Econ 21: Microeconomics W. Gick

Solution to the Practice Midterm Exam 1 – July, 2007

1. Charlie is a Cobb-Douglas consumer. His utility function is $u(x_1, x_2) = 2x_1^3 x_2^5$.

a) Find his individual demand function for each good by using the Lagrange method. Use $p_1 x_1 + p_2 x_2 = m$ for the budget line.

Setting up the Lagrangian leads to:

$$L(u_1, u_2, \lambda) = 2x_1^3 x_2^5 + \lambda(m - p_1 x_1 - p_2 x_2)$$

$$\frac{\partial L}{\partial x_1} = 6x_1^2 x_2^5 - \lambda p_1 = 0 \qquad (1)$$

$$\frac{\partial L}{\partial x_2} = 10x_1^3 x_2^4 - \lambda p_2 = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = m - p_1 x_1 - p_2 x_2 = 0. \qquad (3)$$

Dividing (1) by (2) gives the tangency condition $\frac{3x_2}{5x_1} = \frac{p_1}{p_2}$, from which $x_2 = \frac{5}{3}\frac{p_1}{p_2} x_1$.

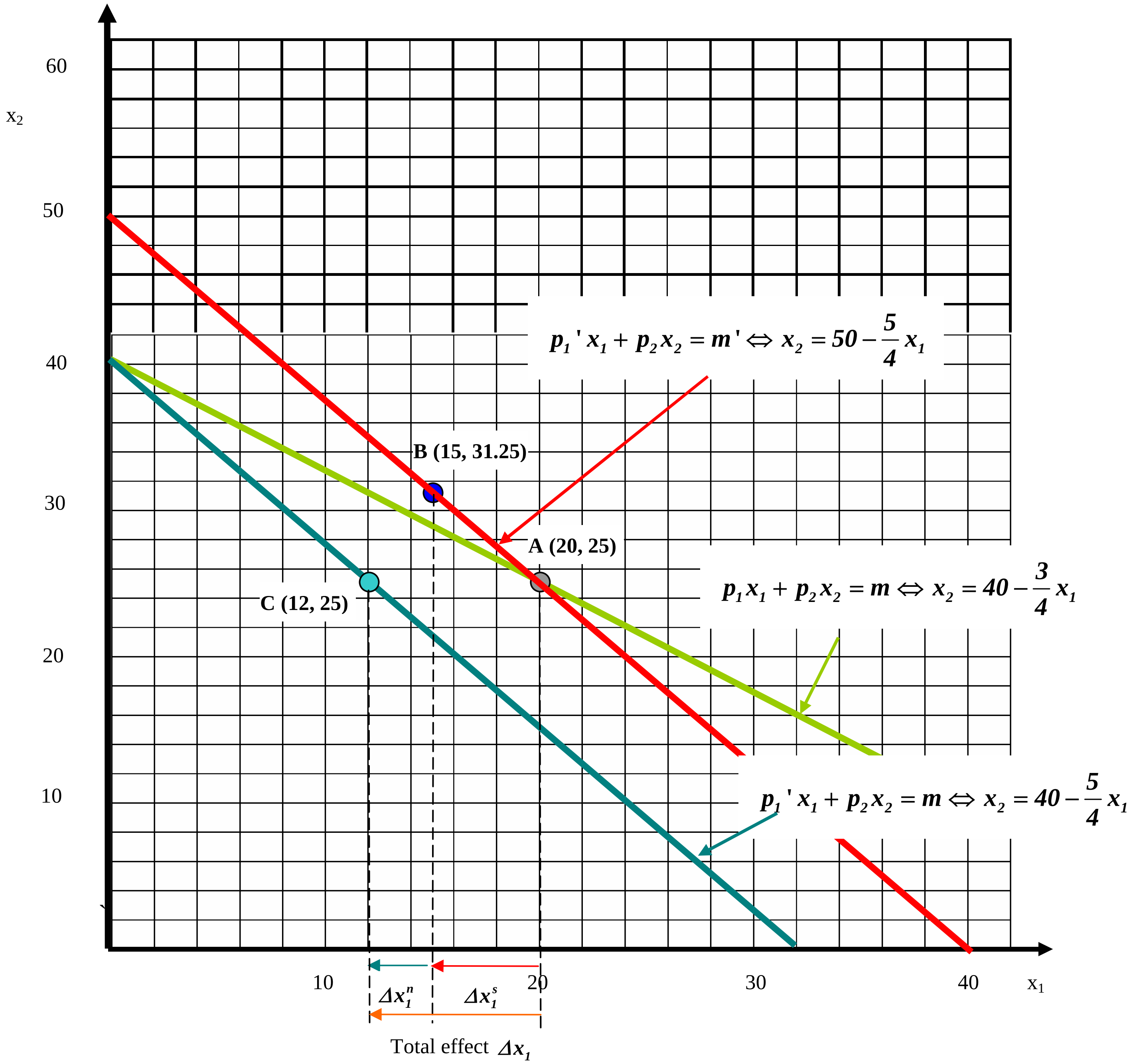
Using budget constraint (3) we derive individual demands:

