7 Revealed preference – a theory of choice

Motivation: During this course we moved from "Preferences" to "Utility," and to "Choice."

At the center of an economic theory of consumer behavior is the theory of consumer choice. It is not always necessary to regard "true utility" as a meaningful concept (rather as a meta-theory).

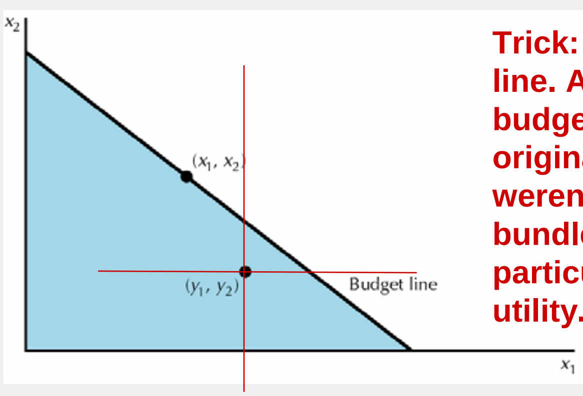
New concepts involve a choice-theoretic treatment of welfare theory.
Revealed preference theory is a choice theory.

It is a useful tool to better understand why e.g.
- the Substitution Effect is always negative, and why
- a price increase always reduces demand if the consumer is a net buyer of a good (Slutsky equation, buying and selling, ch. 9)

What's behind the idea or RP?

Assume: Strictly convex preferences (indifference curves are inward bowed). Then, we always get a unique equilibrium (bundle).

Consider the following two bundles, with bundle (x_1,x_2) being the optimal bundle (this is known, together with the budget line):



Trick: Think along the budget line. All the bundles on the budget line are worse than the original bundle, so if they weren't preferred to the original bundle, the (y_1, y_2) bundle in particular will give a lower utility.

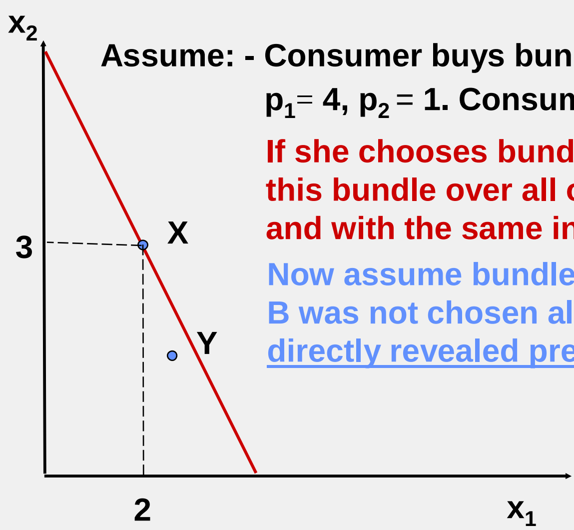
A numeric example

Assume: - Consumer buys bundle X(2,3).

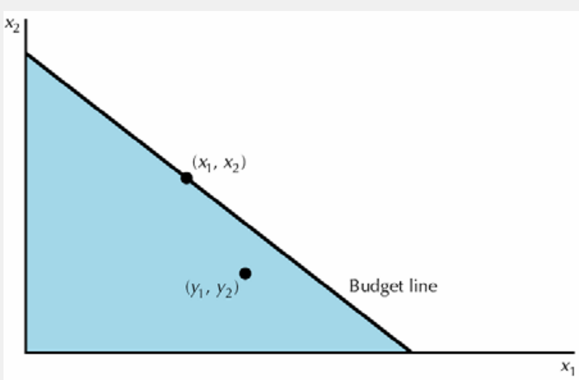
$p_1 = 4$, $p_2 = 1$. Consumer pays 1(2)+3(4)=$14.

If she chooses bundle X, she reveals to prefer this bundle over all others at the same prices and with the same income of $14.

Now assume bundle Y that is affordable. Since B was not chosen although affordable: X is directly revealed preferred over Y.



