-reported motivations, the most satisfying approach directly tests the predicted relationship between survey respondents' preferences, their expectations, and the likelihood of choosing their second-most-preferred candidate. For illustrative purposes, we review a representative set of empirical papers adopting this latter approach: Cain (ibid), Abramson et. al (1992), Blais and Nadeau (1996), Ordeshook and Zeng (1997), Alvarez and Nagler (2000), and Alvarez et al. (2006).

Although distinguished by the electoral setting, statistical technology, and types of data they employ, all 6 papers purport to uncover evidence that voters' expectations affect their eventual choice in exactly the manner posited by an expected utility approach: choosing one's second-most preferred candidate 2 becomes more likely when one's first choice 1 is not a viable contender, and when one's preference for 1 over 2 (2 over 3) is small (large). However, behind these parallel findings lies an unresolved methodological issue which threatens these papers' empirical viability: namely, that of *identifying the* *proper pool of survey respondents upon which to test strategic voting hypotheses*. To elucidate this issue, consider the following typology of voter choice:

(Table 1 here)

Beginning with the Table's top row, notice the set of column markers {**C1,C2,…,C6**}. Each of these markers is associated with a particular *expectation profile*, a ranking of 1, 2, and 3 according to their expected success in the election. For each profile, the candidate listed at the top is expected to place 1st, the candidate listed in the middle is expected to place 2nd, and the candidate listed at the bottom is expected to place 3rd. For example, voters have strategic profile **C1** if they believe 1 will place 1st, 2 will place 2nd, and 3 will place 3rd: $\left(\begin{smallmatrix}1\\2\\3\end{smallmatrix}\right)$. Similarly, voters have profile **C5** if they believe 2 will place 1st, 3 will place 2nd, and 1 will place 3rd: $\left(\begin{smallmatrix}2\\3\\1\end{smallmatrix}\right)$. Moving to the Table's far left-hand side, V_j denotes the choice to vote for one's j^{th} preference; for example any voter who chooses 1 will find herself somewhere in the row denoted V_1. Coupling these two dimensions yields a 24 cell typology which classifies voters according to their vote choice and strategic profile. A voter with strategic profile **C6** who chooses her second-most preferred candidate 2 would thus occupy the cell labeled "Strategic Voting" in the Table's rightmost column, where **C6** and row V_2 intersect; a voter with strategic expectations **C1** who chooses her most-preferred candidate 1 would fall into the cell labeled "Straightforward" in column **C1**.[2]

Most past studies (including Cain, Abramson et. al, and Alvarez and Nagler from the list above) have tested the COV's strategic hypotheses on the entire sample of voters, i.e. voters from all six columns. However, consider a voter with expectation profile **C1**,

[2] The individual behavioral labels in Table 1's various cells are discussed more extensively below.

who believes her most-preferred candidate 1 will place 1st in the election, and her second and third preferences will place 2nd and 3rd in the election respectively. As well, recall that strategic voting is defined as abandoning one's most-preferred candidate when this candidate has little chance of winning, so as to prevent the victory of genuinely disliked candidate. Thus, almost by definition voters in **C1** should not cast strategic votes for their second-most preferred candidate 2, since 1 is a perfectly viable candidate and 3 is expected to place last. The upcoming theoretical derivation in Section III provides a formal basis for this intuition, demonstrating that the necessary conditions for strategic voting in the COV are *never satisfied among voters from **C1***.

What does this imply for empirical analysis? In short, it implies that empirical tests of strategic voting which include respondents with expectation profile **C1**, and furthermore any other expectation profile for which strategic voting is not possible, are mis-specified. For example, the three studies listed above (Cain, Abramson et al., Alvarez and Nagler) all test the hypothesis that the likelihood of choosing 2 should increase as 1's vote share goes down. Among potential strategic voters this should in fact be true: as 1's chances of winning become lower and lower, its chances of competing with the remaining two candidates becomes lower, in turn making it easier to choose 2 for strategic reasons. However, among voters from **C1** strategic voting should *never* be optimal according to the COV. To the extent that statistical results regarding strategic behavior include such voters, they will be clouded by the choice for 2 among voters who are clearly not behaving strategically in the traditional sense. As such, it is not possible to interpret results from a large segment of empirical papers as clear evidence for the presence of strategic voter behavior.

Recent contributions begin to address this problem, but without a rigorous theoretical derivation their analyses provide competing methodological prescriptions. For example, Ordeshook and Zeng (1997) and Alvarez et al. (2000) argue that rigorously specified tests of strategic voting should be restricted to voters whose *most-preferred candidate is expected to place last in the election*, i.e. that correctly specified strategic hypotheses should be restricted to voters with expectation profiles **C5** and **C6** from Table 1. On the other hand, Blais and Nadeau (ibid) argue that tests of strategic voting should be applied to any voters whose *most-preferred candidate is expected to place lower than their second-most preferred candidate*, regardless of whether or not this most-preferred candidate is expected to come in last place. In words, they assert that any voters with profile **C3**, **C5**, or **C6** should be included in strategic voting regressions. To settle this difference, we must return to McKelvey and Ordeshook's original model to establish the precise formal condition for strategic voting in 3-candidate elections. This will in turn allow us to identify the subset of voters to which strategic voting hypotheses should be restricted, a necessary step in establishing the importance of expected utility maximization as a model of voter choice.

