

Again the list....

May 27, 05

- Final Exam: 2 hour exam, 50% of total grade (midterm 35%).

Friday, June 3, 8:30-10:30

Silby 28

- Exercises are on this slide, and a Test Final that will be available by Tue.
- Tuesday: **Finishing up all new topics in chapter 28/29 incl exercises.**
- Wednesday: **Last day: Test Final Exam w solutions.**

Important for final:

- Everything what was on midterm except ch 12 and congruous sets. You need to know how to draw payoffs for mixed best responses. Examples will only be on supports with 2 pure strategies (except really trivial ones..)

- New: Ch 14, 15, 24, 28, 29. No SPMSNE, only games that are easy to solve by backward induction.

-BNE, Bayesian Rationalizability, and solving for PBE.

Arthur's Game of Life (or: The Vogon Game)

We will now show that Arthur could have saved his life.

Description of the game:

- List of Players: {Vogon (Player 1), Arthur (Player 2)}
- Player 1 (Vogon) can be of two types: **bad** and **evil**.
- Nature as non-strategic player draws the type of Vogon before the Vogon moves.
- Arthur does not know the type of the Vogon, but knows that usually Vogons are bad with probability 4/5, and evil with probability 1/5.
- Sequential play: First, the Vogon reads, then Arthur is asked to tell.
- List of strategies:

$S_1 = \{\text{read bad poetry, read horrible poetry}\}$.

$S_2 = \{\text{tell that poetry was good, tell that poetry was the best ever heard}\}$.

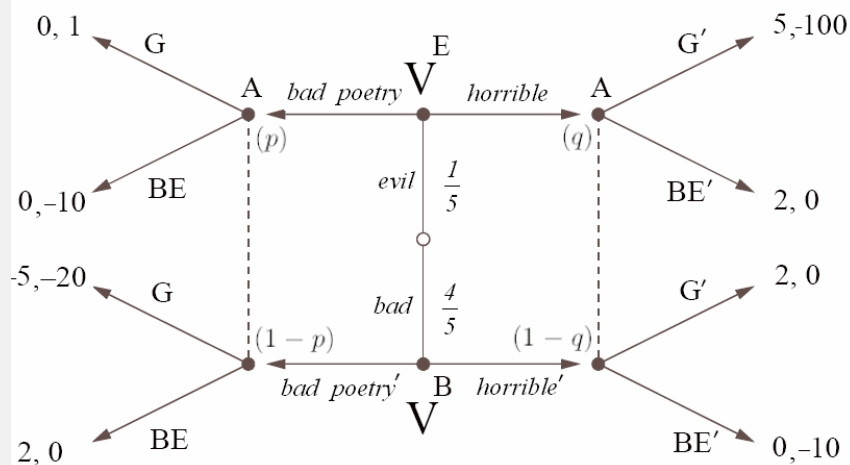
Payoffs if the Vogon is evil

- Whenever an *evil* Vogon reads bad poetry, he knows he could do better but he reads the poetry nonetheless. He does not care about Arthur's answer. Whatever his answer is, the Vogon gets zero utility. However, since the Vogon is evil, he wants to punish a too submissive and pleasing answer of "best ever" and punishes Arthur by putting him into jail (payoff -10 because of the disgusting prison food on Vogosphere - green slime balls). If Arthur's answer is "good", he is set free and receives an additional payment (\$1).
- Whenever an evil Vogon reads horrible poetry, he seeks confirmation for what he does. To tell that the poetry was good is an offense that triggers the death penalty for Arthur (-100), and gives a utility of 5 to the evil Vogon (joy, reputation, satisfaction). If Arthur tells what the Vogon wants to hear, namely that his horrible poetry is the best ever, the Vogon is pleased (payoff of 2) and Arthur is released into freedom (zero payoff).

Payoffs if the Vogon is of type bad

- Whenever a bad Vogon reads bad poetry, he reads what he likes to write. He expects Arthur to honor this by telling it's the best ever. In this case Arthur is freed (payoff=0) and the Vogon enjoys a utility of 2. Should Arthur tell that the poetry was good, he offends the Vogon. Double prison with slimeballs (-20), while the Vogon, not evil, is just sad and in grief about the outcome and wished he had been detected by Arthur as a great poet instead (thus, the Vogon only gets -5).
 - Whenever a bad Vogon reads horrible poetry, he knows that his poetry never has the quality of a real evil Vogon and accepts the answer that his poetry was just "good." In this case, Arthur is released (zero payoff) and the bad Vogon gets 2.
- If Arthur tells the Vogon that his horrible poetry is the best ever, the Vogon suspects mockery and puts him to jail (zero payoff for the Vogon, but -10 for Arthur because of the slime balls...)

The Vogon game in horse form



The two types of Vogons behave the same way

- Pooling equilibria are usually easy to find: We assume that the uninformed second player can only reply with one action, here either G or BE (strategy GG' or BE BE')

Let's now check if the Vogon game has a pooling equilibrium.

- Check the horse: If both Vogons read bad poetry, Arthur knows only that $p=1/5$ and consequently $1-p=4/5$.

Whether he chooses G or BE as his best reply depends on probability p :

$$p - 20(1-p) = -10p \text{ from which } p = 20/31$$

Since $p=1/5$ he will reply with BE. Thus, the pooling candidate is (bb', BE BE').

NOTE: Other than in separating equilibria, it is not known what player 2 would choose if player 1 plays horrible. The q -cutoff is $-100q = -10(1-q)$

We need to distinguish 2 cases:

- a) $q < 1/9$. Then, player 2 would play G on the right side. In this case, the evil type of Player 1 would not play b, but h.

Continued: Candidate (hb' , BE BE')

- b) $q > 1/9$. Then, player 2 would choose action BE on the right side. Again, the evil type of Player 1 would not play b , but h .

Thus, this candidate is not a pooling equilibrium.

We now check if there is a pooling equilibrium if Player 1 plays hh' .

- This sets q to $1/5$. From above we know that Player 2 would choose BE .
- We still need to check what Player 2 would play on the left side:
Case a) $p < 20/31$. In this case, Player 2 replies with BE . **This is not an equilibrium because the bad type would like to deviate to bad poetry.**
Case b) $p > 20/31$. Then, Player 2 chooses action G . **This indeed is an equilibrium since no type of player 1 deviates: ($2 > 0$ and $0 > -5$).**

This game has a pooling equilibrium with (hh' , BE BE') for $p > 20/31$.

(Note: If we permit mixed strategy replies, we have additional equilibria. For reasons of simplicity we don't permit mixed strategy replies.)

So, can Arthur save his life??

Yes. If he's able to tell horrible from bad poetry. And if the Vogons and Arthur possess a sufficiently high degree of rationality (and know the payoffs and strategies of course..).

- The separating equilibrium (hb' , BE BE') tells him that it's his equilibrium strategy to reply always with "best ever."
- The pooling equilibrium tells him that whenever both types read horrible poetry, he again best replies with choosing action BE BE' .
- The game has no further equilibrium in pure strategies.