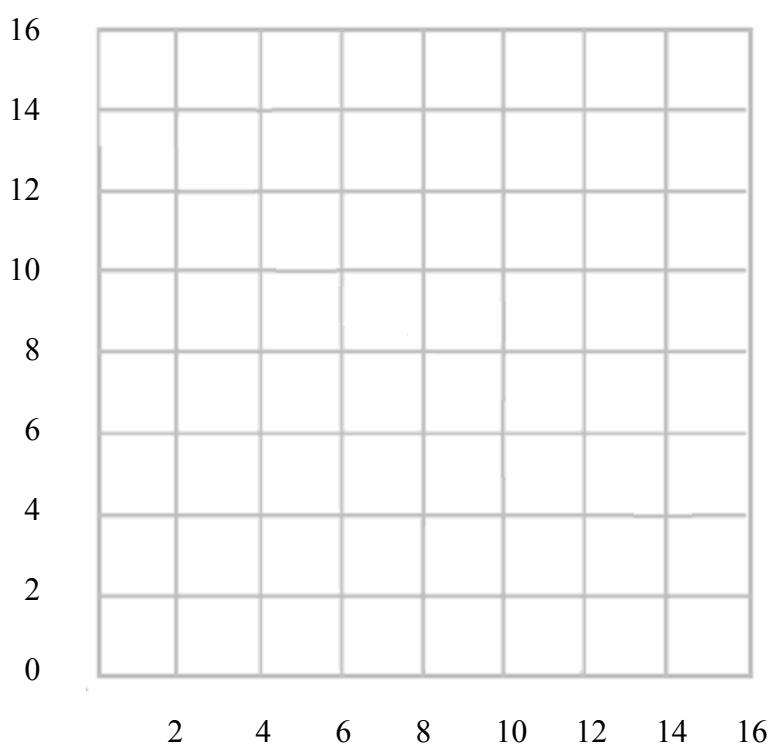


1) The representative consumer of widgets has a utility function of $u(x_1, x_2) = 3\sqrt[3]{x_1} + x_2$. Good 2 is a composite good (sum of all other goods). The consumer has a budget m and faces market prices of p_1 and p_2 , with $p_2 = 1$.

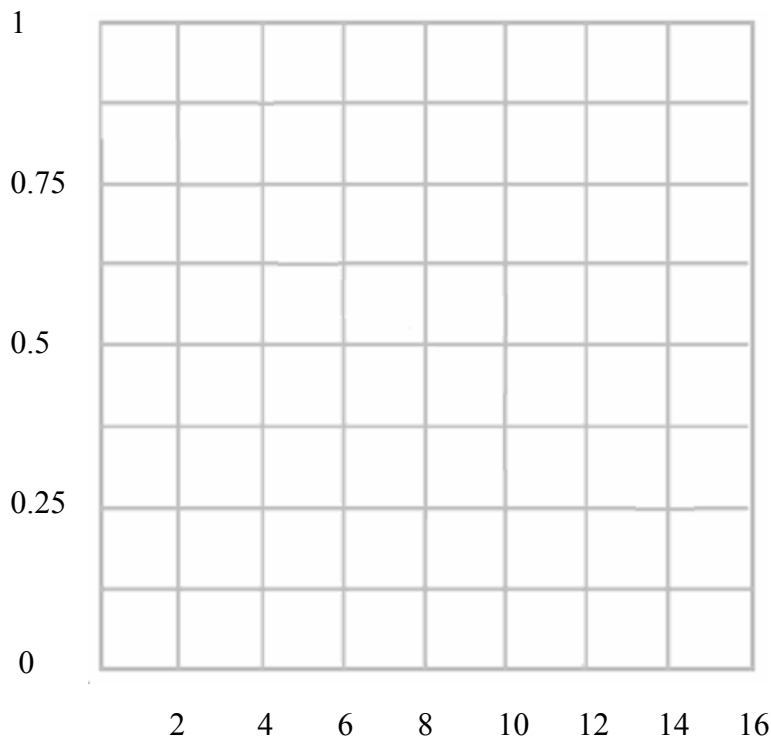
- Use the Lagrange method to find individual demands.
- Find the consumer's optimal bundle using the following numeric values: $p_1 = 0.25$, $m = 12$. Find the utility in this optimum.
- Graph the indifference curves for $u=16$ using 4-5 value pairs for x_1 and x_2 into the diagram below. Label the axes. Graph the initial budget line and the bundle you found in b) and mark this initial bundle with "A".



d) Now price for widgets doubles to $p_1' = 0.5$. Find the new equilibrium algebraically and mark the new budget line into the above diagram. Mark this new equilibrium with "C".