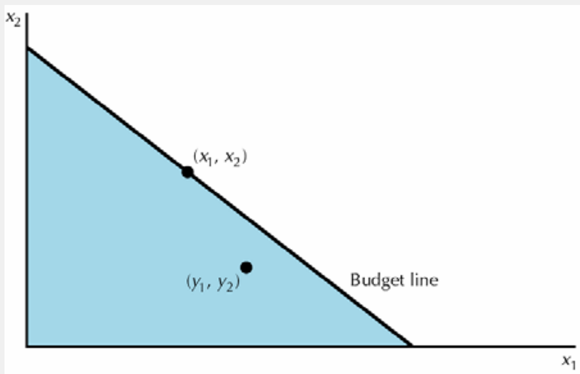
Weak Axiom of Revealed Preferences



If (x_1, x_2) *is directly revealed preferred to* (y_1, y_2) *and the two bundles are not the same, then it cannot happen that* (y_1, y_2) *is directly revealed preferred to* (x_1, x_2).

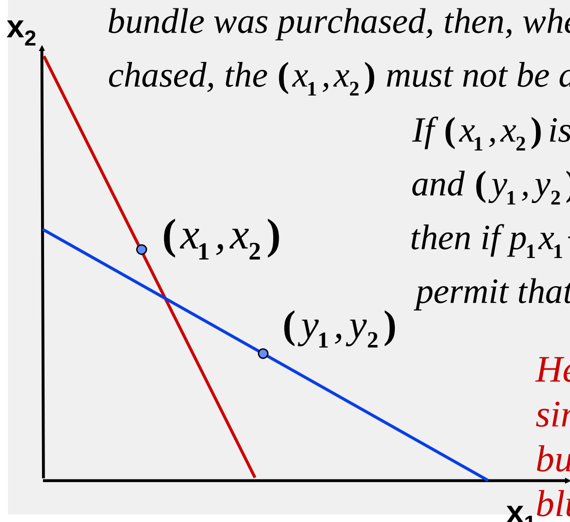
Weak Axiom of Revealed Preferences

In words: if the (y_1, y_2) *bundle was affordable when the* (x_1, x_2) *bundle was purchased, then, when the* (y_1, y_2) *bundle was purchased, the* (x_1, x_2) *must not be affordable.*



How to check: a case satisfying WARP

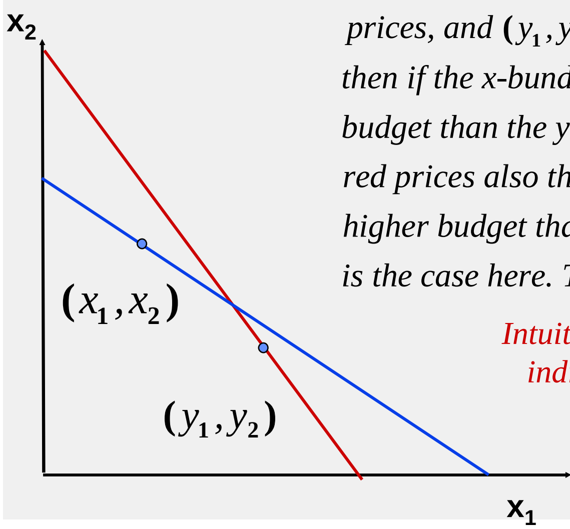
Again: if the (y_1, y_2) *bundle was affordable when the* (x_1, x_2) *bundle was purchased, then, when the* (y_1, y_2) *bundle was purchased, the* (x_1, x_2) *must not be affordable.*



If (x_1, x_2) *is purchased at prices* (p_1, p_2), *and* (y_1, y_2) *is purchased at prices* (q_1, q_2), *then if* $p_1x_1+p_2x_2 \geq p_1y_1+p_2y_2$ *we cannot permit that also* $q_1y_1+q_2y_2 \geq q_1x_1+q_2x_2$.

Here indeed, WARP is satisfied since at red prices, the y-bundle is not affordable and at blue prices, the red bundle.

