

DARTMOUTH COLLEGE  
Department of Economics

Economics 36  
Theory of Finance

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**SOLUTIONS TO PROBLEM SET II**

1. You own an oil pipeline, which will generate a \$2 million cash return over the coming year. The pipeline's operating costs are negligible and it is expected to last for a very long time. Unfortunately, the volume of oil shipped is declining, and cash flows are expected to decline by 4% per year. The discount rate is 10%.

(a) What is the present value of the pipeline's cash flows if its cash flows are assumed to last forever?

Instead of a “growing” perpetuity we have a “shrinking” perpetuity. The discounted stream of cash flows is:

$$PV = \frac{\$2m}{(1+r)} + \frac{\$2m(1+g)}{(1+r)^2} + \frac{\$2m(1+g)^2}{(1+r)^3} + \dots = \frac{\$2m}{r-g}$$

Using this formula, the value of the pipeline’s cash flow is \$14.29m

(b) What is the present value of the cash flows if the pipeline is scrapped after 20 years?

Here we have a shrinking annuity with C=\$2m, T=20, r=10%, and g=-4%:

$$PV = C \left[ \frac{1}{r-g} - \frac{1}{r-g} \left( \frac{1+g}{1+r} \right)^T \right] = \$2m \left[ \frac{1}{.1+.04} - \frac{1}{.1+.04} \left( \frac{1-.04}{1+.1} \right)^{20} \right] = \$13.35m$$

2. Mr. Cyrus Clops, the president of Giant Enterprises, has to make a choice between two mutually exclusive investments:

	Cash flow, thousands of dollars				
Project	time 0	time 1	time 2		IRR%
A	-400	241	293		21
B	-200	131	172		31

The opportunity cost of capital is 9%. Mr. Clops is tempted to take B, which has the higher IRR.

(a) Explain to Mr. Clops why this is not the correct procedure.

Because Project A involves a larger capital outlay, it can have a lower IRR and a higher NPV, as compared to Project B. (In fact, NPV<sub>A</sub> is greater for all discount rates less than 10 percent.) Because the goal is to maximize shareholder wealth, NPV is the correct criterion:

Giant's shareholders would rather be richer (have a higher stock price) than have the satisfaction of a 31 percent IRR.

(b) Show him how to adapt the IRR rule to choose the best project.

To use the IRR rule for mutually exclusive projects, calculate the IRR on the incremental flows:

	<u>C<sub>0</sub></u>	<u>C<sub>1</sub></u>	<u>C<sub>2</sub></u>	<u>IRR</u>
A-B	-200	+110	+121	10%

Because the IRR on the incremental flows exceeds the cost of capital, the additional investment in A is worthwhile.

(c) Show him that this project also has the highest NPV.

There are really two alternatives here. The basic full computations of both NPVs, or you could use the short-cut approach of showing that the NPV of the incremental cash flows is positive at a discount rate of 9%, thus proving that the NPV of A must exceed the NPV of B. Nonetheless, here's the long way:

$$NPV_A = -400 + 241/(1.09) + 293/(1.09)^2 = 67.71$$

$$NPV_B = -200 + 131/(1.09) + 172/(1.09)^2 = 64.95$$

- Pepe's Ski Shop is contemplating replacing its ski boot foam injection equipment with a new machine. The old machine has been completely depreciated but has a current market value of \$2000. The new machine will cost \$25,000 and have a life of ten years and have no value after this time. The new machine will be depreciated on a straight-line basis assuming no salvage value. The new machine will increase annual revenues by \$10,000 and increase annual nondepreciation expenses by \$3,000.

(a) What is the additional after-tax net cash flow realized by replacing the old machine with the new machine? (Assume a 50% tax rate for all net income, including capital gains on the sale of the old machine. Draw a time line.)

Step 1: Calculate taxes:

Revenue	10,000	
Depreciation		2,500
Expenses		3,000
Total deductible expenses	5,500	
Taxable income	4,500	
Tax	2,250	

Step 2: Calculate cash flow:

Revenue	10,000
Expenses	<u>3,000</u>
EBT	7,000
Taxes	<u>2,250</u>
Cash flow	4,750

Step 3: Calculate initial cost:

Sell old machine for \$2,000, but pay \$1,000 tax. So net \$1,000. Net cost of replacement is \$25,000 - 1,000 = \$24,000.

Time line				
time 0	1	...		10
-24,000	4,750	...		4,750

(b) At a cost of capital of 12%, what is the present value of this cash flow stream? At this cost of capital of 12%, is the project worthwhile?

$NPV = \$2,838.56$ . Yes.

4. The treasurer of Takamatsia, Inc. has submitted a proposal to the board of directors that, he argues, will increase profits for the all-equity company by a whopping 55%. The project requires an initial outlay of \$900 and has an expected life of five years, with no salvage value or liquidation value. The project will save \$290/year in labor costs for each of the five years, but it will have no effect on revenues. The firm has a 50% tax rate and uses straight-line depreciation over the asset's five-year life. Income statements for the firm *without* adoption of the project are given below.

