DARTMOUTH COLLEGE
Department of Economics

Economics 36 — Katerina Simons
Theory of Finance

SOLUTIONS TO PROBLEM SET 3

1. Recall that $(1 + r_{nominal}) = (1 + r_{real})\ (1 + \text{inflation rate})$

a. The real return on the S&P 500 in each year was:

1993:	7.0%
1994:	-1.4%
1995:	34.0%
1996:	19.2%
1997:	30.9%

b. Given the numbers in Part (a), the average real return was 17.9 percent.

c. The risk premium, defined as the return over and above the Treasury bill rate, was:

1993:	7.1%
1994:	-2.6%
1995:	31.8%
1996:	17.9%
1997:	27.8%

d. Given the numbers in Part (c), the average risk premium was 16.4 percent.

e. The standard deviation (σ) of the risk premium may be calculated as follows:

$$\sigma^2 = \left(\frac{1}{5-1}\right)\left[(0.071 - 0.164)^2 + (-0.026 - 0.164)^2 + (0.318 - 0.164)^2\right.$$

$$\left. + (0.179 - 0.164)^2 + (-0.278 - 0.164)^2\right]$$

$$\sigma^2 = \left(\frac{1}{4}\right)[0.081686]$$

$$\sigma^2 = 0.0204$$

$$\sigma = 0.143, \text{ or } 14.3\%$$

2. Suppose financial analysts believe that there are four equally likely states of the economy: depression, recession, normal and boom times. The predicted returns on investment in Supertech Company and in Slowpoke Company for each state of the economy are given in the table below:

	Supertech Returns	Slowpoke Returns
Depression	-20%	5%
Recession	10%	20%
Normal	30%	-12%
Boom	50%	9%

a. Calculate the expected return (mean) for each company. This is the average return that an investor can expect.

Supertech: 0.25*(-0.20) + 0.25*(0.10) + 0.25*(0.30) + 0.25*(0.50) = 0.175
Slowpoke: 0.25*(0.05) + 0.25*(0.20) + 0.25*(-0.12) + 0.25*(0.09) = 0.055

b. Calculate the variance of the returns for each company.

Supertech: $0.25*(-0.20 - 0.175)^2 + 0.25*(0.10 - 0.175)^2 + 0.25*(0.30 - 0.175)^2 + 0.25*(0.50 - 0.175)^2 = 0.067$

Slowpoke: $0.25*(0.05 - 0.055)^2 + 0.25*(0.20 - 0.055)^2 + 0.25*(-0.12 - 0.055)^2 + 0.25*(0.09 - 0.055)^2 = 0.013$

c. Calculate the standard deviation of the returns for each company.

Supertech: $\sqrt{0.067} = 0.259$
Slowpoke: $\sqrt{0.013} = 0.115$

d. Calculate the covariance between the returns for the two companies.

Covariance: = 0.25*(-0.20-0.175)(0.05-0.055) + 0.25*(0.10-0.175)(0.20-0.055) + 0.25*(0.30-0.175)(-0.12-0.055) + 0.25*(0.50-0.175)(0.09-0.055)
= -0.004875

e. Calculate the correlation coefficient (correlation) between the returns for the two companies.

$$\text{Correlation} = \frac{\text{Covariance}}{SD_1 * SD_2} = \frac{-0.004875}{0.259 * 0.115} = -0.1639$$

3. You are a wheat farmer. The return on your investment in your farm depends on the weather. If the weather is normal, you will earn 8%; if it is bad, you will earn 5%; if it is unusually good you will earn 15%. The probability of normal weather is 0.6; of bad weather is 0.2; of unusually good weather is 0.2.

a. What is your expected return. What is the standard deviation of that return?

$EV = 0.6 \times 8\% + 0.2 \times 5\% + 0.2 \times 15\% = 8.8\%$

$\text{Variance} = 0.6 \times (8\text{-}8.8)^2 + 0.2 \times (5\text{-}8.8)^2 + 0.2 \times (15\text{-}8.8)^2 = 10.96$

$SD = \sqrt{\text{Variance}} = \sqrt{10.96} = 3.3$

b. Over a period of 20 years the weather was (n=normal, g=good, b=bad)

b g g b g n g g n b n n n g n n g n n b

What is the average return and the standard deviation for this period?

$$\text{Ave} = \frac{4 \times 5 + 7 \times 15 + 9 \times 8}{20} = 9.85$$

$$\text{Variance} = \frac{4 \times (5\text{-}9.85)^2 + 7 \times (15\text{-}9.85)^2 + 9 \times (8\text{-}9.85)^2}{19} = 16.34$$

$SD = \sqrt{\text{Variance}} = \sqrt{16.34} = 4.04$

c. Why do your answers to parts A and B differ?