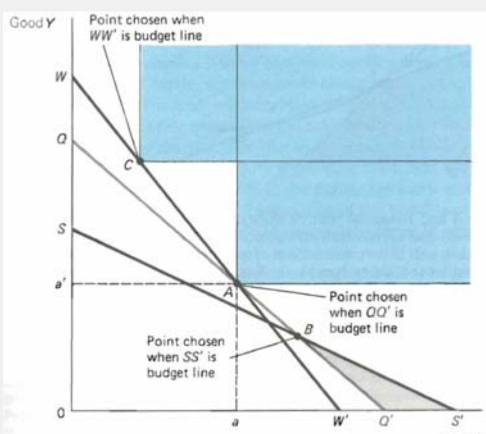
A case where WARP is violated



Check here: if (x_1, x_2) is purchased at blue prices, and (y_1, y_2) is purchased at red prices then if the x-bundle is gives a weakly higher budget than the y-bundle, it cannot be that at red prices also the y bundle gives a weakly higher budget than the x-bundle. This however is the case here. Thus, WARP is violated.

Intuition: you cannot draw consistent indifference curves through the two bundles that fulfill tangency condition for each budget line. They will intersect.

Trapping the indifference curve

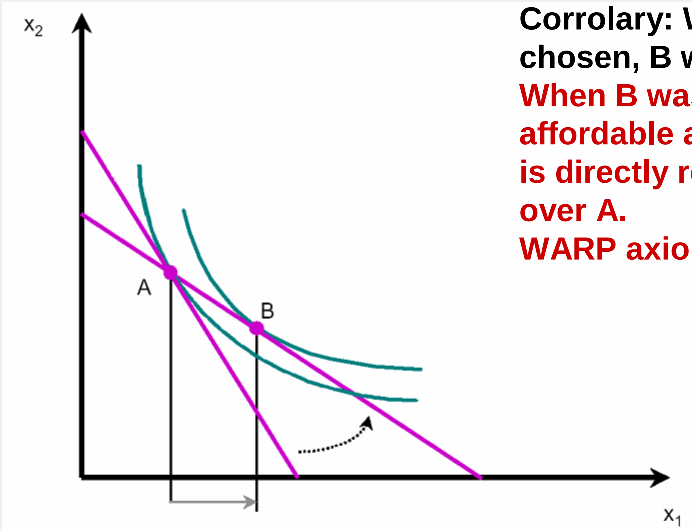


If we observe more than one chosen bundle at known prices, we can further narrow down regions that can be chosen and that cannot be chosen.

Ex: Along QQ' A is chosen over B. However, there could be a budget illustrated by SS' that makes the consumer choose B.

If this is known, we can eliminate the grey triangle: Since A was chosen over along QQ', and B along SS', no point in the grey area will be chosen along any of the given budget lines.

WARP: Negativity of the Substitution Effect



Corrolary: When A was chosen, B was not affordable.
When B was chosen, A was affordable at new prices, but B is directly revealed preferred over A.
WARP axiom holds.

Ch 9: Endogenizing income *m*

Recall: farmer, “endowed” with an initial bundle that usually is not optimal given its money value.

Buying and Selling: Next step toward a more realistic treatment.

More broadly: Consumers earn their income by e.g. working, producing and selling their assets in markets.

Thus, income is also determined “by the market.”

Consumer has an endowment of goods that are priced by the market price.

Endowment, net and gross demands

E.g. A farmer goes to market with carrots and potatoes.

$$\omega_1, \omega_2$$

Net demand:

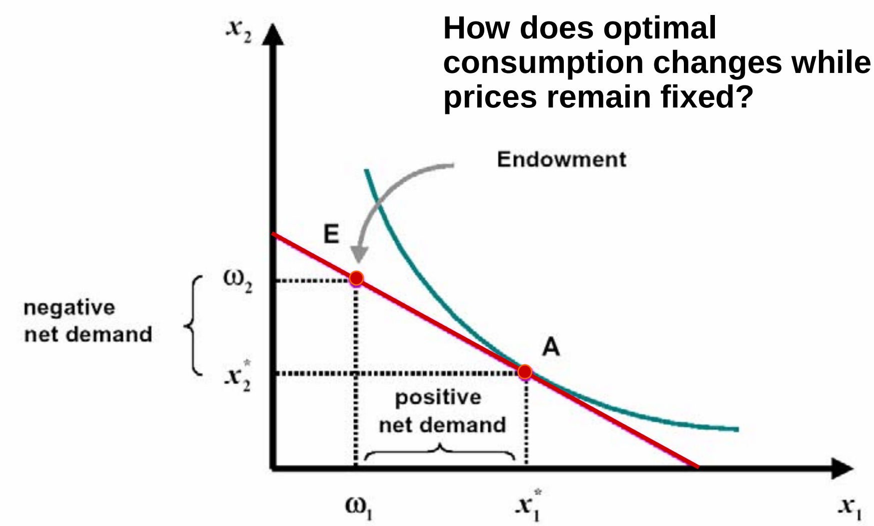
$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2 \quad \Leftrightarrow$$
$$p_1(\underbrace{x_1 - \omega_1}_{\text{net demand}}) + p_2(\underbrace{x_2 - \omega_2}_{\text{net demand}}) = 0$$