III. Which Voters Can Be Strategic: A Direct Proof

To determine the conditions for strategic voting in the decision-theoretic framework, one simply compares the expected utility of choosing 1 to the expected utility for choosing 2 in order to discover the conditions under which strategically choosing 2 is a utility maximizing choice. Cain (ibid) undertakes this process, but his derivation does not adopt the precise mechanics made available by McKelvey and Ordeshook's COV, and as such makes many assumptions not consistent with expected utility maximization:

for example ignoring the possibility of 1st place ties in his primitive utility statements, and equating the probability of *breaking* a tie with the probability of *creating* a tie between 2 candidates. These simplifications render the expectation parameters in his derivation difficult both to interpret and operationalize, and prevent him from deriving the precise result which must guide empirical tests.[3]

Black's derivation (ibid) cleverly derives the conditions for strategic voting without engaging in an explicit comparison of the expected utility for choosing candidates 1 and 2. To do so, he adopts the turnout equations (see (1) above) derived originally by McKelvey and Ordeshook and uses them to derive a result indirectly. Consider the following defintions:

$$E_i^{\,1} - E_i^{\,0} \;=\; \Delta^1 \;, \quad \textit{and} \quad E_i^{\,2} - E_i^{\,0} \;=\; \Delta^2 \qquad \textbf{(2)}$$

where above $E_i^{\,j}$ represents voter i's expected utility for choosing $\mathrm{j} \in \{1,2\}$, and Δ^j represent the expected utility differentials between choosing j and choosing abstention (again see (1) above). To generate an expected utility comparison for choosing 1 as opposed to 2, Black simply subtracts the latter equation from the former, producing a result of the following form: $E_i^{\,1} - E_i^{\,2} = \Delta^1 - \Delta^2$. Though not identical, this result contains a similar intuition to the one that we derive below. Our direct theoretical proof is nonetheless far from merely academic. Formal precision is a value in and of itself; and many of the methodological insights which motivate this paper and our more general theoretical project emerged from process of derivation itself (Kselman and Niou 2007).

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[3] Similarly, Gutkowski and Georges (ibid) present a model of strategic choice which yields predictions somewhat similar to those below, but without undertaking an explicit derivation of the expected utility parameters which empirical work must operationalize.

Assume as above that a voter i strictly prefers $1 \succ 2 \succ 3$. If only $j \in \{1,2,3\}$ compete in a winner-take all contest, there are seven possible outcomes in the set α :[4]

$$\alpha \in \begin{cases} \{1\}......................1\_wins \\ \{2\}......................2\_wins \\ \{3\}......................3\_wins \\ \{12\}..................1,2\_tie \\ \{13\}..................1,3\_tie \\ \{23\}..................2,3\_tie \\ \{123\}................1,2,3\_tie \end{cases} . \qquad \textbf{(3)}$$

Define $q_\alpha{}^j$ as the probability the outcome α occurs given that i votes for candidate j, while $q_\alpha{}^0$ is the probability that outcome α occurs if she abstains. Further define $U_1 > U_2 > U_3$ as the utility the voter i obtains from an electoral victory by 1, 2, or 3 respectively. These parameters are sufficient to state $E_i{}^j$, the expected utility for choosing any candidate $j \in \{1,2,3\}$:

$$E_i{}^j = \left\{q_1{}^j \cdot U_1 + q_2{}^j \cdot U_2 + q_3{}^j \cdot U_3 + q_{12}{}^j \cdot U_{12} + q_{13}{}^j \cdot U_{13} + q_{23}{}^j \cdot U_{23} + q_{123}{}^j \cdot U_{123}\right\} - C \qquad \textbf{(4)}$$

Now write the expected utility comparison between 1 and 2 as follows:

$$E_i{}^1 - E_i{}^2 = \begin{Bmatrix} \left(q_1{}^1 - q_1{}^2\right) \cdot U_1 + \left(q_2{}^1 - q_2{}^2\right) \cdot U_2 + \left(q_3{}^1 - q_3{}^2\right) \cdot U_3 + \\ \left(q_{12}{}^1 - q_{12}{}^2\right) \cdot U_{12} + \left(q_{13}{}^1 - q_{13}{}^2\right) \cdot U_{13} + \left(q_{23}{}^1 - q_{23}{}^2\right) \cdot U_{23} + \\ \left(q_{123}{}^1 - q_{123}{}^2\right) \cdot U_{123} + \end{Bmatrix} . \qquad \textbf{(5)}$$

Each term in (5) represents the probability i is *pivotal* in securing a particular outcome multiplied by i's utility for the outcome. When $E_i{}^1 - E_i{}^2 > 0$ the voter should choose 1,

[4] As with the original COV, the model here could be quite naturally extended to study strategic choice in environments with more than three parties. In more complicated strategic environments, it may be the case that one chooses a party ranked even lower than second in one's preference ordering, but for the same reasons as those derived below: because this candidate has a more viable chance of defeating an even less preferred alternative.

while if $E_i^1 - E_i^2 < 0$ the voter should strategically choose 2. Note that the parameter C drops out of the expected utility comparison between choosing 1 and choosing 2. Thus, only a voter's utilities for candidates and expectations over outcomes appear in equation (5), supporting the idea that once in the voting booth voters ought to choose the utility maximizing candidate. To arrive at a more precise and parsimonious result, we now demonstrate the following Lemma, which provides the first explicit statements for the controversial *pivotal probabilities* so important for the above debate on rational voting:

Lemma 1:

a.) $q_1^1 - q_1^2 = q_{12}^0 + q_{12}^2 + q_{13}^2 + q_{123}^0 + q_{123}^2$;

b.) $q_2^1 - q_2^2 = -\left(q_{12}^0 + q_{12}^1 + q_{23}^1 + q_{123}^0 + q_{123}^1\right)$;

c.) $q_3^1 - q_3^2 = q_{23}^2 - q_{13}^1$.