$x_1 = \frac{3}{8}\frac{m}{p_1}$, plugged into tangency condition again yields $x_2 = \frac{5}{8}\frac{m}{p_2}$.

b) Now use the following values: $m=160$, $p_1 = 3$ and $p_2 = 4$. Find his initial consumption bundle A and mark it in the graph below:

$$x_1^A = x_1(3,4,160) = \frac{3}{8}\frac{160}{3} = 20.$$

$$x_2^A = x_2(3,4,160) = \frac{5}{8}\frac{160}{4} = 25.$$



c) Assume that price of good 1 now increases $p_1' = 5$. Find Charlie's Slutsky income compensation that he needs to receive to still be able to consume bundle A at new prices.

Solution: We look for the value of Δm that Charlie needs to be purchasing-power compensated through A at new prices. Or: how much additional money does he need should he be able to still consume 20 apples and 25 potatoes if the price for apples has increased to $p_1' = 5$?

This is $\Delta m = (p_1' - p_1)x_1^A = (5-3)20 = 40$. His compensated income would thus be $m' = m + \Delta m = 160 + 40 = 200$. (see red budget line in the graph above.

d) Find his compensated bundle B and his final bundle at new prices and original income. Marke the bundles in the graph above.

We now compute the new values in B using individual demand function:

$$x_1^B = x_1(5,4,200) = \frac{3}{8}\frac{200}{5} = 15.$$
$$x_2^B = x_2(5,4,200) = \frac{5}{8}\frac{200}{4} = 31.25.$$

which yields Point B (15, 31.25) (see graph above).

Finding his final allocation: taking off the Δm compensation reduces income back to *m*, but at new prices, with the following demands:

$$x_1^C = x_1(5,4,160) = \frac{3}{8}\frac{160}{5} = 12.$$
$$x_2^C = x_2(5,4,160) = \frac{5}{8}\frac{160}{4} = 25.$$

This yields Point C(12,25).

e) Write down the Slutsky equation including substitution and income effect and compute both effects. Mark them in the diagram above. You need to mark it along the correct axis and with the correct length and direction.

Slutsky equation reads: $\Delta x_1^{total} = \Delta x_1^s + \Delta x_1^n$

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m) = x_1^B - x_1^A$$
$$\text{with } x_1^B = 15, x_1^A = 20.$$
$$\text{Thus, } \Delta x_1^s = x_1^B - x_1^A = 15 - 20 = -5.$$
This is the substitution effect (red arrow).

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m') = x_1^C - x_1^B.$$
$$\text{with } x_1 = x_1^C = 12, \text{ and } x_1^B = 15$$
$$\text{Thus, } \Delta x_1^n = x_1^C - x_1^B = 12 - 15 = -3.$$
This is the income effect (green arrow)

2. Debbie's new preferences follow the function $u(x_1, x_2) = (x_1 + 10)x_2$

(a) Find her optimal demand functions using the general expression for the budget line $p_1x_1 + p_2x_2 = m$ by using the Lagrange method.

