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Department of Economics

Economics 36 Katerina Simons
Theory of Finance

PROBLEM SET 3

1. Here are inflation rates and stock market and Treasury bill returns between 1993 and 1997:

YEAR	INFLATION	S&P 500 RETURN	T-Bill Return
1993	+2.8%	+10.0%	+2.9
1994	+2.7	+1.3	+3.9
1995	+2.5	+37.4	+5.6
1996	+3.3	+23.1	+5.2
1997	+1.7	+33.4	+5.3

a. What was the real return on the S&P 500 in each year?
b. What was the average real return?
c. What was the risk premium in each year?
d. What was the average risk premium?
e. What was the standard deviation of the risk premium?

2. Suppose financial analysts believe that there are four equally likely states of the economy: depression, recession, normal and boom times. The predicted returns on investment in Supertech Company and in Slowpoke Company for each state of the economy are given in the table below:

	Supertech Returns	Slowpoke Returns
Depression	-20%	5%
Recession	10%	20%
Normal	30%	-12%
Boom	50%	9%

a. Calculate the expected return (mean) for each company. This is the average return that an investor can expect.

b. Calculate the variance of the returns for each company.

c. Calculate the standard deviation of the returns for each company.

d. Calculate the covariance between the returns for the two companies

e. Calculate the correlation coefficient (correlation) between the returns for the two companies.

3. You are a wheat farmer. The return on your investment in your farm depends on the weather. If the weather is normal, you will earn 8%; if it is bad, you will earn 5%; if it is unusually good you will earn 15%. The probability of normal weather is 0.6; of bad weather is 0.2; of unusually good weather is 0.2.

a. What is your expected return. What is the standard deviation of that return?

b. Over a period of 20 years the weather was (n=normal, g=good, b=bad)
b g g b g n g g n b n n n g n n g n n b
What is the average return and the standard deviation for this period?

c. Why do your answers to parts A and B differ?

4. Assume you wish to hold a portfolio of asset A and a riskless asset. Asset A has a beta of 1.2 and an expected return of 18%. The risk-free rate is 7%. Calculate portfolio expected returns and portfolio betas for the six portfolios in the following table:

% Invested in Asset A	% Invested in Risk-free Asset	Portfolio Expected Return (%)	Portfolio Beta
0	100		
25	70		
50	50		
75	25		
100	0		
125	-25		

5. The table below shows standard deviations and correlation coefficients for seven stocks. Calculate the variance of a portfolio invested 40 percent in Compaq, 40 percent in McDonald's, and 20 percent in McGraw-Hill.

Standard deviations and correlation coefficients for a sample of seven stocks.

	Correlation Coefficients							
	AT&T	Biogen	Coca-Cola	Compaq	General Electric	McDonlad's	McGraw-Hill	Standard Deviation
AT&T	1	.13	.40	.08	.42	.27	.26	21%
Biogen		1	.22	.34	.45	.28	.18	51
Coca-Cola			1	.24	.48	.34	.32	22
Compaq				1	.17	.14	.17	44
General Electric					1	.48	.54	20
McDonald's						1	.39	22
McGraw-Hill							1	19

6. Our eccentric Aunt Claudia has left you $50,000 in General Electric shares plus $50,000 cash. Unfortunately her will requires that the General Electric stock not be sold for 1 year, and the $50,000 cash must be entirely invested in one of the stocks shown in the table above. What is the safest attainable portfolio under these restrictions?

7. You are considering investing in the stocks of some small companies. The average return for each company is 12%, with a standard deviation of 20%. If you diversify by putting 1% of your money into each of 100 such stocks, what is your expected return? What is the standard deviation, assuming that the returns on the different stocks are uncorrelated?

8. A company is deciding whether to raise money for an investment project which has the same risk as the market and an expected return of 20%. If the risk free rate is 10 percent and the expected return on the market is 15 percent the company should go ahead
   a. Unless the company beta is greater than 2
   b. Unless the company's beta is less than 2
   c. Whatever the company's beta.

   Which answer is correct? Explain briefly why.