INCOME WITHOUT ADOPTION OF THE PROJECT					
	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Revenue	1,000	1,000	1,000	1,000	1,000

Variable Cost	500	500	500	500	500
Depreciation	<u>300</u>	<u>300</u>	<u>300</u>	<u>300</u>	<u>300</u>
Taxable Income	200	200	200	200	200
Taxes	<u>100</u>	<u>100</u>	<u>100</u>	<u>100</u>	<u>100</u>
Net Income	100	100	100	100	100

- (a) Complete a second set of income statements for the situation if the firm *does* adopt the project to verify that net income will increase by 55% relative to the situation if the firm does not adopt the project.

INCOME WITH ADOPTION OF THE PROJECT					
	<u>Year 1</u>	<u>Year 2</u>	<u>Year 3</u>	<u>Year 4</u>	<u>Year 5</u>
Revenue	1,000	1,000	1,000	1,000	1,000
Variable Cost	210	210	210	210	210
Depreciation	<u>480</u>	<u>480</u>	<u>480</u>	<u>480</u>	<u>480</u>
Taxable Income	310	310	310	310	310
Taxes	<u>155</u>	<u>155</u>	<u>155</u>	<u>155</u>	<u>155</u>
Net Income	155	155	155	155	155

- b. If the project has a 10% cost of capital, should the firm adopt the project?

Cash flow = revenue - VC - tax

Without the project: 1,000 - 500 - 100 = 400

With the project: 1,000 - 210 - 155 = 635

So adds 235 per year.

$$NPV = -900 + 235 \frac{1}{0.10} \left[ 1 - \frac{1}{(1.10)^5} \right] = -9.17$$

Do not do the project. Note: we could also have calculated the NPV separately with and without the project.

5. Borgia Pharmaceuticals has \$1 million allocated for capital expenditures. Which of the following projects should the company accept to stay within the \$1 million budget? How much does the budget limit cost the company in terms of its market value? The opportunity cost of capital for each project is 11 percent.

	Investment,	NPV,	IRR,
Project	Thousand of Dollars	Thousands of Dollars	Percent

1	300	66	17.2
2	200	-4	10.7
3	250	43	16.6
4	100	14	12.1
5	100	7	11.8
6	350	63	18.0
7	400	48	13.5

This is a case of capital rationing so we can use the profitability index. Using the fact that the Profitability Index is equal to the present value of cash flows divided by investment, we find that:

<u>Project</u>	<u>Profitability Index</u>
1	1.22
2	0.98
3	1.17
4	1.14
5	1.07
6	1.18
7	1.12

Given the budget of \$1 million, the best the company can do is to accept projects 1, 3, 4, & 6. [Notice that because of the scale problem, the IRR rule would have yielded a different answer (choosing project 7 before project 4).]

If the company accepted *all* positive NPV projects, the market value (compared to the market value under the budget limitation) would increase by the NPVs of projects 5 and 7, or  $\$7 + \$48 = \$55$ , or \$55,000. Thus, the budget limit costs the company \$55,000 in terms of its market value.

6. Consider the following projects:

Project	C0	C1	C2	C3	C4	C5
A	-1,000	+1,000	0	0	0	0
B	-2,000	+1,000	+1,000	+4,000	+1,000	+1,000
C	-3,000	+1,000	+1,000	0	+1,000	+1,000

If the opportunity cost of capital is 10 percent, which projects have a positive NPV?

$$NPV_A = -1,000 + 1,000/(1.1) = -90.91$$

$$NPV_B = -2,000 + 1,000/(1.1) + 1,000/(1.1)^2 + 4,000/(1.1)^3 + 1,000/(1.1)^4 + 1,000/(1.1)^5 \\ = \$4,044.73$$

$$NPV_C = -3,000 + 1,000/(1.1) + 1,000/(1.1)^2 + 1,000/(1.1)^4 + 1,000/(1.1)^5 = \$39.47$$

Calculate the payback period for each project.

$$\text{Payback}_A = 1 \text{ year}; \text{ Payback}_B = 2 \text{ years}; \text{ Payback}_C = 4 \text{ years}$$

Which project(s) would a firm using the payback rule accept if the cutoff period were three years?

A and B.

7. The treasurer of Davids, Inc., has projected the cash flows of projects *A*, *B*, and *C* as follows. Suppose the relevant discount rate is 12 percent a year.

Year	Project A	Project B	Project C
0	\$100,000	\$200,000	\$100,000
1	70,000	130,000	75,000
2	70,000	130,000	60,000

a. Compute the profitability indices for each of the three projects.

$$PI_A = (\$70,000 / 1.12 + \$70,000 / 1.12^2) / \$100,000 = 1.183$$

$$PI_B = (\$130,000 / 1.12 + \$130,000 / 1.12^2) / \$200,000 = 1.099$$

$$PI_C = (\$75,000 / 1.12 + \$60,000 / 1.12^2) / \$100,000 = 1.148$$

b. Compute the NPVs for each of the three projects.

$$NPV_A = -\$100,000 + \$118,303.57 = \$18,303.57$$

$$NPV_B = -\$200,000 + \$219,706.63 = \$19,706.63$$

$$NPV_C = -\$100,000 + \$114,795.92 = \$14,795.92$$

c. Suppose these three projects are independent. Which projects should Davids accept based on the profitability index rule?