We solve

$$\max_{x_1,x_2} \quad (x_1+10)x_2 \qquad \text{s.t.} \quad p_1x_1+p_2x_2=m$$

We set up the Lagrangian

$$L(x_1,x_2,\lambda)=\underbrace{x_1x_2+10x_2}_{=(x_1+10)x_2}+\lambda(m-p_1x_1-p_2x_2)$$

with

$$\frac{\partial L}{\partial x_1}=x_2-\lambda p_1=0 \qquad (1)$$

$$\frac{\partial L}{\partial x_2}=x_1+10-\lambda p_2=0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda}=m-p_1x_1-p_2x_2=0 \qquad (3)$$

Dividing from (1) by (2) yields the tangency condition

$\frac{x_2}{x_1+10}=\frac{p_1}{p_2}$, or, solved for x_2:

$$x_2=\frac{p_1}{p_2}(x_1+10)$$

Plugging into budget line:

$$p_1x_1+p_2\left(\frac{p_1}{p_2}(x_1+10)\right)=p_1x_1+p_1(x_1+10)=2p_1x_1+10p_1=m \Leftrightarrow$$

$$x_1=\frac{m-10p_1}{2p_1}=\frac{m}{2p_1}-5.$$

Plugging into tangency condition yields individual demand for Good 2:

$$x_2=\frac{p_1}{p_2}\left(\frac{m}{2p_1}+5\right)=\frac{m}{2p_2}+5\frac{p_1}{p_2}$$

(b) Use the individual demand functions that you found in (a) to plug in the following values: $p_1=3, p_2=4, m=70$. What is the utility that Debbie has in the optimum?

$$x_1(p_1=3,p_2=4,m=70)=\frac{70-30}{6}=\frac{40}{6}=6.\overline{6}$$

$$x_2(p_1=3,p_2=4,m=70)=\frac{70}{8}+5\cdot\frac{3}{4}=8.75+3.75=12.5$$

Debbie's utility in this equilibrium is $u(x_1,x_2)=(x_1+10)x_2=(6.\overline{6}+10)\cdot 12.5=208.\overline{3}$

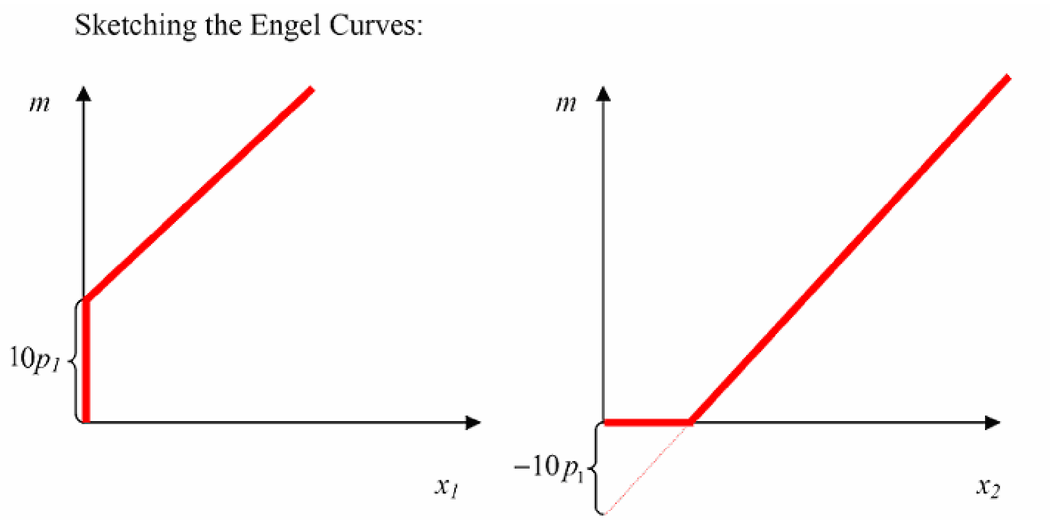
(c) Compute and draw the Engel curves for each good. Use one for each good and label the axes. Do the Engel curves differ from Engel curves of Cobb-Douglas demands? If so, how?