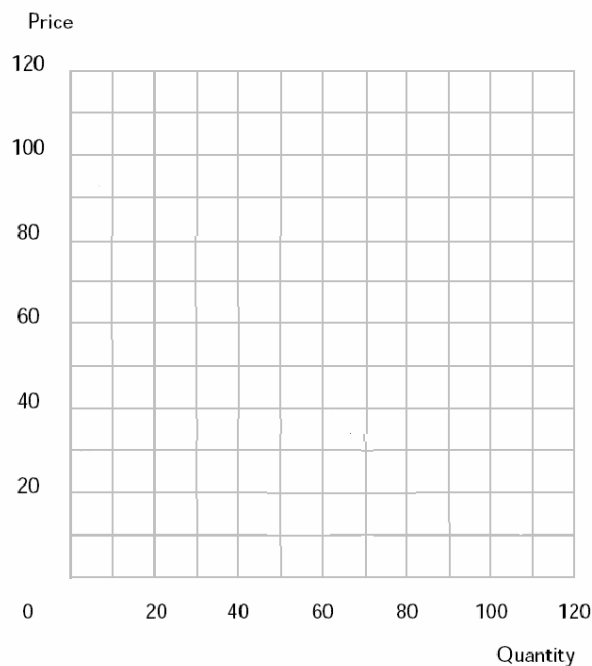
e) Find the value of the Slutsky substitution and income effect for widgets. You may round the result. Mark Point "B" (new optimum at new prices, income compensated through the initial bundle) into the diagram and mark the two effects along the x_1 -axis. Draw all budget lines into the diagram and explain your findings.

f) By using the points A and C, sketch the demand curve into the diagram below, together with two additional value pairs. Mark the loss of consumer surplus that the consumer faces through the price increase. Label the axes. Compute the loss of consumer surplus.



2) An online publisher is considering offering articles to readers by email. Their market research division indicates that there are two types of potential users in the market, students and executives. There are the same number of students and executives in the market. Let x be the number of articles that a user requests per year.

The executives have an inverse demand function $PE(x) = 120 - x$ and the students have an inverse demand function $PU(x) = 100 - x$. (Prices are measured in cents.) The publisher has a zero marginal cost of sending articles via email. Draw these demand functions in the graph below and label them.



a) Suppose that the seller can tell each type apart and uses price discrimination. What is the optimal pricing strategy? What are the quantities sold, and what is the profit-maximizing price?

b) Now assume that the seller cannot tell the two consumer groups apart. An executive can always buy the package size designed for the student. Assume the size is not changed compared to part a): what are the prices now for each bundle? Explain why the seller gives a "discount" to one type of buyers. What would happen if he does not?

c) It is known that the seller can improve its profit by reducing the package size for the students. Find the optimal package size if there is one consumer of each type. Find price and the publisher's profit.

d) Now assume there are twice as many students in the market as executives. What is now the optimal package size for the students? What does the publisher charge now for the different bundles? What is the publisher's profit?

e) Imagine the ratio between students and executives changes to 1:4, that is, there are 4 times as many executives as students. Again, find optimal package size for students and the new price for the two different packages. Compute the seller's profit assuming there are indeed 4 executives and 1 student.

In this situation, would the seller be better off by offering two different packages or only one package? Find price and profits if the seller only caters to the executives. Explain.

3) Note: this problem replaces the old one on the "part 1" version of the practice final.