The full proof of Lemma 1 is contained in Appendix A. In short, to derive explicit statements for the pivotal probabilities contained in (5), we undertake an exhaustive case by case analysis of the strategic conditions under which a voter can affect the outcome of an election by altering his or her vote choice. This involves cataloguing all situations in which a voter is effective in either creating or breaking electoral ties. Substituting into (5) using Lemma 1 yields the following expression:

$$E_i^1 - E_i^2 = \left\{ \begin{array}{l} \left(q_{12}^0 + q_{12}^2 + q_{13}^2 + q_{123}^0 + q_{123}^2\right) \cdot U_1 - \left(q_{12}^0 + q_{12}^1 + q_{23}^1 + q_{123}^0 + q_{123}^1\right) \cdot U_2 + \\ \left(q_{23}^2 - q_{13}^1\right) \cdot U_3 + \left(q_{12}^1 - q_{12}^2\right) \cdot U_{12} + \left(q_{13}^1 - q_{13}^2\right) \cdot U_{13} \\ + \left(q_{23}^1 - q_{23}^2\right) \cdot U_{23} + \left(q_{123}^1 - q_{123}^2\right) \cdot U_{123} \end{array} \right\} \quad \textbf{(6)}$$

Rearranging terms and assuming 2-way and 3-way ties are decided by chance, such that $U_{jk} = \frac{1}{2}\cdot\left(U_j + U_k\right)$ and $U_{ijk} = \frac{1}{3}\cdot\left(U_i + U_j + U_k\right)$, now restate (6) in the following form:

$$E_i^1 - E_i^2 = \left\{ \begin{array}{l} \left(q_{12}{}^0 + \frac{1}{2} q_{12}{}^1 + \frac{1}{2} q_{12}{}^2 + q_{123}{}^0 + \frac{1}{3} q_{123}{}^1 + \frac{1}{3} q_{123}{}^2\right)\left(U_1 - U_2\right) + \\ \left(\frac{1}{2} q_{13}{}^1 + \frac{1}{2} q_{13}{}^2 + \frac{1}{3} q_{123}{}^2\right)\left(U_1 - U_3\right) - \\ \left(\frac{1}{2} q_{23}{}^1 + \frac{1}{2} q_{23}{}^2 + \frac{1}{3} q_{123}{}^1\right)\cdot\left(U_2 - U_3\right) \end{array} \right\} \quad \textbf{(7)}$$

From (7) we can state the criteria which must obtain in order for a voter i to strategically abandon 1 for 2:

*** Proposition 1: The conditions for strategic voting in 3-Candidate elections are:**

$$E_i^{\ 2} - E_i^{\ 1} > 0 \quad iff:$$

$$\left(\frac{1}{2} q_{23}{}^1 + \frac{1}{2} q_{23}{}^2 + \frac{1}{3} q_{123}{}^1\right)\left(U_2 - U_3\right) > \quad \textbf{(8)}$$

$$\left(q_{12}{}^0 + \frac{1}{2} q_{12}{}^1 + \frac{1}{2} q_{12}{}^2 + q_{123}{}^0 + \frac{1}{3} q_{123}{}^1 + \frac{1}{3} q_{123}{}^2\right)\left(U_1 - U_2\right) + \left(\frac{1}{2} q_{13}{}^1 + \frac{1}{2} q_{13}{}^2 + \frac{1}{3} q_{123}{}^2\right)\left(U_1 - U_3\right).$$

If, as in McKelvey and Ordeshook's original analysis, one assumes that voters discount 3-way ties (i.e. $q_{123}{}^j = 0$), then Proposition 1 can be stated in a simplified form:

$$E_i^{\ 2} - E_i^{\ 1} > 0 \quad iff: \quad \textbf{(9)}$$

$$\left(q_{23}{}^1 + q_{23}{}^2\right)\cdot\left(U_2 - U_3\right) > \left(2q_{12}{}^0 + q_{12}{}^1 + q_{12}{}^2\right)\cdot\left(U_1 - U_2\right) + \left(q_{13}{}^1 + q_{13}{}^2\right)\cdot\left(U_1 - U_3\right).$$

It is from this condition that empirical hypotheses regarding the likelihood of strategic voting must be derived. Proposition 1's comparative static implications are straightforward and well-known: choosing 2 over 1 becomes more likely as the probability of a 1st place tie between 2 and 3 increases, and as the probability of 1st place ties between 1 and $j \in \{2,3\}$

decrease. Furthermore, choosing 2 over 1 becomes more likely as the utility differential between 1 and 2 decreases; and as the utility differential between 2 and 3 increases.

A less straight-forward implication which arises from Proposition 1 is that *not all voters can be strategic*. Consider once again a voter with profile **C1** who believes that her first-preference 1 will win the election, her second-preference 2 will place 2nd, and her least-preferred candidate will finish last. Among such voters, Proposition 1 can never be satisfied, as the *inequality's right-hand side will always be greater than its left-hand side* (see Appendix A). As argued above, including voters for whom the inequality in Proposition 1 cannot be satisfied in empirical analysis leads to mis-specified statistical regressions. Among which voters may this inequality be satisfied? Most obviously, under certain conditions be satisfied for voters with expectation profile's **C5** and **C6**, *who believe 1 will place last*, as the probability terms q_{12} and q_{13} will tend to be low among such voters. Indeed, Alvarez et al. and Ordeshook and Zeng from above claim that properly specified strategic voting hypotheses should be restricted to such voters.