Solution: We know from Problem set 2 that the Marshallian demands are:

$$x_1^* = \frac{m}{2p_1} - 5, \text{ and } x_2^* = \frac{m}{2p_2} + 5\frac{p_1}{p_2}$$

Solving the functions for m yields

$$m = \underbrace{10p_1}_{\text{intercept}} + \underbrace{2p_1}_{\text{slope}} x_1, \text{ and for good 2: } m = -\underbrace{10p_1}_{\text{intercept}} + \underbrace{2p_2}_{\text{slope}} x_2$$

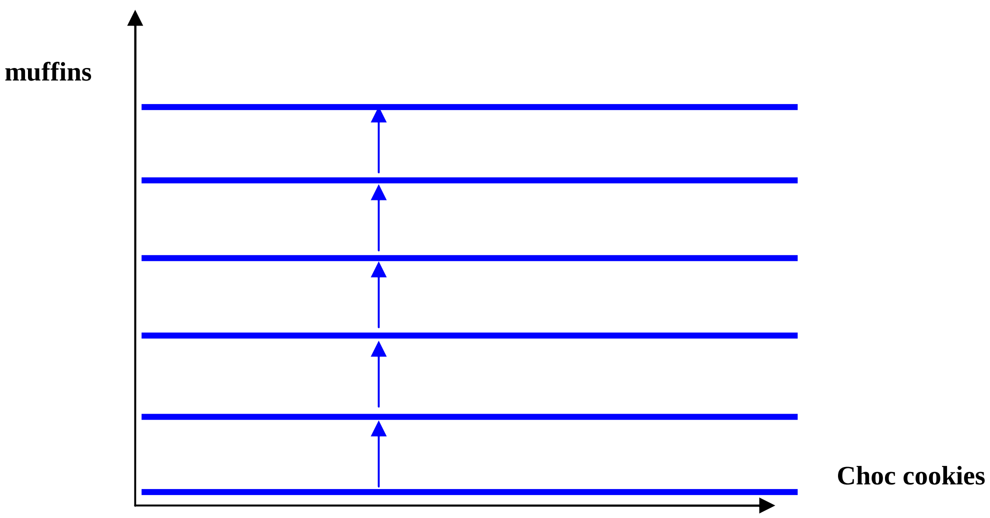


Other than with a Cobb-Douglas function, the Engel curve does not go through the origin but has an intercept. This means that the given utility function $(x_1 + \mathbf{10})x_2$ does not lead to homothetic preferences.

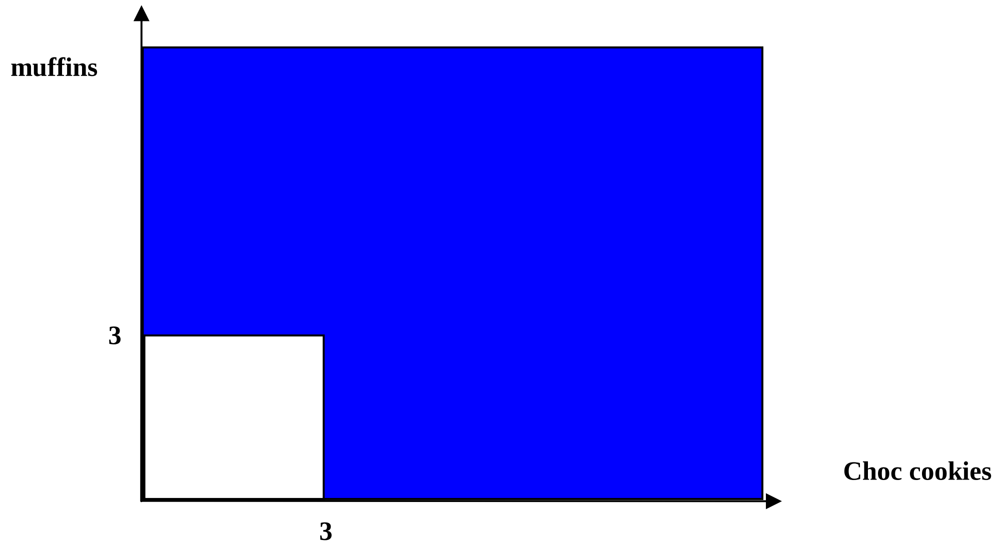
(3) Sketch the following indifference curves of an individual consuming two commodities: chocolate cookies and muffins.