Assume an inferior good 1 and a price decrease of good 1

Graphically illustrate substitution effect, income effect, and total effect for

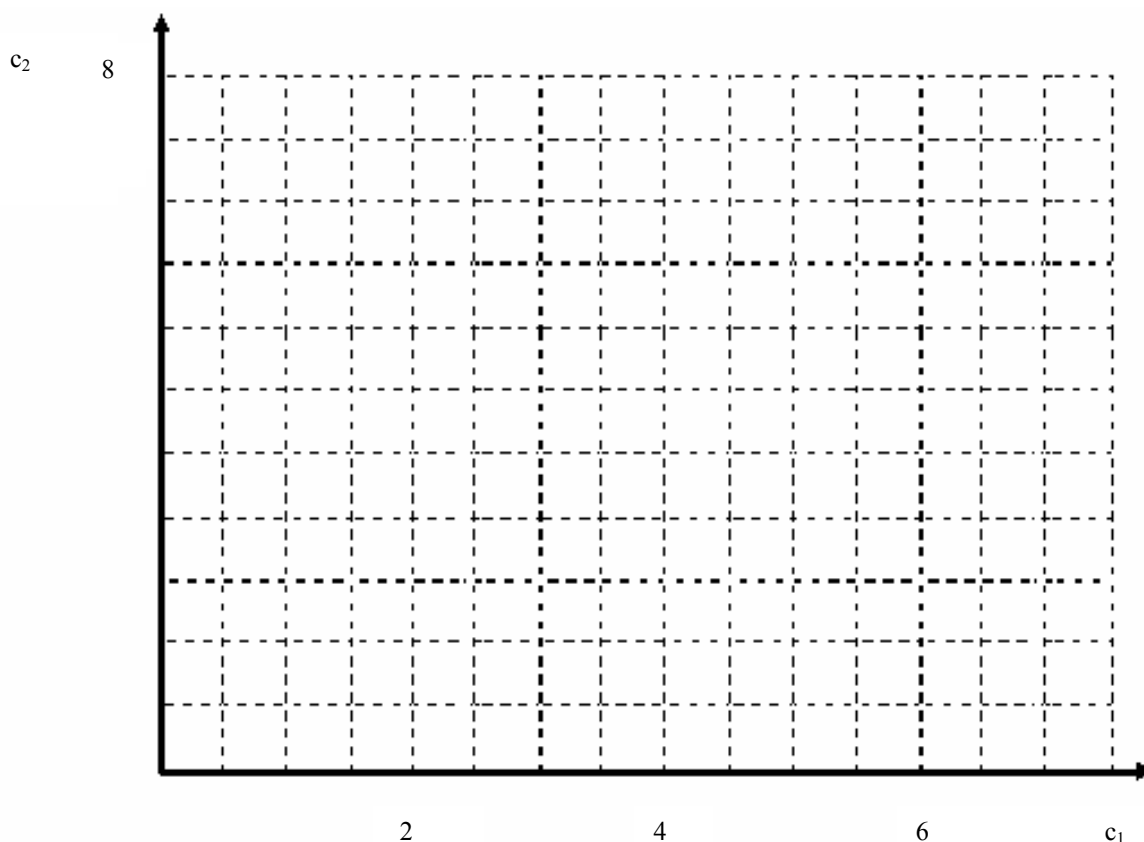
(a) good 1 not being a *Giffen* good,

(b) good 1 being a *Giffen* good.



4) Carla Cobb has to decide about her present and future consumption. She has preferences that follow the utility function $u(c_1, c_2) = c_1^2 c_2^2$. In her first period, she earns an income $m > 0$.

a) Find her individual demands by either using the Lagrange method or any other algebraic approach. Draw the indifference curve for $\bar{u} = 81$ into the chart below by using a few (>4) points.



b) Assume now that Carla instead of earning m , is endowed by a present income $m_1 = 4$, and a future income $m_2 = 2$. Assume an interest rate of $r = 0$. Draw the budget line into the chart as well, and mark her endowment point. Assume there are no credit markets: According to her preferences, does Carla have an equilibrium?

c) Now introduce a positive interest rate r and a positive income m_2 . Assume we have credit markets and thus both borrowing and lending would become possible. Write down Carla's budget line for the case she is a lender.

d) Assume now that Carla has access to credit markets, and that $r = 16\%$. Draw the new budget line into the above chart. In this situation, will Carla be a borrower or a lender? Is there a chance that this decision may change with a further increase of the interest rate?