However, careful examination reveals that *voters with profile **C3** may also under some conditions cast strategic votes.*[5] Voters in column **C3** believe their second-preference 2 will win, their first-preference 1 will place second, and their least-preferred candidate 3 will finish last. Consider someone who sees very little difference between 1 and 2, and who believes that 2's lead over both 1 and 3 is small (for example that 2 will win with probability 35%, 1 with probability 33%, and 3 with probability 32%). In such circumstances it is perfectly possible that strategically choosing 2 is a utility-maximizing choice. In other words, such voters choose the leading candidate 2 because this candidate

[5] We thank an anonymous reviewer for encouraging us to flesh out the following argument.

has the best chance of defeating one's least-preferred candidate 3, who is in fact quite viable. Restricting empirical analysis to voters with strategic profiles **C5** and **C6** would thus neglect a subset of voters whose incentives may be perfectly strategic (i.e. those with profile **C3**). Finally, it is straight-forward to show that among voters with profiles **C2** and **C4**, like those with profile **C1**, Proposition 1 cannot be satisfied. These results, all derived formally in Appendix A, are summarized in the following Proposition:

*** Proposition 2: A necessary condition for strategic voting is that the voter's second-most preferred candidate has a better chance of winning than does his/her most-preferred candidate. That is, strategic voting is only applicable to voters with profiles C3, C5, and C6 in Table 1.**

This Proposition formally establishes that voters with profiles **C3, C5,** and **C6**, those who believe 2 will finish ahead of 1, constitute the set of potential strategic voters. Studies of strategic voting which include voters from **C1, C2,** and **C4** are would thus be incorrectly specified. Furthermore, it provides the theoretical underpinning necessary to support Blais and Nadeau's approach to identifying the pool of potentially strategic voters as against the competing methodology proposed by Ordeshook and Zeng (1997) and Alvarez et al (2000). Restricting empirical analysis to voters with strategic profiles **C5** and **C6** neglects a subset of voters whose incentives may be perfectly strategic (i.e. those with profile **C3**), thus generating a systematically biased sample.

We now present a series of aggregate data to demonstrate tangibly the importance of our methodological prescriptions, and more particularly to demonstrate that there do indeed exist many voters who choose their second-most preferred candidate for reasons having little to do with strategic voting in the traditional sense. Table 2 classifies survey respondents from the 1988 Canadian Federal National Election study survey according to their preferences, expectations, and vote choice as in the typology above:[6]

(Table 2 here)

To measure voters' relative preferences over candidates we follow most previous work in using survey respondents' Feeling Thermometer ratings. In this survey respondents were asked to assign each candidate competing in their district a 'Probability of Winning', which allows us to rank the candidates according to their expected finish. Finally, we used respondents' pre-election vote 'intention' to capture their vote choice, so as to avoid problems of misreporting in post-election surveys (Alvarez and Nagler ibid).

The data are complicated by the fact that many respondents assigned two or more candidates an equal probability of winning, thus falling into Table 2b, which groups them according to their proper strategic profile. However, such respondents can in fact be assigned *strategically equivalent* profiles to those in Table 2a, which in turn match the typological categories from Table 1.[7] For example, voters with profile **C12** in Table 2b believe that their most-preferred candidate 1 will place last, and assign their second-most preferred candidate 2 and least preferred candidate 3 an equal probability of winning. These voters find themselves in a situation which is qualitatively identical to those of columns **C5** and **C6** from Table 2a, who also believe that 1 will place last while 2 and 3

[6] To remain consistent with the 3-party example developed throughout the paper, we exclude respondents from the 4-party Quebecois districts. What follows is not intended to be an exhaustive empirical analysis of strategic voting in the 1988 Canadian election, but merely to suggest the importance of excluding voters with profiles **C1, C2,** and **C4** from tests of strategic voting, while including those with profile **C3.** Investigating voters from Quebecois districts would neither upset nor strengthen the following argument, but would require a rather lengthy theoretical proof of Proposition 1's equivalent conditions in the case of 4-party competition. We reserve this and other extensions for future work.

[7] A significant number of voters also reported being indifferent between one or more of the election's candidates. Each indifference profile could similarly be assigned a strategically equivalent profile from the fundamental typology in Table 1. As with the data on Quebecois respondents, including such respondents would neither upset nor strengthen the empirical analysis' basic message, but would multiply combinatorially the number of categories in table 2 in order to capture the many indifference profiles associated with each of its 13 strategic profiles. Such an analysis is beyond our current scope.

are competing for 1st place. Proposition 2 applies as well to voters in Table 2b such that, regardless of expected ties, it is among voters who believe 2 will place ahead of 1 that Proposition 1 may be satisfied (**C3**, **C5**, **C6**, **C10**, and **C12**).