(a) Abby loves muffins but does not care about choc cookies. She always prefers muffins, independent of the number of cookies she is offered.

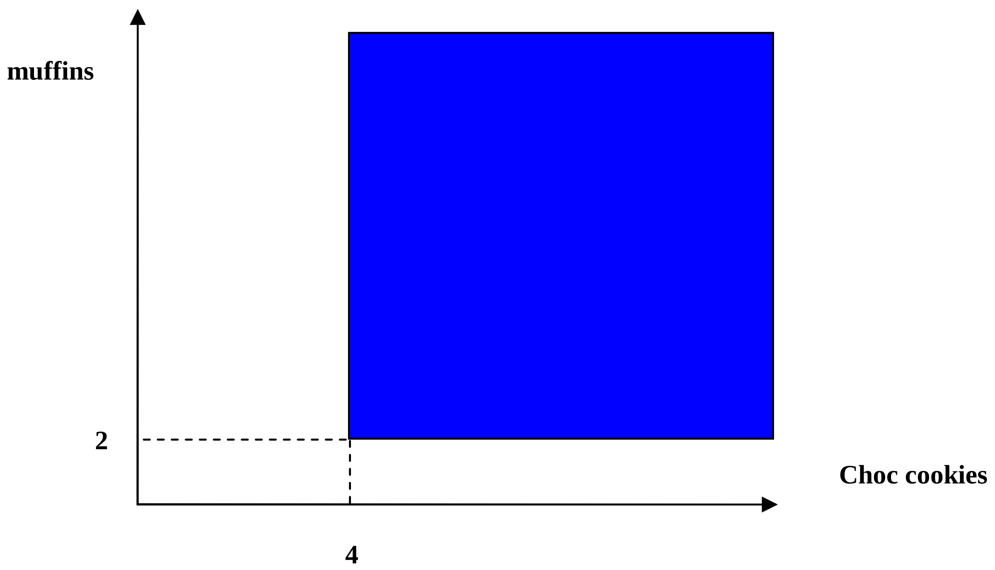
Intuition: Her utility increases only toward muffin, choc cookies are a "neutral"



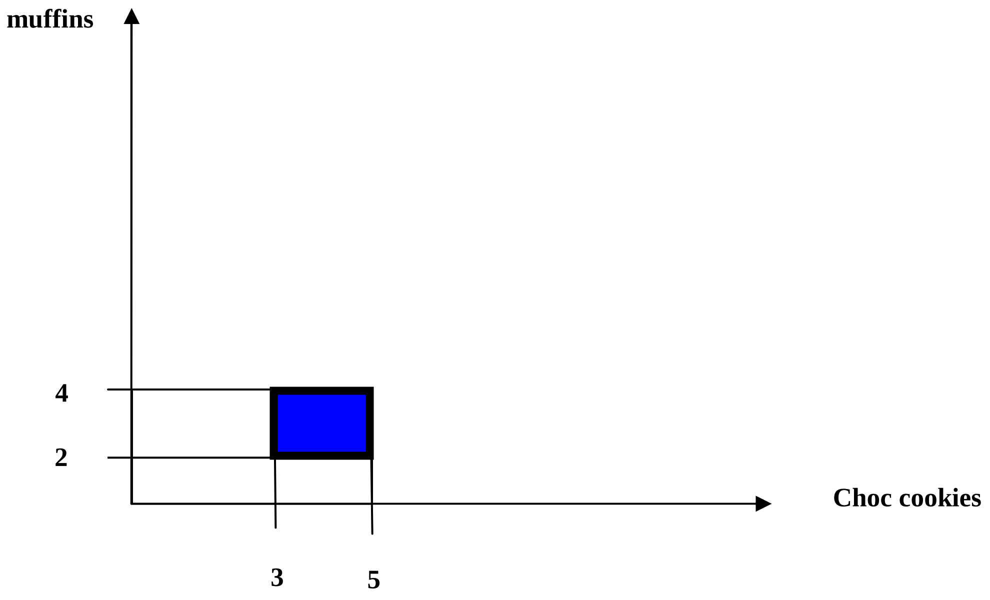
(b) Betsy is indifferent between two bundles that contain at least 3 choc cookies OR 3 muffins. Her preferences do not change should she consume additional units of at least one of the goods in combination.