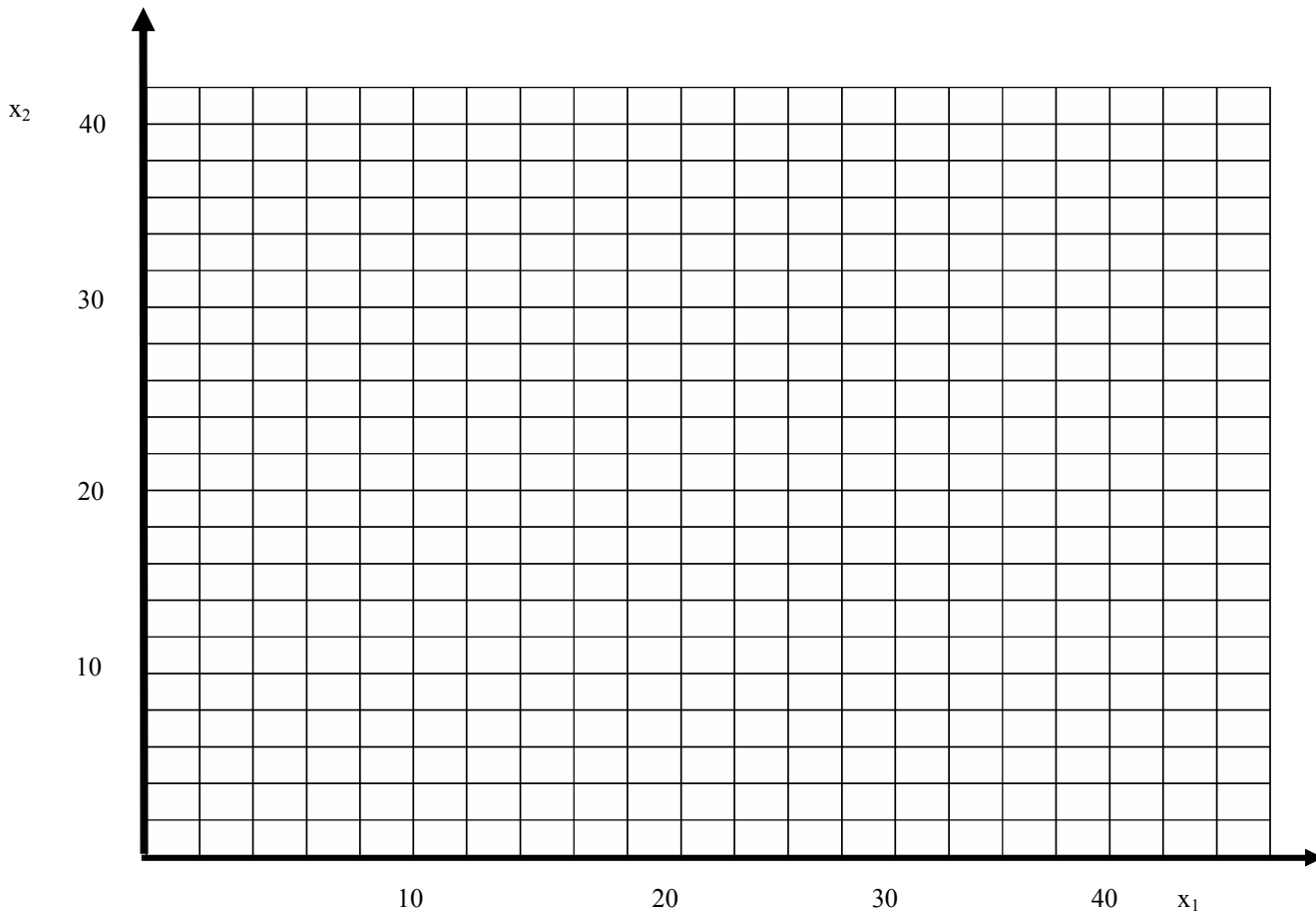
e) Derive the c_1 and c_2 intercepts algebraically and interpret their meaning.

f) Write down the Slutsky equation for Carla's intertemporal choice and mark the signs of the derivatives. What can you tell for Carla's choice if Carla is a borrower and if she is a lender?

5) On planet Melmac, they mainly produce and consume two goods: catsup (good 1) and green slimeballs (good 2). Alf as a citizen of this planet has a utility function of

$$u(x_1, x_2) = 2\sqrt{x_1} + x_2.$$

- Find Alf's individual demands using Lagrangian.
- Write down the mathematical definition for substitutes and complements. By using demand for good 1 show why Alf regards these goods as substitutes.
- Find his price elasticity for good 1 and for good 2.
- As written down in Melmac's constitution, each citizen has the right to receive an endowment of 20 gallons of catsup and 10 slimeballs. Alf receives this endowment too. Draw this bundle into the graph below together with the budget line, assuming that $p_1 = 0.5$ and $p_2 = 1$. Find his initial income m . Also, graph the four indifference curves with $u = 22$, $u = 23$, $u = 31$, and $u = 41$ into the same graph (it is sufficient to find the values for $x_1 = 0, 1, 4, 9, 25$ and say 36 or 40 and to connect the points).



