

DARTMOUTH COLLEGE
Department of Economics

Economics 36
Theory of Finance

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First Mid-Term Exam Solutions

1. (5 points) You work at Bell Atlantic at an annual salary of \$102,000, paid monthly. You are offered a severance package that will pay 6 months salary now or you and your heirs \$7000 per year forever. If your required return is 12 percent, which should you choose, and why?

**Answer: take the perpetuity! 6 months salary = $6(102,000/12) = \$51,000$
Perpetuity: $PV=C*(1/r) = 7000/.12 = \$58,333$**

2. (5 points) Your portfolio contains three stocks: Sybase, Dell, and Microsoft. At the beginning of the year you bought one share of each at \$10, \$50, and \$100 respectively. At the end of the year you sold them at \$20, \$60, and \$110 respectively. No dividends were paid. What is the holding period return on your portfolio?

Beginning value=100+50+10=160. Ending value=110+60+20=190. Holding period return=(190-160)/160=18.75%

3. (10 points) You manage Dartmouth College Endowment. You are considering moving a quarter of your \$2 billion US equity portfolio into the Japanese stock market. The expected return and standard deviation of returns in the US market are 10% and 14% respectively. The expected return and standard deviation of the Japanese market's returns, measured in US dollars, are 12% and 17%, respectively. The correlation coefficient of returns between the two markets is 0.6. If you moved the money, what would the portfolio's standard deviation of returns be?

13.49%

The 2-asset portfolio variance is:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 \sigma_1 w_2 \sigma_2 \rho_{12}$$

$$w_1 = 0.75, w_2 = 0.25, \sigma_1 = 0.14, \sigma_2 = 0.17, \rho_{12} = 0.6$$

$$\sigma_p^2 = 0.01818625, \sigma_p = 0.1349$$

4. (6 points) Suppose you are offered an investment that pays 11.55% continuously compounded. How long will it take for your money to double?

Solve the following equation for t: $\$1(e^{0.1155t}) = \2 . Solve for by taking the natural log of 2 and dividing by .1155; the solution is 6.00 years.

5. (6 points) An investment costs \$125 and produces cash inflows of \$15 per year in perpetuity. What is the IRR of this investment?

IRR is the discount rate that makes NPV=0. Thus, IRR is the rate r which solves the following equation. $NPV = 0 = -\$125 + \$15/r$ $r = \$15/\$125 = .12$ or 12%

6. (7 points) Mr. DiCaprio has \$100 today and the market interest rate is 10% per year. Mr. DiCaprio also has an investment opportunity in which he can invest \$50 today and receive \$60 next year. Suppose Mr. DiCaprio consumes \$20 this year and invests in the project. What is the maximum amount he can consume next year?

Consumption next year is what's left over from this year's income (\$100-\$50-\$20) invested at 10% plus next year's \$60. Therefore, consumption next year = $(100-20-50)*1.1 + 60=93$.

7. (10 points) You have been asked to value orange groves belonging to Roll Corporation. The groves produce 1.6 billion oranges per year. Oranges currently sell for \$.10 per 100. With normal maintenance, this level of production can be sustained indefinitely (assuming no unexpected hard freezes occur). Variable costs (primarily upkeep and harvesting) are \$1.2 million per year. Fixed costs are negligible. The nominal discount rate is 18%, and the inflation rate is 10%. Assuming that orange prices and the variable costs change with inflation, what is the value of the groves? (Ignore taxes and depreciation).

Revenues from sales of \$1.6 billion oranges at \$.001 apiece, are equal to \$1.6 million. Annual cash flow is equal to revenues minus expenses (\$1.6million - \$1.2 million)=\$.4 million per year. Note: it would be wrong to treat the \$.4 million as a perpetuity, and divide by .18 to calculate the present value. The \$.4 million annual cash flow is in current dollars, therefore does not reflect future inflation. The 18% discount rate does reflect future inflation, so we are being inconsistent by dividing a real cash flow by a nominal discount rate. To be consistent, we must either use nominal cash flows or real discount rate. We'll do it both ways. The real discount rate is

$$\frac{1.18}{1.10} - 1 = 7.2727\% \quad \text{The PV of perpetuity is } \$400,000 / .072727 = \$5.5 \text{ million.}$$

Alternatively, the nominal cash flow grows by 10% per year. The year 1 cash flow is thus $\$400,000 * 1.1 = \$440,000$. We can use growing perpetuity formula

$$PV = \frac{C}{r - g} = \frac{\$440,000}{.18 - .1} = \$5.5 \text{ million}$$

8. (10 points) Your savings account at Ledyard Bank pays interest of 4% APR, compounded semi-annually. If you want to have a balance of \$100,000 in an account after 15 years, what constant amount must you deposit in each and every six month period (first deposit in 6 months)?