Sample versus process (i.e. sampling error).

4. Assume you wish to hold a portfolio of asset A and a riskless asset. Asset A has a beta of 1.2 and an expected return of 18%. The risk-free rate is 7%. Calculate portfolio expected returns and portfolio betas for the six portfolios in the following table:

% Invested in Asset A	% Invested in Risk-free Asset	Portfolio Expected Return (%)	Portfolio Beta
0	100	7.00	0.00
25	70	9.75	0.30
50	50	12.50	0.60
75	25	15.25	0.90
100	0	18.00	1.20
125	-25	20.75	1.50

5. The table below shows standard deviations and correlation coefficients for seven stocks. Calculate the variance of a portfolio invested 40 percent in Compaq, 40 percent in McDonald's, and 20 percent in McGraw-Hill.

Standard deviations and correlation coefficients for a sample of seven stocks.

	Correlation Coefficients							
	AT&T	Biogen	Coca-Cola	Compaq	General Electric	McDonlad's	McGraw-Hill	Standard Deviation
AT&T	1	.13	.40	.08	.42	.27	.26	21%

Biogen	1	.22	.34	.45	.28	.18	51
Coca-Cola		1	.24	.48	.34	.32	22
Compaq			1	.17	.14	.17	44
General Electric				1	.48	.54	20
McDonald's					1	.39	22
McGraw-Hill						1	19

The formula for the variance of a three stock portfolio is:

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_1X_2\rho_{12}\sigma_1\sigma_2 + 2X_1X_3\rho_{13}\sigma_1\sigma_3 + 2X_2X_3\rho_{23}\sigma_2\sigma_3$$

Let Compaq be security 1, McDonald's be security two and McGraw-Hill be security three. Then:

$$\sigma_p^2 = [(0.4)^2(0.44)^2] + [(0.4)^2(0.22)^2] + [(0.2)^2(0.19)^2] + 2(.4)(.4)(.44)(.22)(.14) + 2(.4)(.2)(.44)(.19)(.17) + 2(.4)(.2)(.22)(.19)(.39)$$

$$\sigma_p^2 = 0.049383$$

6. Our eccentric Aunt Claudia has left you $50,000 in General Electric shares plus $50,000 cash. Unfortunately her will requires that the General Electric stock not be sold for 1 year, and the $50,000 cash must be entirely invested in one of the stocks shown in the table above. What is the safest attainable portfolio under these restrictions?

"Safest" means lowest risk; in a portfolio context, that means lowest variance of return. Half the money is invested in the General Electric stock, and half the money must be invested entirely in one of the other securities given. Thus, we must calculate the portfolio variance for six portfolios (half the money in GE and half the money in AT&T, half in GE and half in Biogen, etc.) to see which is the lowest. Buying stock in McGraw-Hill results in the lowest portfolio variance:

Stocks	Portfolio Variance
GE & AT&T	0.029845
GE & Biogen	0.097975
GE & Coca Cola	0.032660
GE & Compaq	0.065880
GE & McDonald's	0.032660
GE & McGraw-Hill	0.029285

7. You are considering investing in the stocks of some small companies. The average return for each company is 12%, with a standard deviation of 20%. If you diversify by putting 1% of your money into each of 100 such stocks, what is your expected return? What is the standard deviation, assuming that the returns on the different stocks are uncorrelated?

Look at this in general:
Find ER and SD of average of T values of R_X.
Suppose that correlation is zero.

$$ER_p = \sum_{t=1}^{T} E(\frac{1}{T} R_x) = \frac{1}{T}\sum_{t=1}^{T} ER_x = \frac{1}{100} * 100 * 12\% = 12\%$$

$$var(R_p) = \sum_{t=1}^{T} var(\frac{1}{T} R_x) = \sum_{t=1}^{T}\left[\frac{1}{T}\right]^2 var(R_x) = 100 * (.01)^2 * (0.2)^2 = .0004$$

$SD(R_p) = .02 = 2\%$

8. A company is deciding whether to raise money for an investment project which has the same risk as the market and an expected return of 20%. If the risk free rate is 10 percent and the expected return on the market is 15 percent the company should go ahead

a. Unless the company beta is greater than 2
b. Unless the company's beta is less than 2
c. Whatever the company's beta.

(c) is correct. This is because the opportunity cost of capital depends on the *project's* risk. With the same risk as the market (i.e., a beta of one), the project offers an expected return of 20 percent, while with a beta of one we would only require 15 percent.

9. Here are betas estimated from 1990 to 1994 for several well-known common stocks. The historical market risk <u>premium</u> is 8.4%.