All told 84.9% of respondents choose their most-preferred candidate. These voters come in two basic 'types'. Within the context of the traditional COV studied here, many *straight-forward* respondents see no conflict between expectations and preferences: if their first-preference 1 is expected to place ahead of their second-preference 2, choosing 1 will always be a utility-maximizing choice. The respondents directly above potential strategic voters in columns **C3, C5, C6, C10,** and **C12** are either straight-forward or *sincere*: some voters in this group may choose 1 because it is a straight-forward utility maximizing choice (i.e. the condition for choosing 2 in Proposition 1 is not satisfied); on the other hand, some voters may 'sincerely' choose 1 for expressive reasons despite the fact that casting a strategic vote for 2 would in fact be the utility maximizing choice.

Roughly 7.9% of respondents chose candidate 2 (121 of 1,525). Among these, only 54.5% (66 of 121) of respondents (those from **C3, C5, C6, C10,** and **C12**) choose their second-preference when this candidate is expected to place ahead of 1, and should be analyzed in empirical tests of strategic voting. Thus, in empirical tests which included voters from all columns, *nearly half* of the respondents that choose candidate 2 must be doing so for 'non-strategic' reasons. Statistical results from regressions which included such voters would thus be largely spurious: their regression coefficients regarding the likelihood of strategic voting would be influenced by the choice for 2 among voters who are by definition 'non-strategic'. Furthermore, among the 54.5% of voters who may indeed be choosing 2 for strategic reasons, over 40% come from column **C3** (28 of 66).

Excluding such voters from empirical analyses as prescribed by Alvarez et al. (ibid) and Ordeshook and Zeng (ibid) would thus neglect an important subset of potentially strategic voters, and generate regression coefficients based on a systematically biased sample. In summary, the heterogeneous strategic profiles of respondents who choose 2 reinforces the importance for future empirical studies of isolating all voters whose most-preferred candidate is trailing her second-most-preferred. Not doing so leads to one or the other specification problems described above, which in turn prevents an accurate empirical appraisal of strategic voter behavior.

IV. Concluding Discussion

This paper extends McKelvey and Ordeshook's COV by providing a direct proof of the conditions under which voters should strategically abandon their most-preferred candidate, 1, and vote for their second-most preferred candidate, 2. A useful theoretical implication of our model is that only voters who believe 2 has a better chance of winning than does 1, i.e. voters with profiles **C3**, **C5**, and **C6** in the typology above, might choose their second-most-preferred candidate. These theoretical results allow us to both identify a recurring methodological problem in past empirical studies and provide a solution applicable to future empirical work in the field.

However, even studies which followed this paper's basic methodological prescription would be confronted with a number of additional complications in attempting to interpret their empirical results. The decision-theoretic approach embodied in the COV is only one of many possible models which explain why voters choose their second-most-preferred candidate 2. Voters in columns **C3**, **C5**, and **C6** might choose 2 for any number of reasons not associated with the COV; similarly, it would be hasty to label voters who choose 2 from columns **C1**, **C2**, and **C4** as 'non-rational' simply because their behavior is not compatible with decision-theoretic expected utility maximization.

For example, Niou (2001) studies the incentives for strategic coordination when a 3-candidate election's plurality winner is also the election's Condorcet loser. In other words, the candidate ranked first by the largest number of voters would nonetheless lose in a pair-wise contest against either of the other 2 candidates. This situation arises when one large party competes against two smaller parties, and the 2 smaller parties' supporters rank the largest party last in their preference profile. This is possible for voters with strategic profiles **C4** and **C6**, who expect their least-preferred-candidate 3 to win the election, and their most-preferred and second-most-preferred candidates 1 and 2 to come in second and third place (for voters in **C4** 1 is in second place and 2 is in third, and vice versa for voters in **C6**). The analysis highlights that under these circumstances elites from the two smaller parties find themselves in a *Battle of the Sexes coordination game*: proponents of both smaller parties prefer a victory by either small party over the least-preferred plurality winner; but each would also prefer that, from among the two small parties, their particular party be the eventual coordination choice. This model generates insights not captured in the COV. While voters in **C4** never cast strategic votes of the type modeled in Proposition 1, they may yet choose 2 when this second-preference becomes the *coordination choice of a Battle of the Sexes* game played by political elites. Similarly, we should avoid rushing to the conclusion that voters with profile **C6** are behaving 'non-instrumentally' if statistical analysis fails to support the COV's strategic hypotheses; their behavior may simply be the result of a distinct instrumental calculation based on the outcome of elite coordination.

Thus, without adequately accounting for the predictions of competing explanatory models, one risks making incomplete inferences about the basic motivations which determine voter choice. This point becomes especially salient as the diversity of explanatory approaches to tactical voter choice moves beyond the traditional strategic voting framework. Kselman and Niou (2007) model the conditions under which voters with profiles **C1-C4,** who believe their most-preferred candidate 1 will place no lower than 2nd in the election, will cast *protest votes* for 2: votes which register a *signal of dissatisfaction* with their most-preferred party. In Britain, for example, traditional Labour voters who feel Labour has moved too far to the ideological left may choose their second-most preferred party, the centrist Liberals, in the hope of inducing Labour leaders back towards the political center. This becomes an optimal strategy when the short-term instrumental costs of defecting to one's second-preference 2 are low and/or when the expected impact of protest voting on 1's future behavior is high.