Signs: positive or negative, one must be negative.

Positive: consumer is a net buyer (net demander)
Negative: consumer is a net seller (net supplier).

Endowment

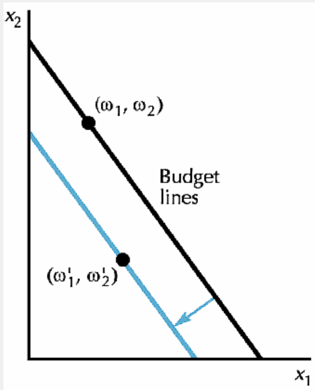
How does optimal consumption changes while prices remain fixed?

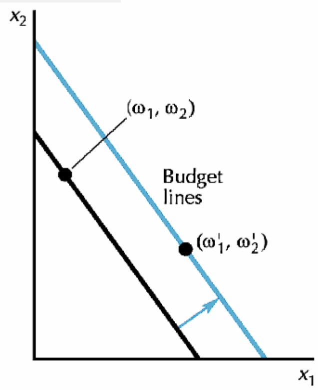


Changing the endowment

We now ask: How does optimal consumption change as the endowment changes?

Assume first that prices are fixed: $p_1\omega_1 + p_2\omega_2 \gtrless p_1\omega_1' + p_2\omega_2'$



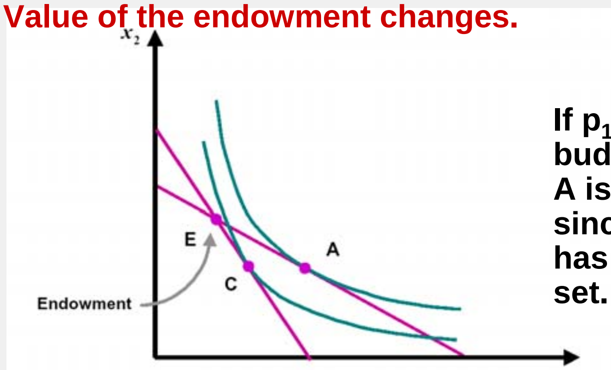


Changing the endowment II

Now price changes: $p_1\omega_1 + p_2\omega_2 \gtrless p_1'\omega_1 + p_2'\omega_2$

A price change implies that the budget line pivots:

Value of the endowment changes.



If p_1 increases, the budget line pivots in E. A is no longer affordable, since the price increase has reduced the budget set. => C.

Remember from this graph: At the end, you need to apply the new prices also to E !!

Changing the endowment: theory

- Since we had a price change, moving from A to C involves again both a substitution and an income effect:

- Recall that we treated the income effect as an implicit change of income following the fact that any price increase decreases purchasing power and vice versa ("holding money income fixed").

- Additionally to this known ordinary income effect we now get a second income effect, the endowment income effect.

This endowment income effect is the product of

(1) the change in demand divided by change in income
times
(2) the change in income due to the price change