Stock	Beta
Hewlett-Packard	1.81
Thermo Electron	1.29
Niagara Mohawk	.69
Merrill Lynch	1.81
Tyson Foods	1.04

(a) Estimate the expected rate of return using the CAPM formula. The risk free rate was 6 percent.

We are given the risk-free rate (6 percent); assume that the expected market risk premium is the historical one of 8.4 percent. Using the security market line, $r_j = r_f + \beta_j (r_m - r_f)$, we can compute all the expected rates of return:

Hewlett-Packard: $r = 0.06 + 1.81\ (0.084) = .212$ or 21.2%
Thermo-Electron: $r = 0.06 + 1.29\ (0.084) = .168$, or 16.8%
Niagara Mohawk: $r = 0.06 + 0.69\ (0.084) = .118$, or 11.8%

Merrill-Lynch:	r = 0.06 + 1.81 (0.084) = .212, or 21.2%
Tyson Foods:	r = 0.06 + 1.04 (0.084) = .147, or 14.7%

(b) The standard deviation of Tyson Foods' stock was about 26 percent per year. Thermo Electron's standard deviation was about 24 percent. Yet the CAPM says Tyson Foods was the safer investment. Explain why this makes sense.

Although the variability of return (i.e., the total risk) was higher for Tyson Foods, Tyson had a lower non-diversifiable measure of risk (beta) and hence was a safer investment.

10. Mark Harrywitz proposes to invest in two shares, X and Y. He expects a return of 12 percent from X and 8 percent from Y. The standard deviation of returns is 8 percent for X and 5 percent for Y. The correlation coefficient between the returns is .2.

(a) Compute the expected return and standard deviation of the following portfolios:

Portfolio	Percentage in X	Percentage in Y
1	50	50
2	25	75
3	75	25

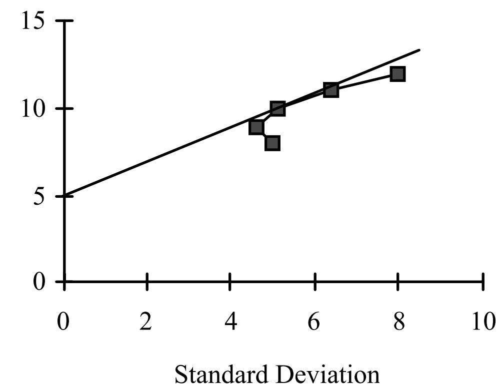
(b) Sketch the set of portfolios composed of X and Y.

(c) Suppose that Mr. Harrywitz can also borrow or lend at an interest rate of 5 percent. Show on your sketch how this alters his opportunities. Given that he can borrow or lend, what proportions of the common stock portfolio should be invested in X and Y? (Do not solve for the equation of this line. Use your sketch to identify the approximate proportions.)

(d) Explain how, in principal, you would determine the optimal portfolio of risky and risk-free assets in this diagram.

a.

Portfolio	$E(R_p)$	σ_p
1	10%	5.1
2	9%	4.6
3	11%	6.4



b. See figure at right. The set of portfolios (plotted with five points, i.e., the three specified plus the portfolios with weights of {1,0} and {0,1}) is represented by the curved line.

c. Again, see the figure. The best opportunities lie along the straight line. From the diagram, the optimal portfolio of risky assets is portfolio 1, and so Mr. Harrywitz should invest 50 percent in X and 50 percent in Y.

d. To find the optimal portfolio of risky and risk-free assets we need to know something about Mr. Harrywitz's preferences for risk. These preferences would be represented by indifference curves. Tangency with the frontier would identify the optimal portfolio.

11. Percival Hygiene has $10 million invested in long-term corporate bonds. This bond portfolio's expected annual rate of return is 9 percent, and the annual standard deviation is 10 percent.

Amanda Reckonwith, Percival's financial advisor, recommends that Percival consider investing in an index fund which closely tracks the Standard and Poor's 500 index. The index has an expected return of 14 percent, and its standard deviation is 16 percent.

a. Suppose Percival puts all his money in a combination of the index fund and treasury bills. Can he thereby improve his expected rate of return without changing the risk of the portfolio? The Treasury bill yield is 6 percent.

Percival's current portfolio provides an expected return of 9 percent with an annual standard deviation of 10 percent. We must first find the portfolio weights for a combination of Treasury bills (security 1; standard deviation of 0 percent) and the index fund (security 2; standard deviation of 16 percent) such that the portfolio standard deviation is 10 percent. Then we can assess whether Percy's return improves with the change. In general, for a two-security portfolio:

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\rho_{12}\sigma_1\sigma_2$$

So, we solve the following: $(0.1)^2 = 0 + 0 + x_2^2(0.16)^2$

$x_2 = 0.625$, and so $x_1 = 0.375$

Further: $r_P = x_1r_1 + x_2r_2 = 0.375\ (0.06) + 0.625\ (0.14) = 0.11$ or 11%.