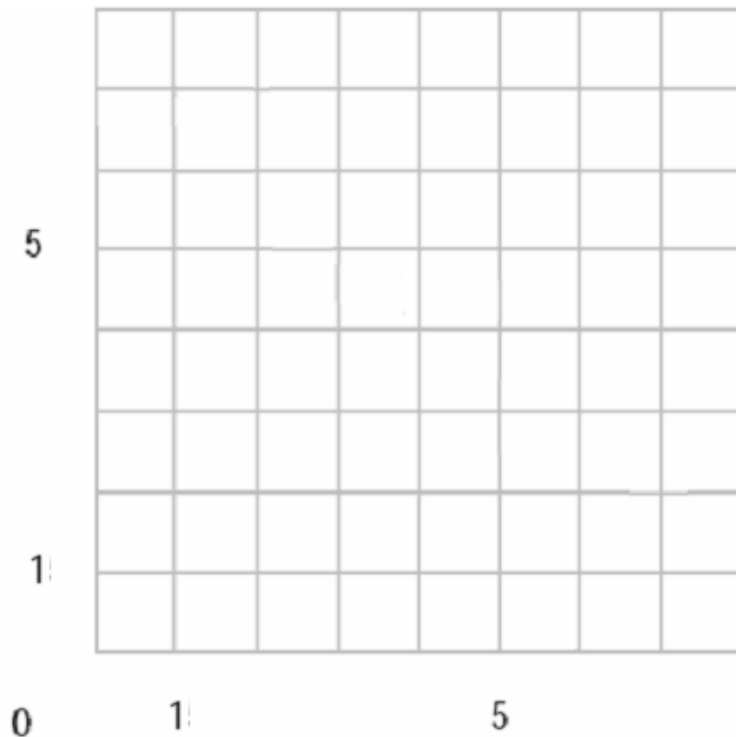
- Find his initial consumption bundle A. Mark this point into the chart above. Is Alf a net seller or net buyer of good 1?

f) Now assume that the price of good 1 increases to $p_1' = 1$. Find his Slutsky income compensation and his compensated bundle B and mark it in the graph. Find his other consumption bundles that include the ordinary income effect and the endowment income effect, at new prices. Mark all budget lines and equilibria in the graph above.

g) Compute Alf's substitution effect. Mark it in the graph above. What is the magnitude of each single income effect? What does the total effect consist of?

h) Write down Alf's Slutsky equation and explain whether or not you can predict the result by only checking the sign of the single parts of the Slutsky equation.

i) For this part assume that prices of each good are 1. Find the formula to compute the Engel curves for each good and draw these into the two charts below. Label the axes. Explain.

