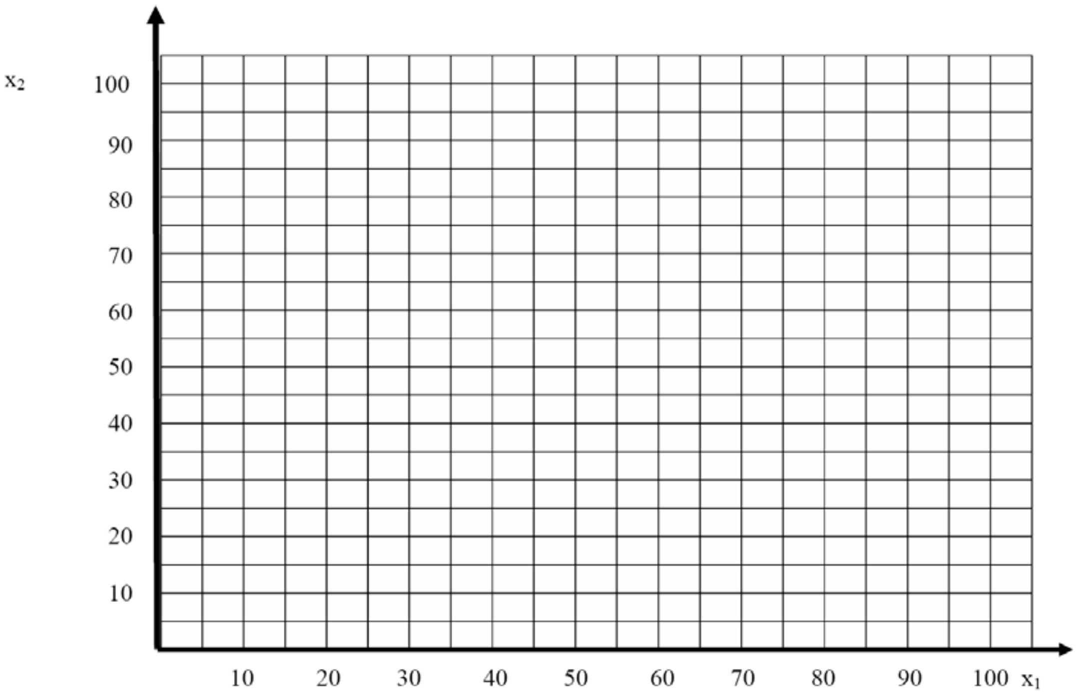
Practice Midterm #2

(1) Show that the demand curve $x(p) = ap^{-e}$ is isoelastic, which means it shows a constant price elasticity of demand for any x.

(2) Remember Kelly Umpleby from the last problem set. Here is the story of her sister Kim. She is a Cobb-Douglas consumer as well. Her preferences follow the utility function $u(x_1, x_2) = x_1 x_2$. Good 1 (gasoline) has a price of p_1. Good 2 is consumption, its price being normalized to one. The representative consumer's budget is m.

a) Find optimum consumption (individual demands). Explain verbally why the utility function $\tilde{u}(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ leads to the same individual demands. Show that this is the case.



b) Now use the following initial numeric values: price of gasoline is $p_1 = 1$, budget is $m = 100$. Compute demands and utility for this initial bundle. Graphically illustrate the solution by drawing the budget line and this allocation (Point A) into the chart below.