As well, though not included in Table 1 (which is explicitly designed to catalogue forms of instrumental or outcome-oriented behavior), scholars of electoral 'tides' and voter band-wagoning might argue that respondents from **C3** and **C5**, whose second-preference 2 is the expected plurality winner, choose this second-preference not to effectuate some tangible political outcome, but simply out of a desire to 'choose the winning team' (Bartels 1988 and Schuessler 2000). Furthermore, consider voters with profile **C3**, for whom 2 is the expected plurality winner. Among such voters, Proposition 1 above and the logic of electoral band-wagoning generate over-lapping predictions: they should become more likely to choose 2 as her lead over 1 increases. Since both arguments make the same prediction, evidence to this effect is insufficient to precisely identify voters' basic behavioral motivations.

The typology developed in Table 1 thus provides a general analytic framework within which to both classify and compare alternative theoretical explanations of tactical voter choice. Future empirical analysis should be grounded in theoretical advances which identify empirical procedures to account for explanatory heterogeneity, and which address the individual, institutional, and contextual factors that may impel voters to adopt one behavioral pattern over another. Where models generate parallel predictions, analysts must find additional survey items which help to identify the precise motivations which guide tactical choice. Indeed, theoretical precision is only a first step towards improved empirical analysis. Deepening our theoretical understanding of tactical behavior will in turn help us in developing survey instruments suited to identifying novel behavioral forms such as coordinated voting, voter signaling, and voter band-wagoning. As a whole, this research program will allow us a more nuanced and accurate appraisal of instrumental rationality as a paradigm for the study of voter behavior.

Table 1: A Typology of Choice in 3-Candidate Plurality Elections

Preferences/Viabilities

Vote Choice

	C1 1 2 3	C2 1 3 2	C3 2 1 3	C4 3 1 2	C5 2 3 1	C6 3 2 1
Vote for 1	Straight-Forward Vote	Straight-Forward Vote	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote/ Sincere Vote
Vote for 2	Protest Vote	Protest Vote	Strategic Vote/ Protest Vote	Protest Vote/ Coordination Vote	Strategic Vote	Strategic Vote/ Coordination Vote
Vote for 3	Irrational Vote	Irrational Vote	Irrational Vote	Irrational Vote	Irrational Vote	Irrational Vote
Abstention	Abstention	Abstention	Abstention	Abstention	Abstention	Abstention

Table 2: Distribution of Voters with Strict Preferences in the 1988 Canadian NES

Table 2a.) Respondents with Strict Expectations.

	C1 1 2 3	**C2** 1 3 2	**C3** 2 1 3	**C4** 3 1 2	**C5** 2 3 1	**C6** 3 2 1	**Total**
$V = P_1$	427	231	103	81	28	34	904
$V = P_2$	20	8	28	5	13	8	82
$V = P_3$	7	5	0	2	1	7	22
A	20	10	6	4	4	5	49
Total	474	254	137	92	46	54	**N=1057**

Table 2b.) Respondents with Weak Expectations.

	C7 12 3	**C8** 13 2	**C9** 1 23	**C10** 2 13	**C11** 3 12	**C12** 23 1	**C13** 123	**Total**
$V = P_1$	93	67	133	33	37	15	13	391
$V = P_2$	11	1	6	11	4	6	0	39
$V = P_3$	1	2	2	0	3	0	0	8
A	10	1	7	5	3	2	2	30
Total	115	71	148	49	47	23	15	**N=468**

* In Table 2b, respondents assign two or more candidates an equal probability of winning. A respondent in column **C7** assigns 1 and 2 an equal likelihood of winning, and assigns 3 a lesser likelihood. A respondent in column **C10** believes 2 to be the front-runner, and believes that the two trailing candidates 1 and 3 have the same probability of winning. A respondent in column **C13** believes all 3 candidates have an equal likelihood of winning.

Appendix A

A1.) Proof of Lemma 1 and Proposition 1

Begin with the proof of Lemma 1 part (a). $q_1^{1} - q_1^{2}$ is the probability that '1 wins' given a vote for 1 as opposed to 2, i.e. the probability that choosing 1 as opposed to 2 is *pivotal* for 1's victory. The choice between 1 and 2 is pivotal for the outcome '1 wins' if a vote for 2 as opposed to 1 either *creates* or *upsets* this outcome. In moving his or her choice from 1 to 2, a voter i can obviously not *create* the outcome '1 wins' ($\alpha = 1$). We are thus looking for the competitive situations in which choosing 2 as opposed to 1 *upsets* the outcome $\alpha = 1$, a victory by the voter's first-preference. The following demonstration identifies the conditions under which switching a vote from 1 to 2 changes the outcome from '1 wins' ($\alpha = 1$) to one of the other 6 outcomes in α.

-Case 1) '1 wins' to '2 wins'
Shifting a vote from 1 to 2 changes the outcome from $\alpha = 1$ to $\alpha = 2$ only if:

a.) all 3 candidates are tied given the voter's abstention, denoted by q_{123}^{0}; or
b.) 1 and 2 are tied given the voter's abstention, denoted by q_{12}^{0}.

For example, in a 10-person electorate, if the vote shares among 1, 2, and 3 are (3,3,3) or (4,4,1) with the voter abstaining from voting, then the voter is pivotal in deciding whether 1 or 2 would win.

-Case 2) '1 wins' to '3 wins'
Under no circumstances will shifting a vote from 1 to 2 change the outcome from $\alpha = 1$ to $\alpha = 3$. By assumption, 1 is the plurality winner given a vote for 1, which implies that 1 must have at least one more vote than 3. By switching her vote to 2, the voter can thus at best put 3 in a 1st place tie with 1, but under no circumstances will this switch lead to an outright victory by 3.