Slutsky and Endowment – Ch. 9.6 p. 196

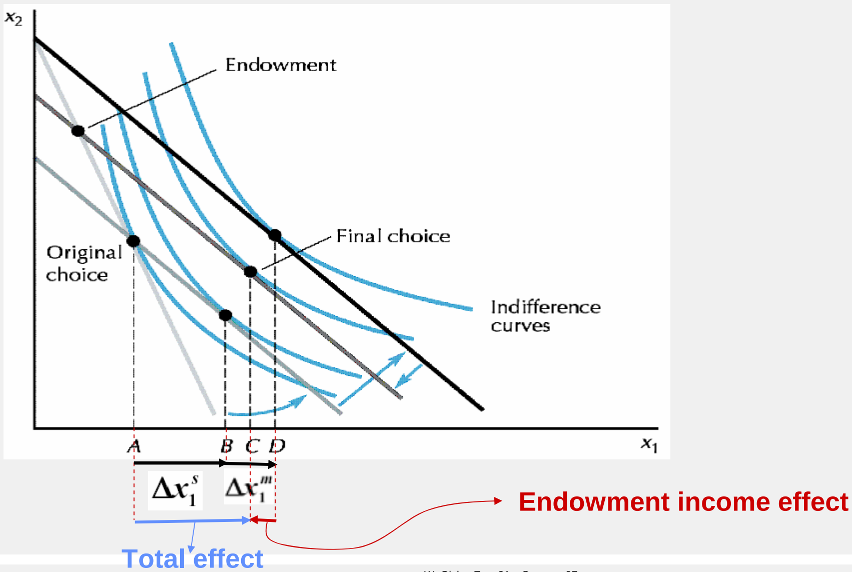
$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1 + \underbrace{\frac{\Delta x_1^m}{\Delta m}\frac{\Delta m}{\Delta p_1}}_{\text{Endowment income effect}} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1 + \underbrace{\frac{\Delta x_1^m}{\Delta m}\omega_1}_{\text{Endowment income effect}}$$

Thus:

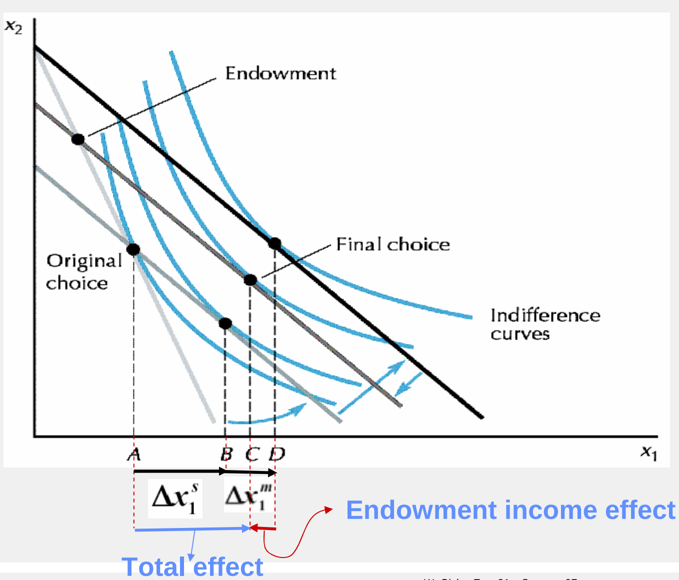
$$\underbrace{\frac{\Delta x_1}{\Delta p_1}}_{?} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{-} + \underbrace{(\omega_1 - x_1)}_{?}\underbrace{\frac{\Delta x_1^m}{\Delta m}}_{+} \quad \text{or} \quad \underbrace{\frac{\partial x_1}{\partial p_1}}_{?} = \underbrace{\frac{\partial x_1^s}{\partial p_1}}_{-} + \underbrace{(\omega_1 - x_1)}_{?}\underbrace{\frac{\partial x_1^m}{\partial m}}_{+}$$

Note that the signs differ (we don't have inferior goods because if Cobb-Douglas): Sign of EIE depends on whether the consumer is a net demander or a net supplier of the good in question:

Graph:



Graph:



Endowment point defined an income m according to the black and the steep grey budget line.
Price of good 1 decreases (flatter budget line) and B is consumed (substitution effect).
The price decrease harms the consumer once: Endowment bundle contains good 1 (EIE), but benefits her as well (OIE).

Signs and results:

- If the consumer is net demander and we observe a normal good, a price increase leads to a reduced demand. (WARP)

(that is: we can tell from just checking the Slutsky equation)

- If the consumer is net supplier and we observe a normal good, she may react with an increase in demand, if the endowment income effect is large enough.

Reason: The endowment income effect outweighs or overcompensates the ordinary income effect.

And we need to compute the magnitude of the two effects.