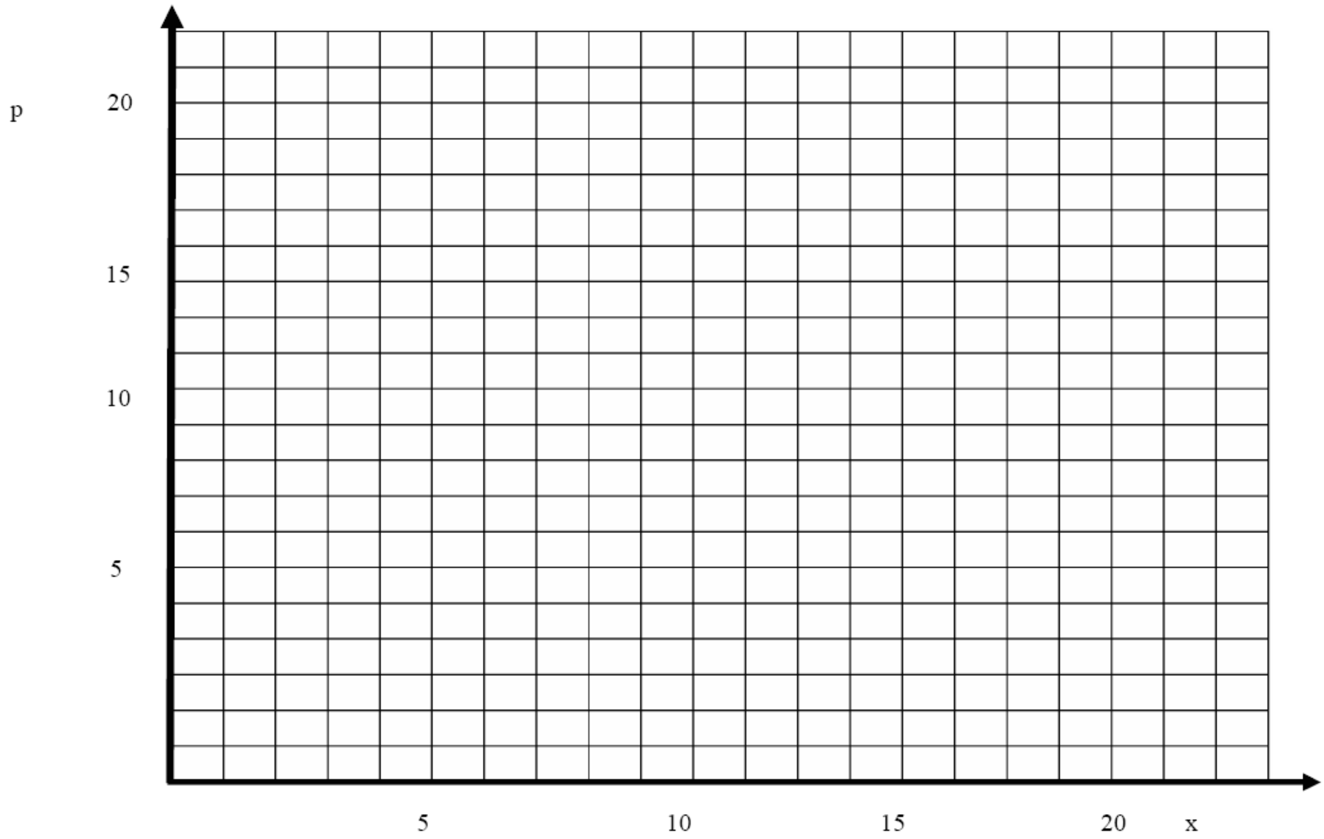
c) Government is preparing a quantity tax on gasoline of $t = 1$. Assume the representative consumer carries the entire tax burden. Compute quantity of gasoline consumed and utility after the tax and illustrate this change in the diagram above.

c) Find the Slutsky substitution and income effect mathematically and illustrate graphically.

d) Mathematically find equivalent and compensating variation (EV and CV) to the tax, as well as net consumer surplus.

(3) A monopolist faces an inverse demand curve given by $p = 20 - x$. The cost function is $C(x)=10+8x$.

(a) Derive and graph demand curve, marginal revenue, average cost and marginal cost. Find and mark profit-maximizing quantity, price, total cost, and the profit. Compute the Lerner index.



