

Problem Set 1 Solutions (For January 13)

1. The average atomic mass is just the weighted sum of individual isotopic masses with natural isotopic abundances (expressed as fractions) for weighting factors. Thus,

$$\begin{aligned}\text{Atomic mass of Fe} &= (0.058)(53.9396 \text{ amu}) + (0.918)(55.9499 \text{ amu}) + \\ &\quad (0.021)(56.9354 \text{ amu}) + (0.003)(57.9333 \text{ amu}) \\ &= 55.9 \text{ amu}\end{aligned}$$

There's a bit of a significant figure problem here, since one of the abundances, 0.3%, was quoted to only one significant figure. The product $(0.003)(57.9333)$ therefore has only one significant figure ($0.003 \times 57.9333 = 0.2$, not 0.1738), so that when all the products are summed, the final answer is correctly expressed with only one figure to the right of the decimal point. You'll note that the text reports 55.85 as the Fe atomic weight. This is due to more precise abundance measurements than were quoted in the problem.

2. We're told that three oxides of manganese (Mn) contain 63.2%, 69.6%, and 77.4% Mn by weight. We're asked to deduce two possible atomic masses for Mn and the corresponding the molecular formulas. The first calculation follows the iron oxides calculations we did in lecture. Imagine we have 100 g of each oxide:

Mass Mn/g	Mass O/g	Mass O/Mass Mn	Combining ratios
63.2	36.8	0.582	$0.582/0.292 = 2$
69.6	30.4	0.437	$0.437/0.292 = 1.5$
77.4	22.6	0.292	$0.292/0.292 = 1$

so the weights of O combining with a fixed amount of Mn are in the ratio 1:1.5:2 or 2:3:4. There are thus two possibilities: either we have the oxides MnO, MnO_{1.5}, (which we should write as Mn₂O₃), and MnO₂ or we have the oxides MnO₂, MnO₃, and MnO₄. With an oxygen molar mass of 16.00 g mol⁻¹, the first set predicts a molar mass for Mn of

$$(16.00 \text{ g mol}^{-1}) \left(\frac{77.4 \text{ g Mn}}{22.6 \text{ g O}} \right) \left(1 \frac{\text{mol O}}{\text{mol Mn}} \right) \cong 55 \text{ g mol}^{-1}$$

while the second predicts a value twice as large. The first choice turns out to be the correct one, but additional information would be needed to prove this.

3. Conservation of charge says the total number of excess elementary negative charges on the oxygen ions must equal the total number of excess elementary positive charges on all the iron atoms (where "excess" means those charges produced by ionization of the neutral atoms). Suppose we have a sample of this compound, Fe_{0.930}O_{1.000}, that has 1.00 mol of oxygen ions (O²⁻). The number of moles of elementary negative charges is therefore 2.00 mol, and the molecular formula tells us that 0.930 mol of iron ions of *both* charge states are in our sample. Of these, let x mol represent the amount of Fe³⁺ ions so that $(0.930 - x)$ mol is the amount of Fe²⁺ ions. The total number of elementary positive charges from all iron ions is thus $[3x + 2(0.930 - x)]$ mol. We write the charge conservation equation:

$$\begin{array}{ccc} [3x + 2(0.930 - x)] \text{ mol} & = & 2 \text{ mol} \\ \text{total number of} & & \text{total number of} \\ \text{positive charges} & & \text{negative charges} \end{array}$$

and solve for x , finding $x = 0.140$ mol. The percentage of Fe³⁺ in the sample is the ratio of this amount, 0.140 mol, to the total iron amount, 0.930 mol, multiplied by 100 %:

$$\frac{0.140 \text{ mol}}{0.930 \text{ mol}} = 100 \% = 15.1 \% \text{ Fe}^{3+}$$