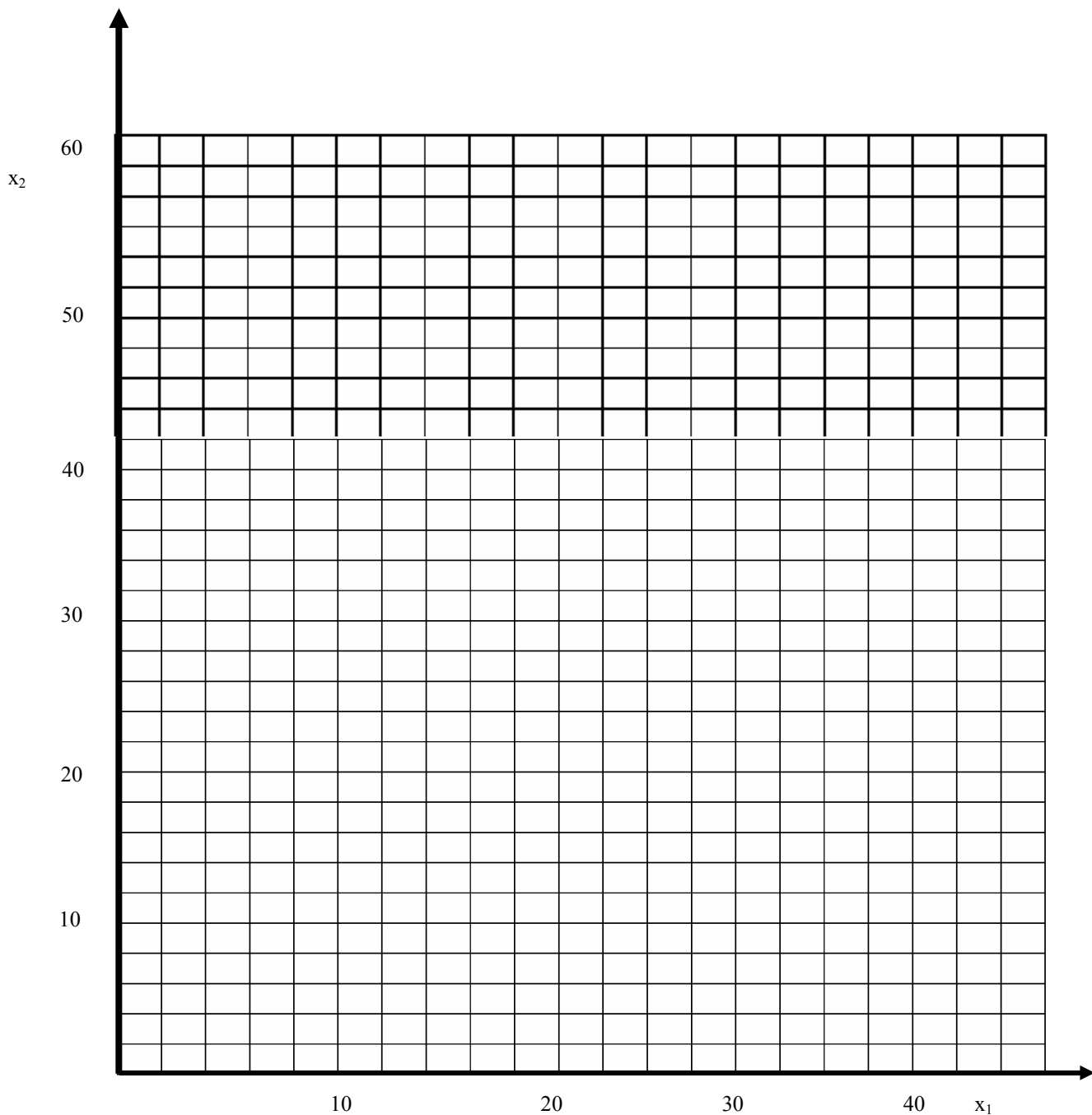


6) This is a new version of Castaway. As you know, Castaway survives on Coconut Island by eating fish that he catches (good 1) and coconuts (good 2). In one week, he is able to catch 8 fish and harvest 28 coconuts. He is of course a self supporter since he lives alone on his island.

(a) Luckily he finds himself stranded on an island surrounded by other islands with a population of really friendly aborigines (henceforth called his trade partners). They actually discover him on their usual joyrides while jetskiing with their noisy motorboats. Realistically you should not think you could meet “real” aborigines anywhere anymore; even the remotest place in the world is already blessed with modern pastime technologies. The trade partners use a currency unit of 1 Lira. Communication is somewhat difficult, and his trade partners think of him as funny guy that likes staying on this island. They believe that he likes trading fish and coconuts as much as they do, so they decide to visit him regularly. Castaway actually has given up explaining that he is stranded but makes the best out of his situation by trading with them as well. Castaway has the following preferences: $u(x_1, x_2) = x_1 x_2$. His trade partners pay him 10 Lire for each coconut and 5 Lire for each fish.

Mark his endowment point E and both algebraically and graphically find his choice A . Is Castaway a net seller or net buyer of fish?

(b) Now assume that next time his trade partners are more interested in fish and pay him now 10 Lire per fish. Algebraically and graphically find his individual demand for fish after substitution effect, after his ordinary income effect, and after his endowment income effect.



(c) Mark all points that represent the change according to the substitution effect and the two income effects in the chart above. Use arrows to mark the effects along the x_1 -axis.

(d) Write down the Slutsky equation in its appropriate form. By using only your information about the position of E and A , can you predict the outcome (reduction/increase of consumption of good 1) from the Slutsky equation? Explain.

e) (additional:) Algebraically find Hicks demands for Good 1 and Good 2.