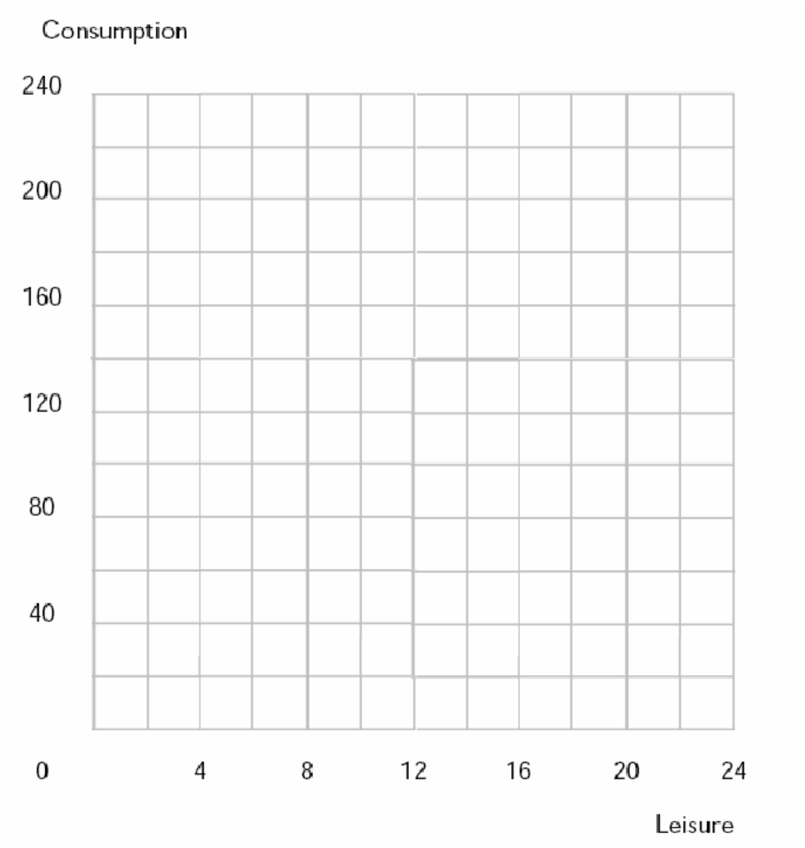
(b) Find the price elasticity of demand for the profit maximizing quantity.

Compute consumer surplus and compare the consumer surplus of the monopoly with the one under perfect competition (the efficient quantity). Mark both areas in the chart.

c) Assume the monopolist introduces a new technology of production that reduces marginal cost to MC=6. Find the new profit maximizing quantity and the new price. Did the Lerner index change? Did the consumer surplus change?

4) Bill Brown works at Jenny's Roadside Cafe. He can work as many hours per day as he wishes at a wage rate of *w*. Let C be the number of dollars he spends on consumer goods and let R be the number of hours of leisure that he chooses.

(a) Bill is paid $13 an hour and has 18 hours per day to devote to labor or leisure. He furthermore receives a daily nonlabor income of $ 6 from dividends (his portfolio management was ok, but not that good in the past). Price of Consumption is $1. Write down the equation for his budget between consumption and leisure and draw this line into the chart below. Mark his endowment point.



b) Assume that Bill has Cobb-Douglas preferences for leisure and consumption of the style $U(R,C)=2R^3C^2$. How many hours will he work (you may round the result)? What will be his amount of Consumption worth in $?

c) Write down Bill's Slutsky equation for labor and leisure and mark the signs of each term. Please use the correct notation for the variables. Can you predict without knowing the magnitude of each effect whether he will reduce or increase labor supply? Why or why not? Explain.

d) Jenny's Roadside Cafe now faces competition from a chic Seattle-based coffee chain. To save the business, all employees agree to work henceforth at a lower wage of w=10, so does Bill. Write down the equation for the new budget line and draw this new budget line into the graph on page 1. Mathematically find his new allocation. Does he increase or decrease labor supply? Thus, which effect has dominated, the substitution or the (total) income effect? Explain.

(5) Tom's inverse demand function for running shoes is $p(q) = \frac{1}{8q^3}$.

a) Find his price elasticity of demand for price p.

b) If the price of running shoes is $1, what is his price elasticity of demand?

(6)** A monopolist faces a revenue function of $R(p) = \frac{p}{(p+1)^2}$. There are no marginal costs, so there is no difference between the monopolist finding the revenue maximizing price and the profit maximizing price.

Find the revenue maximizing price by setting $MR(p)$ to zero (first-order condition). Check the second order condition to see that by setting $MR(p)$ to zero you indeed find a maximum.