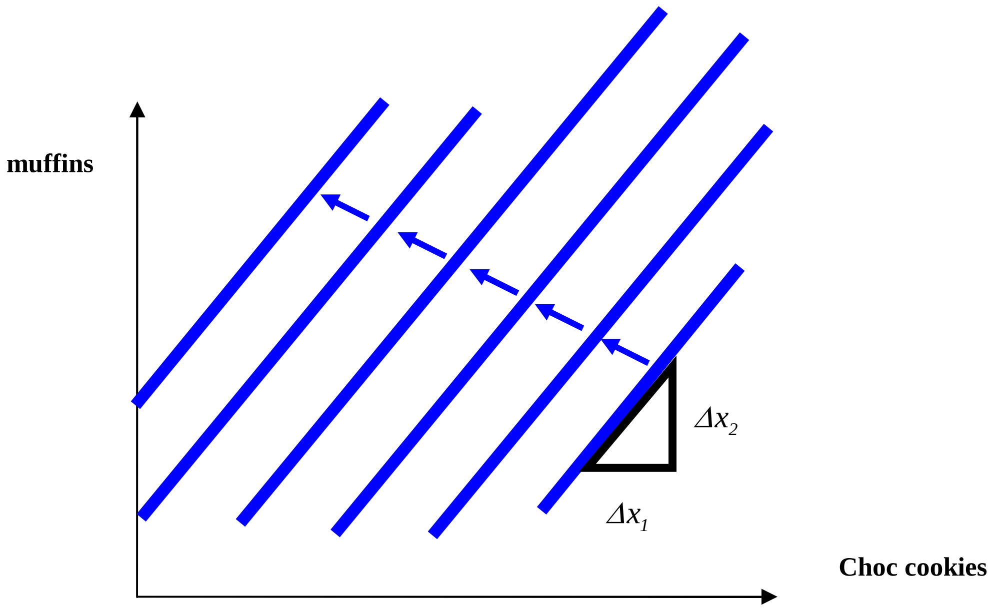
(c) Her sister Connie is indifferent between two bundles that contain at least 4 marshmallows AND 2 muffins. Her preferences do not change should Connie consume additional units of at least one of the goods in combination.



(d) Debbie prefers muffins and chocolate cookies only in the following combination: Any 2-4 muffins together with 3-5 chocolate cookies give her the same utility, otherwise her utility is zero.



(e) Emily likes muffins. She unfortunately is allergic to chocolate cookies. If you want he to eat a chocolate cookie, you need to compensate her by giving her 0.75 muffins. (Illustrate the MRS).



The MRS can be computed using the slope triangle: $MRS = \frac{\Delta x_2}{\Delta x_1} = \frac{0.75}{1} = 0.75$

4. Charlize has a utility function of $u(x_1, x_2) = 2\sqrt{x_1} + x_2$. Assume p_1 and p_2 to be positive.

a) Algebraically solve her optimization problem supposing that she acts rationally.