9. Here are betas estimated from 1990 to 1994 for several well-known common stocks. The historical market risk premium is 8.4%.

Stock	Beta
Hewlett-Packard	1.81
Thermo Electron	1.29
Niagara Mohawk	.69
Merrill Lynch	1.81
Tyson Foods	1.04

(a) Estimate the expected rate of return using the CAPM formula. The risk free rate was 6 percent.

(b) The standard deviation of Tyson Foods' stock was about 26 percent per year. Thermo Electron's standard deviation was about 24 percent. Yet the CAPM says Tyson Foods was the safer investment. Explain why this makes sense.

10. Mark Harrywitz proposes to invest in two shares, X and Y. He expects a return of 12 percent from X and 8 percent from Y. The standard deviation of returns is 8 percent for X and 5 percent for Y. The correlation coefficient between the returns is .2.

(a) Compute the expected return and standard deviation of the following portfolios:

Portfolio	Percentage in X	Percentage in Y
1	50	50
2	25	75
3	75	25

(b) Sketch the set of portfolios composed of X and Y.

(c) Suppose that Mr. Harrywitz can also borrow or lend at an interest rate of 5 percent. Show on your sketch how this alters his opportunities. Given that he can borrow or lend, what proportions of the common stock portfolio should be invested in X and Y? (Do not solve for the equation of this line. Use your sketch to identify the approximate proportions.)

(d) Explain how, in principal, you would determine the optimal portfolio of risky and risk-free assets in this diagram.

11. Percival Hygiene has $10 million invested in long-term corporate bonds. This bond portfolio's expected annual rate of return is 9 percent, and the annual standard deviation is 10 percent.

Amanda Reckonwith, Percival's financial advisor, recommends that Percival consider investing in an index fund which closely tracks the standard and Poors 500 index. The index has an expected return of 14 percent, and its standard deviation is 16 percent.

a. Suppose Percival puts all his money in a combination of the index fund and treasury bills. Can he thereby improve his expected rate of return without changing the risk of the portfolio? The Treasury bill yield is 6 percent.

b. Could Percival do even better by investing equal amounts in the corporate bond portfolio and the index fund? The correlation between the bond portfolio and the index fund is +0.1.

12. Suppose that the current risk-free rate is 7.6 percent. Potpourri Inc. stock has a beta of 1.7 and an expected return of 16.7 percent. (Assume CAPM is true.)

a. What is the risk premium on the market?

b. Magnolia Industries stock has a beta of 0.8. What is the expected return on Magnolia stock?

c. Suppose that you invested $10,000 in some combination of Potpourri and Magnolia stocks. The beta of this portfolio is 1.07. How much did you invest in each stock? What is the expected return on the portfolio?

13. Butler Company has developed the following data regarding the possible return on a new project for each state of the economy:

State (i)	Probability [Prob(i)]	Market Return (R_m)	Project Return (R_x)
1	0.10	-0.30	-0.30
2	0.25	0.20	0.05
3	0.30	0.15	0.20
4	0.15	0.20	0.25
5	0.20	0.25	0.30

Calculate:

(a) The expected return on the project
(b) The variance of the project returns and the variance of the market returns
(c) The standard deviation of the project returns and the standard deviation of the market returns
(d) The covariance of the project returns with the market returns
(e) The correlation coefficient between the project returns and the market returns
(f) The beta coefficient (a measure of the systematic risk) of the project:

$$\beta = \frac{Cov(R_x, R_m)}{Var(R_m)}$$

14. There are few, if any, real companies with negative betas. But suppose you found one with $\beta = -.25$.

a. How would you expect this stock's price to change if the overall market rose by an extra 5 percent? What if the market fell by an extra 5 percent?

b. You have $1 million invested in a well-diversified portfolio of stocks. Now you receive an additional $20,000 bequest. Which of the following actions will yield the safest overall portfolio return?

(i) Invest $20,000 in Treasury bills (which have $\sigma = 0$).
(ii) Invest $20,000 in stocks with $\beta = 1$.
(iii) Invest $20,000 in the stock with $\beta = -.25$.

Explain your answer.