8.5 More about the Rates of Change

Recall from Ch.9 that the Slutsky equation is not only expressed as

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

but holds as well as a rate:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

Recall also that in the new form including buying and selling, Slutsky equation reads:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1 + \frac{\Delta x_1^m}{\Delta m} \frac{\Delta m}{\Delta p_1}$$

Details

Q: Where does the

$$-\frac{\Delta x_1^m}{\Delta m} x_1$$

term comes from?

Trick 1: Recall that the ordinary income effect is the "movement from B to C," or:

$$\Delta x_1^n = x_1(m, p_1') - x_1(m', p_1'),$$
$$= x_1^C - x_1^B$$

Instead of adding the income effect we subtract the negative of it:

$$\Delta x_1^m = x_1(m', p_1') - x_1(m, p_1')$$
$$= x_1^B - x_1^C$$

Deriving the Slutsky Identity (rates of change)

We now can rewrite Slutsky as

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

Trick 2: How did we find the "Slutsky compensation"?

$$\Delta m = \Delta p_1 x_1$$

Solving for the price difference permits to find the new version:

$$\Delta p_1 = \frac{\Delta m}{x_1}$$

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1$$

Ch. 9.8 Labor Supply: Notation

M	Consumer's nonlabor income
C	Consumtion
w	Wage rate
L	Labor supply
$\overline{L}$	Maximum amount of labor supply
$pC = M + wL$	budget constraint

Labor Supply

Modifying the budget constraint:

$$pC = M + wL \quad \Leftrightarrow \quad pC - wL = M \quad \Leftrightarrow \quad pC + w(\overline{L} - L) = M + w\overline{L}$$

- pC: value paid for consumption
- $w(\overline{L} - L)$: value paid for leisure
- M: nonlabor income
- $w\overline{L}$: value of total time (endowment)

$\overline{C} := \dfrac{M}{p}$ nonlabor consumption (consumption endowment)

$$pC + w(\overline{L} - L) = p\overline{C} + w\overline{L}$$

$R := \overline{L} - L$ Relaxation

$$pC + wR = p\overline{C} + w\overline{L}$$

9.9 The Comparative Statics of Labor Supply

Other view: We now have reached a setup equivalent to a household endowed with two consumption goods:

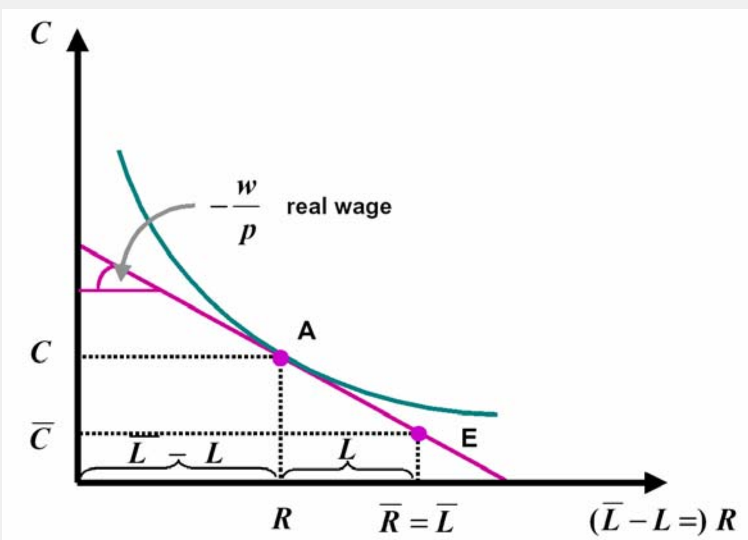
$$\underbrace{pC + wR}_{"p_1 x_1 + p_2 x_2"} = \underbrace{p\bar{C} + w\bar{R}}_{"p_1 \omega_1 + p_2 \omega_2"}$$

Right-hand side: Your implicit income.

From this endowment, the individual under consideration draws an income, according to the market price. <u>But the labor-leisure decision hits: Are you the cool Reggae type staying on Barbados forever or does your "per hour value" hit you once you know it?</u>

Essential thinking: (Real) wage can be interpreted as the opportunity cost of leisure: Price of an extra hour reduces your work income..