-Case 3) '1 wins' to '1 and 2 tie'
Under what circumstances will shifting a vote from 1 to 2 change the outcome from $\alpha = 1$ to $\alpha = 12$? Intuitively, this is the case only if a vote for 2 creates a tie between 1 and 2, q_{12}^{2}. For example, in a 10 personal electorate, the following profile would imply that switching from 1 to 2 changed the outcome from $\alpha = 1$ to $\alpha = 12$: (5,3,2). Switching from 1 to 2 thus creates a 4-4 tie between 1 and 2.

-Case 4) '1 wins' to '1 and 3 tie'
As in Case 3, shifting from 1 to 2 shifts the outcome from $\alpha = 1$ to $\alpha = 13$ only if 1 and 3 tie given a vote for 2: q_{13}^{2}. In a 10 person electorate, consider the following profile: (5,1,4). Switching from 1 to 2 thus creates a 4-4 tie between 1 and 3.

-Case 5) '1 wins' to '2 and 3 tie'
By assumption, a vote for 1 accompanies a plurality victory for 1. Shifting the vote to 2 can at best put 3 in a 1st place tie with 1, but can never put 3 ahead of 1. Thus, under no circumstances will shifting a vote from 1 to 2 give 2 and 3 sole possession of 1st place (implying 3 receives more votes than 1, which is a contradiction).

-Case 6) '1 wins' to '1, 2 and 3 tie'
Shifting one's vote from 1 to 2 changes the outcome from $\alpha = 1$ to $\alpha = 123$ if choosing 2 creates a 3-way 1st place ties: $q_{123}{}^{2}$. Consider the following profile: (5,3,4). 1 would lose its winning position and a 3-way tie would be created if the voter switcher her vote from 1 to 2.

Adding up all of the possible situations in which shifting a vote from 1 to 2 is pivotal in changing the outcome from '1 wins' to some other outcome in α (from Cases 1, 3, 4, and 6 above) yields the following statement of Lemma 1 part (a): $q_1{}^{1} - q_1{}^{2}$ = $q_{12}{}^{0} + q_{12}{}^{2} + q_{13}{}^{2} + q_{123}{}^{0} + q_{123}{}^{2}$. ■

To prove part (b) of Lemma 1, note that $q_2{}^{1} - q_2{}^{2} = -(q_2{}^{2} - q_2{}^{1}) < 0$. In other words, 2 is always more likely to win when the voter chooses 2 rather than 1. The proof that $q_2{}^{2} - q_2{}^{1}$ = $q_{12}{}^{0} + q_{12}{}^{1} + q_{23}{}^{1} + q_{123}{}^{0} + q_{123}{}^{1}$, which yields part (b) follows the same logic as in part (a) of Lemma 1 and is thus omitted.

To prove part (c) of Lemma 1, note again that $q_3{}^{1} - q_3{}^{2}$ denotes the probability that voter i either *creates* or *upsets* the outcome '3 wins' ($\alpha = 3$) by switching her vote from 1 to 2. Among the seven possible outcomes, if '1 wins' (α = 1) given a choice for 1, then by switching the her vote from 1 to 2 voter i could at best move 3 into a tie for the first place with 1, but could never produce the outcome of interest '3 wins'. If α = 2, 12, 23, or 123 given a choice for 1, then a switch from 1 to 2 would either make or leave 2 the winner. Thus, in the cases α = 1, 2, 12, 23, or 123, voter *i* cannot create the outcome $\alpha = 3$ by choosing 2 over 1; 3 has no chance of winning regardless of whether i chooses 1 or 2.

Only if '1 and 3 tie' (α = 13) or '3 wins' (α = 3) when i chooses 1 can i possibly affect the outcome '3 wins' by switching his or her vote from 1 to 2. In the case $\alpha = 13$ given a choice for 1 (which occurs with probability $q_{13}{}^{1}$), in switching her vote from 1 to 2 she *creates* the outcome in question '3 wins'. For example, if the vote distribution is (5,3,5) with the voter voting for 1, 3 would emerge as the winner if the voter switches her vote to 2 because the vote distribution would now become (4,4,5). In the case $\alpha = 3$ given a choice for 1, in switching her vote from 1 to 2 voter i can *upset* this outcome by moving 2 into a first-place tie with 3 (which occurs with probability $q_{23}{}^{2}$). For example, if the vote distribution is (4,4,5) when the voter votes for 1, then the voter would create a tie between 2 and 3 after switching her vote from 1 to 2 because the vote distribution would now become (3,5,5).

The combined wisdom of these individual cases indicates that a vote for 1 as opposed to 2 makes the outcome '3 wins' more likely to the extent that voting for 2 upsets 3's outright victory over 2, creating the outcome '2 and 3 tie'; but it makes the outcome '3 wins' less likely to the extent that choosing 1 eliminates 3's outright victory over 1, creating the outcome '1 and 3 tie'. We can thus state that $q_3^{\ 1} - q_3^{\ 2} = q_{23}^{\ 2} - q_{13}^{\ 1}$, establishing the final part (a) of Lemma 1. ■

A2.) <u>Proof of Proposition 2</u>

Proposition 1 allows us to identify the subsets of voters for whom strategic choice is in fact a viable option. In particular, we will identify the subsets of voter expectation profiles from Table 1 for whom the condition expressed in Proposition 1 can in fact be satisfied. The proof is based on the following simple insight: if candidate 2 is expected to place ahead of candidate 1 (expectation profiles **C3**, **C5**, and **C6**), then candidate 2 will be more likely than candidate 1 to find itself in a 1st place tie with candidate 3: $q_{23}^{\ j} > q_{13}^{\ j}$. If 1 is expected to place ahead of 2 (**C1**, **C2**, and **C4**), the reverse is true. Furthermore, if candidate 2 is expected to place ahead of 1, than $q_{123}^{\ 1} > q_{123}^{\ 2}$: the likelihood of a 3-way tie decreases when choosing the election's expected plurality winner, and thus further distancing her from the remaining electoral competitors. Again, the reverse is true when candidate 1 is expected to place ahead of candidate 2.