Accept all three projects because PIs of all the three projects are greater than one.

d. Suppose these three projects are mutually exclusive. Which project should Davids accept based on the profitability index rule?

*Project A*

- e. Suppose Davids' budget for these projects is \$300,000. The projects are not divisible. Which projects should Davids accept?

*Project B has the highest NPV, while A has the next highest NPV.*

*Take both projects A and B.*

8. The Spectrum Computer Corporation is trying to choose between the following two mutually exclusive design projects:

Year	Cash Flow (I)	Cash Flow (II)
0	-\$15,000	-\$2,000
1	7,000	1,500
2	7,000	1,500
3	7,000	1,500

- a. If the required return is 9 percent and Spectrum Computer applies the profitability index decision rule, which project should the firm accept?

$$PI_I = 1.181 \quad PI_{II} = 1.898$$

- b. If the company applies the NPV decision rule, which project should it take?

$$NPV_I = \$2,719 \quad NPV_{II} = \$1,797$$

- c. Explain why your answers in (a) and (b) are different.

The scale problem with mutually exclusive projects (again).

9. a. What is the payback period on each of the following projects?

Project	CASH FLOWS, DOLLARS				
	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$
A	-5,000	+1,000	+1,000	+3,000	0
B	-1,000	0	+1,000	+2,000	+3,000
C	-5,000	+1,000	+1,000	+3,000	+5,000

$$A=3 \text{ years, } B = 2 \text{ years, } C = 3 \text{ years}$$

- b. Given that you wish to use the payback rule with a cutoff period of 2 years, which projects would you accept? B

- c. If you use a cutoff period of 3 years, which projects would you accept? A,B,C

d. If the opportunity cost of capital is 10 percent, which projects have positive NPVs?

B and C ( $NPV_B = \$3378$ ;  $NPV_C = \$2405$ )

e. "Payback gives too much weight to cash flows that occur after the cutoff date." True or false? False

f. "If a firm uses a single cutoff period for all projects, it is likely to accept too many short-lived projects." True or false? True

g. If the firm uses the discounted-payback rule, will it accept any negative-NPV projects? Will it turn down positive-NPV projects? Explain.

It will accept no negative-NPV projects but will turn down some with positive NPVs. A project can have positive NPV if all future cash flows are considered, but still not meet the stated cutoff period.

10. a. Calculate the net present value of the following project for discount rates of 0, 50, and 100 percent

Cash Flows, Dollars		
$C_0$	$C_1$	$C_2$
-6,750	+4,500	+18,000

$NPV_0 = \$15,750$ ;  $NPV_{50} = \$4,250$ ;  $NPV_{100} = \$0$

b. What is the IRR of the project? 100%

11. Which of the following should be treated as incremental cash flows when deciding whether to invest in a new manufacturing plant? The site is already owned by the company, but existing buildings would need to be demolished.

- a. The market value of the site and existing buildings - yes
- b. Demolition costs and site clearance - yes
- c. The cost of a new access road put in last year – no (sunk cost)
- d. Lost earnings on other products due to executive time spent on the new facility - yes
- e. A proportion of the cost of leasing the president's jet airplane - no
- f. Future depreciation of the new plant - no

- g. The reduction in the corporation's tax bill resulting from tax depreciation of the new plant -yes
- h. The initial investment in inventories of raw materials - yes
- i. Money already spent on engineering design of the new plant - no

12. Machines A and B are mutually exclusive and are expected to produce the following cash flows:

	CASH FLOWS, THOUSANDS OF DOLLARS			
Machine	C0	C1	C2	C3
A	-100	+110	+121	
B	-120	+110	+121	+133

The opportunity cost of capital is 10 percent.

- a. Calculate the NPV of each machine.

$$NPV_A = \$100,000; \quad NPV_B = \$180,000$$

- b. Use present value table to calculate the equivalent annual cash flow from each machine.

$$\text{Equivalent cash flow of A} = 100,000/1.736 = \$57,604$$

$$\text{Equivalent cash flow of B} = 180,000/2.487 = \$72,376$$

- c. Which machine should you buy? B

13. Consider the following statements: “We like to do all our capital budgeting calculations in real terms. It saves making any forecasts of the inflation rate.” Discuss briefly.

Even when capital budgeting calculations are done in real terms, an inflation forecast is still needed because

- a. Some real flows depend on the inflation rate, e.g., real taxes and real proceeds from collecting receivables; and,
- b. Real discount rates are often estimated by starting with nominal rates and “taking out” inflation.