Payment=\$2,464.60. The account pays interest of 2% every 6 months. Thus we have a 30 period annuity with a 2% interest rate.

$$100,000 / (1.02)^{30} = C \times A(30, 2\%)$$

$$55,207.09 = C \times 22.40 \quad C = 55,207.09 / 22.40 = \$2,464.60$$

9. (6 points) The internal rate of return and NPV rules generally lead to the same decision for independent projects. However, in some cases the IRR rule either fails or does not provide an unambiguous result. Describe these circumstances. Give an example. Remember, we are considering only independent projects.

The NPV may fail if the cash flows from a project are “nonconventional”. A cash flow stream is nonconventional if negative cash flows appear after the present period (in such cases there will be as many IRRs as there are changes in sign in the cash flow stream. An example might be closing a reactor down for scheduled maintenance every five years. In such a case every fifth term in the cash flow stream may be negative.

10. (10 points) The following table provides information on returns on a stock in three states of the economy:

State of the economy	Probability of state	Return
Rapid growth	.20	.30
Moderate growth	.60	.15
Recession	.20	-.05

- a. (3 points) Calculate the expected rate of return for the stock.

$$E(R) = (.20)(-.05) + (.60)(.15) + (.20)(.30) = .14 = 14\%$$

- b. 7 (points) Calculate the variance and the standard deviation of returns for the stock.

The variance is calculated using values in the following table:

State of the economy	Probability of state	Return	Deviation	Squared Deviation	Prob*squared deviation
Rapid growth	.20	.30	-.19	.0361	.00722
Moderate growth	.60	.15	.01	.0001	.00006
Recession	.20	-.05	.16	.0256	.00512

The variance is determined by first calculating the deviation from expected return for each state, these deviations are then squared and multiplied by their respective probabilities in each state. The sum of the weighted squared deviations is the variance. Therefore Var= .01240.

$$SD = \sqrt{.01240} = .11136 = 11.14\%$$

11. (10 points) The Cape Cod Cranberry Company has been harvesting cranberries

and producing cranberry sauce since 1749. The demand for cranberry sauce has been remarkably stable over this period, in fact, earnings have been \$1,479 thousand per year for the last 100 years, and no change is expected in the future. However, for the last 20 years the firm has been contemplating the introduction of a new product to celebrate the firm's 250th birthday. The new product will require a \$750 thousand investment two years from now and will produce a 30% return on that investment starting in year three. Since the new product is related to the firm's 250th birthday, income from the product will last only five years. Cape Cod's cost of capital is 6%.

- a. (3 points) What is the value of the firm if the new project is **not** undertaken?

The value of the firm if no new investments are undertaken and earnings remain constant for the foreseeable future is $PV = E/r = \$1,479/.06 = \$24,650$ thousand

- b. (7 points) What is the value of the firm if the new project proceeds as planned?

The value of the firm today depends on the investments NPV.

First, find the NPV of the project two years from now:

$NPV(\text{year } 2) = -\$750 + [\$750(.30)]A(5\text{yr}, 6\%) = -\$750 + \$225 \times 4.2124 = \197.79 thousand

The value of the firm is its value without the project plus the NPV (at time zero) of all future investments' NPVs. The latter value is the NPV of growth opportunities (NPVGO). Thus, the value of the firm is $PV = \$24,650 + 197.79/(1.06)^2 = \$24,826$ thousand

12. (5 points) Based on your answers to the previous question, what is Cape Cod Cranberry Company's price-to-earnings ratio?

To calculate P/E ratio, we divide the total firm value by total earnings. From the previous problem, $PV = \$24,826$ thousand, thus $P/E = \$24,826/\$1,479 = 16.8$. Note that the P/E is the inverse of the cost of capital only when $NPVGO = 0$.

13. (10 points) The Cape Cod Cranberry Company must choose between two bog skimmers. Skimmer 1 is made in Maine, cost \$7,500, will last 5 years and requires \$150 per year maintenance. Skimmer 2 is made in Michigan, costs \$9,900, lasts 7 years and requires only \$127 per year in maintenance. The discount rate is 8%. Which skimmer should Cape Cod Cranberry purchase? Assume that whichever skimmer is purchased, Cape Cod will continue using that type of skimmer forever. Also, assume that all cash flows are after tax.

This is an investment decision involving mutually exclusive projects with unequal lives. Cape Cod Cranberry must compare the equivalent annual costs of ownership

(EAC).

Skimmer 1: PV (costs) = 7,500+150(3.9927)=8,099

EAC = 8,099/3.9927=2,028

Skimmer 2: PV (costs) 9,900+127(5.2064)=10,561

EAC = 10,561/5.2064=2,028

Since both skimmers have the same EAC, Cape Cod Cranberry should be indifferent between the two.