These relationships allow us to establish Proposition 2. To do so, we consider the case in which strategic voting is most likely, namely that in which $U_1 \cong U_2$ (such that $U_1 - U_2 \cong 0$): when voters perceive little difference between their most- and second-most-preferred candidates, the choice for 2 becomes all the more acceptable. In such circumstances, it is straight-forward to show that, among voters for whom 2 is expected to place ahead of 1, the condition for strategic voting expressed in Proposition 1 can be satisfied: since $q_{23}^{\ j} > q_{13}^{\ j}$ and $q_{123}^{\ 1} > q_{123}^{\ 2}$, as long as the difference between $(U_1 - U_3)$ and $(U_2 - U_3)$ is not excessive, the inequality's left-hand side will be greater than its right-hand side, and choosing 2 will be the utility maximizing option. On the other hand, if 1 is expected to place ahead of 2, we know that $q_{23}^{\ j} < q_{13}^{\ j}$ and $q_{123}^{\ 1} < q_{123}^{\ 2}$. Furthermore, by definition $(U_1 - U_3) > (U_2 - U_3)$. As such, even in the most favorable circumstances for strategic voting (i.e. when $U_1 \cong U_2$), if 1 is expected to place ahead of 2 the right-hand side of Proposition 1 will always be greater than its left-hand side, and as such voters with expectation profiles **C1**, **C2**, and **C4** cannot be strategic voters. ■

REFERENCES

-Abramson, Paul R, John H. Aldrich, Phil Paolino, and David W. Rohde. 1992. "'Sophisticated' Voting in the 1988 Presidential Primaries." *American Political Science Review*, vol. 86, no. 1.

-Alvarez, Michael R, and Jonathon Nagler. 2000. "A New Approach for Modelling Strategic Voting in Multiparty Elections." *British Journal of Political Science*, vol. 30, pgs 57-75.

-Black, Jerome H. 1978. "The Multi-candidate Calculus of Voting: Application to Canadian Federal Elections." *American Journal of Political Science*, vol. 22, no. 3.

-Blais, Andre and Richard Nadeau. 1996. "Measuring Strategic Voting: A Two-Step Procedure," *Electoral Studies*, vol. 15, no. 1, pp. 39-52.

-Cain, Bruce E. 1978. "Strategic Voting in Britain." *American Journal of Political Science*, vol. 22, no. 3.

-Converse, Philip. 1964. "The Nature of Belief Systems in Mass Publics," in Apter, editor (1964).

-Cox, Gary. 1994. "Strategic Voting Equilibria Under the Single Non-Transferable Vote." *American Political Science Review*, vol. 88, pgs 608-621.

-Cox, Gary. 1997. *Making Votes Count. Strategic Coordination in the World's Electoral Systems.* Cambirdge University Press: Cambridge.

-Downs, Anthony. 1957. *An Economic Theory of Democracy.* Harper and Row: New York.

-Duverger, Maurice. 1954. *Political Parties.* Wiley: New York.

-Fiorina. Morris P. 1981. *Retrospective Voting in American National Elections.* Yale University Press: New Haven.

-Gutkowski, William, and John Georges. 1993. "Optimal Sophisticated Voting Strategies in Single Ballot Elections Involving Three Candidates." *Public Choice*, vol. 77, no. 2.

-Herndon, James F, and Joseph L. Bernd editors. 1972. *Mathematical Applications in Political Science IV.* University Press of Virginia: Charlottesville.

-Kselman, Daniel, and Emerson Niou. 2007. *Protest Voting: A Theory of Voter Signaling.* Working paper, Duke University.

-McKelvey, Richard D. and Peter C. Ordeshook. 1972. "A General Theory of the Calculus of Voting," in Herndon and Bernd (1972).

-Niou, M. S. Emerson. 2001. "Strategic Voting under Plurality and Runoff Rules," *Journal of Theoretical Politics* 13(2):pp.209-227, April.

-Ordeshook, Peter C, and Langche Zeng. 1997. "Rational Voters and Strategic Voting. Evidence from the 1968, 1980, and 1992 Elections." *Journal of Theoretical Politics*, vol. 9, no. 2.

-Ordeshook, Peter C, and Olga Shvetsova. 1994. "Ethnic Heterogeneity, District Magnitude, and the number of Parties." *American Journal of Political Science*, vol. 38, pgs 100-123.

-Palfrey, Thomas. 1989. "A Mathematical Proof of Duverger's Law." In Peter C. Ordeshook (ed), *Models of Strategic Choice in Politics.* Ann Arbor: Univeristy of Michigan Press.

-Riker, William H, and Peter C. Ordeshook. 1968. "A theory of the calculus of voting." *American Political Science Review*, vol. 62, no. 1.

-Schuessler, Alexander A. 2000. *A Logic of Expressive Choice.* Princeton University Press, Princeton and London.