We solve by using the Lagrange method:

$$L(u_1,u_2,\lambda) = 2\sqrt{x_1} + x_2 + \lambda(m - p_1x_1 - p_2x_2)$$

$$\frac{\partial L}{\partial x_1} = \frac{1}{\sqrt{x_1}} - \lambda p_1 = 0 \qquad (1)$$

$$\frac{\partial L}{\partial x_2} = 1 - \lambda p_2 = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = m - p_1x_1 - p_2x_2 = 0. \quad (3)$$

From (2) we have: $\lambda = \frac{1}{p_2}$; plugged into (1) yields:

$x_1 = \left(\frac{p_2}{p_1}\right)^2$, plugged into (3) yields $x_2 = \frac{m}{p_2} - \frac{p_2}{p_1}$.

b) For a good being a substitute, the cross-price derivative needs to be positive, $\frac{\Delta x_1}{\Delta p_2} > 0$.

Thus, if demand for good 1 increases if price of good 2 increases, good 1 is a substitute for good 2.

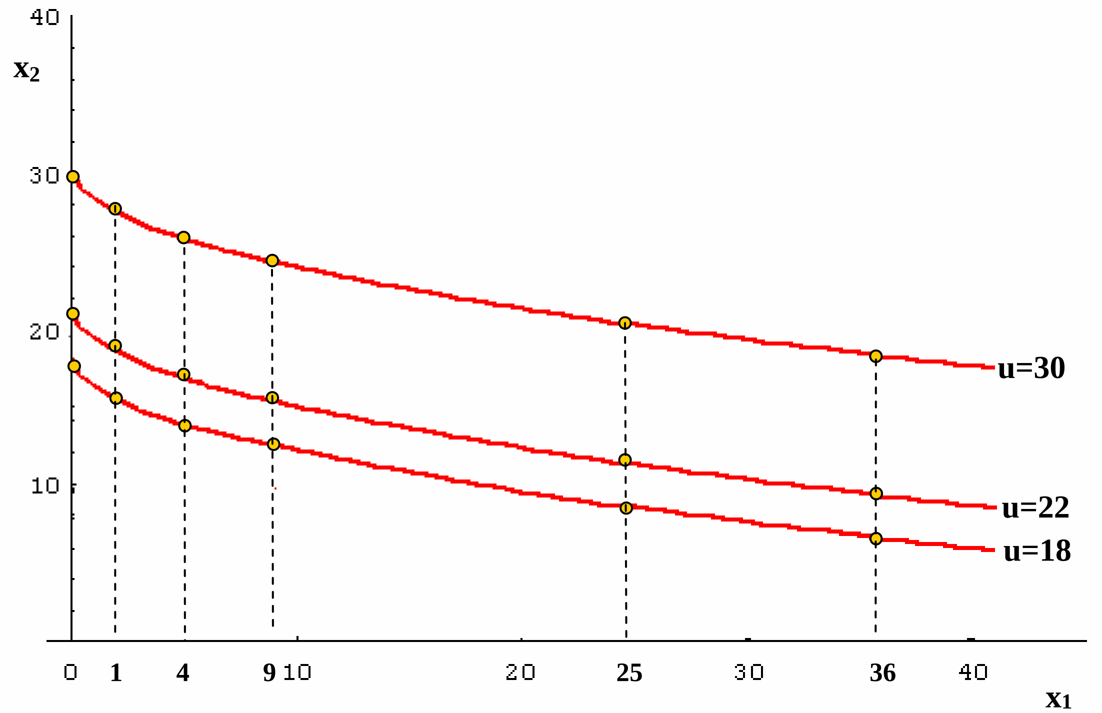
From

$x_1 = \left(\frac{p_2}{p_1}\right)^2$

we can see that demand for good 1 increases (quadratically) with the price of good 2. Thus, the goods are substitutes.

c) We draw the table for $u = 18$, $u = 22$, and $u = 30$. The values for $u=18$ are reached by $2\sqrt{x_1} + x_2 = 18 \Leftrightarrow x_2 = 18 - 2\sqrt{x_1}$.

U	18	22	30
x1	x2	x2	x2
0	18	22	30
1	16	20	28
4	14	18	26
9	12	16	24
16	10	14	22
25	8	12	20
36	6	10	18


40
x_2
30
20
10
0
1
4
9
10
20
25
30
36
40
x_1
u=30
u=22
u=18