So, Percival can improve his annual rate of return without changing the risk of his portfolio.

b. Could Percival do even better by investing equal amounts in the corporate bond portfolio and the index fund? The correlation between the bond portfolio and the index fund is +0.1.

With equal amounts in the corporate bond portfolio (security 1) and the index fund (security 2), the expected return will be: $r_p = 0.5(0.09) + 0.5(0.14) = 0.115$, or 11.5%

The portfolio standard deviation will be: $\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\rho_{12}\sigma_1\sigma_2$

$$\sigma_p^2 = (0.5)^2(0.1)^2 + 2\,(0.5)(0.5)(0.10)(0.16)(0.1) + (0.5)^2(0.16)^2$$
$$= 0.0097$$

So, $\sigma_p = 0.0985$.

Thus, by investing equal amounts in the corporate bond portfolio and the index fund, Percival will increase his portfolio's expected return (11.5 percent versus 11 percent) and he will also decrease his portfolio's standard deviation of return (9.85 percent versus 10 percent). This is a clear improvement.

12. Suppose that the current risk-free rate is 7.6 percent. Potpourri Inc. stock has a beta of 1.7 and an expected return of 16.7 percent. (Assume CAPM is true.)

a. What is the risk premium on the market?

b. Magnolia Industries stock has a beta of 0.8. What is the expected return on Magnolia stock?

c. Suppose that you invested $10,000 in some combination of Potpourri and Magnolia stocks. The beta of this portfolio is 1.07. How much did you invest in each stock? What is the expected return on the portfolio?

We can work backwards from the CAPM to find the risk premium on the market.

Potpourri's stock return = $16.7 = 7.6 + 1.7(R_M - R_f)$

where $(R_M - R_f)$ = the market risk premium.

So, $R_M - R_f = (16.7 - 7.6) / 1.7 = 5.353\%$

b. $R_{Mag} = 7.6 + 0.8\,(5.353) = 11.88\%$

c. $X_{Pot}\beta_{Pot} + X_{Mag}\beta_{Mag} = 1.07 \implies 1.7X_{pot} + 0.8(1-X_{pot}) = 1.07 \implies 0.9X_{pot} = 0.27$

So, $X_{Pot} = 0.3$, $X_{Mag} = 0.7$

That is, $3000 in Potpourri stock and $7,000 in Magnolia stock.

$R_P = 7.6 + 1.07(5.353) = 13.33\%$

13. Butler Company has developed the following data regarding the possible return on a new project for each state of the economy:

State (i)	Probability [Prob(i)]	Market Return (R_m)	Project Return (R_x)
1	0.10	-0.30	-0.30
2	0.25	0.20	0.05
3	0.30	0.15	0.20
4	0.15	0.20	0.25
5	0.20	0.25	0.30

Calculate:

(a) The expected return on the project
$E(R_x) = 0.14$

(b) The variance of the project returns and the variance of the market returns
$Var(R_x) = 0.029$ $Var(R_m) = 0.023$

(c) The standard deviation of the project returns and the standard deviation of the market returns
Std Dev(R_x) = 0.17 Std Dev(R_m) = 0.153

(d) The covariance of the project returns with the market returns
$Cov(R_x,R_m) = 0.0227$

(e) The correlation coefficient between the project returns and the market returns

$Corr(R_x,R_m) = 0.868$

(f) The beta coefficient (a measure of the systematic risk) of the project:

$$\beta = \frac{Cov(R_x, R_m)}{Var(R_m)} = 0.976$$

14. There are few, if any, real companies with negative betas. But suppose you found one with β = -.25.

a. How would you expect this stock's price to change if the overall market rose by an extra 5 percent? What if the market fell by an extra 5 percent?

b. You have $1 million invested in a well-diversified portfolio of stocks. Now you receive an additional $20,000 bequest. Which of the following actions will yield the safest overall portfolio return?

(i) Invest $20,000 in Treasury bills (which have $\sigma = 0$).
(ii) Invest $20,000 in stocks with $\beta = 1$.
(iii) Invest $20,000 in the stock with $\beta = -.25$.
Explain your answer.

. a) In general, we expect a stock's price to change by the amount, beta times the change in the market. With beta equal to -0.25, if the market rises by an extra 5%, the expected change would be -1.25%; if the market falls an extra 5%, it would be 1.25%.

b) "Safest" implies lowest risk. Assuming the well-diversified portfolio is invested in typical securities, the portfolio beta will be around 1. We can reduce this portfolio beta the most if we invest the $20,000 in a stock with a negative beta. Answer (iii) is correct.