The Comparative Statics of Labor Supply



Comparative Statics: Changing M or w

Consider a change of *M*: *("Grandma gives you more money")*

Changing *M* implies a change in the endowment of good 2:
If *R* is a normal good, an increase in M leads to an increase in *R* and to a decrease in L. => Endowment point shifts upward

Consider a change of the wage rate *w*: *(your boss pays you more)*

Changing *w* implies that leisure becomes more expensive. If again leisure is a normal good, an increase in the wage rate *w* will usually lead to a decrease in the demand for leisure.
=> Budget line pivots turns clockwise in E…

Comparative Statics

Q: Does more wage always means more work?
No, because of the endowment income effect:

If the wage rate *w* changes, there must be a change in money income: endowment income effect occurs on top of the ordinary income effect

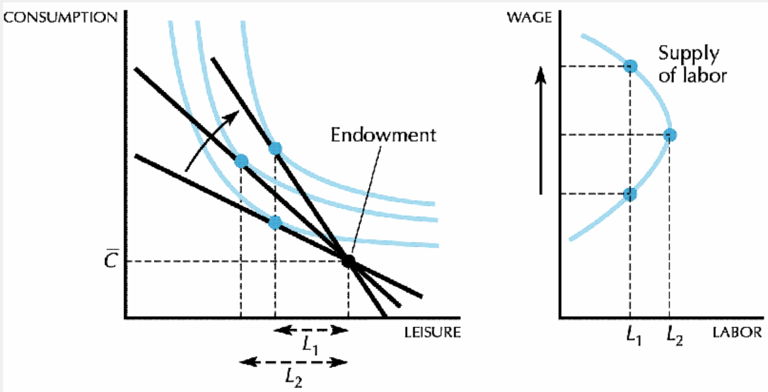
$$\underbrace{\frac{\Delta R}{\Delta w}}_{?} = \underbrace{\frac{\Delta R^s}{\Delta w}}_{-} + \underbrace{(\bar{R} - R)}_{+}\underbrace{\frac{\Delta R^m}{\Delta m}}_{+}$$

$\frac{\Delta R^s}{\Delta w}$ **always negative.**

$\frac{\Delta R^m}{\Delta m}$ **always positive. Well, except if you're a workoholic..**

Result: Maybe backward bending Labor Supply

Slutsky equation tells: At a high $\bar{R} - R$, when supply of labor is large, an **increase in the wage** rate results in a **decrease in the supply of labor.**



A Indifference curves
B Labor supply curve

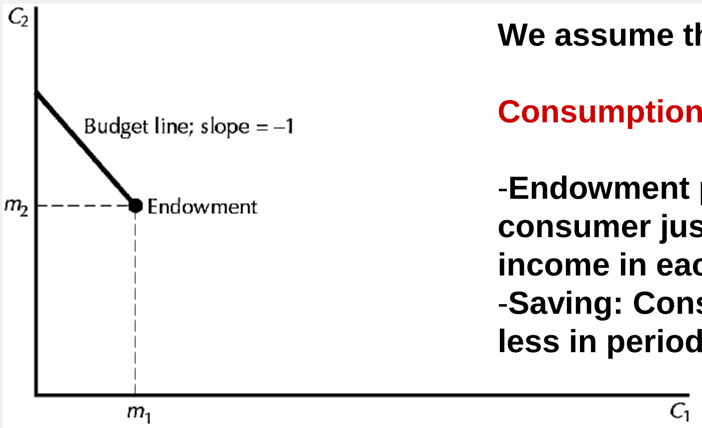
Ch 10: Intertemporal Choice

- What's behind?
  How do consumers allocate their consumption over time?
- Explaining this we have to show why consumers
  borrow (consume more today than tomorrow)
  save (consume less than their today's endowment.
- Intuition: There are preferences over "timed consumption."

Setting: 2 periods *t = 1,2.* We denote consumption by

c_1 consumption in period 1
c_2 consumption in period 2.

First Benchmark: No borrowing possible



We assume that $p_1=p_2=1$.

Consumption possibilities:

-Endowment point: (m_1,m_2): The consumer just consumes his income in each period.
-Saving: Consumer consumes less in period 1, thus $c_1 < m_1$.

Extended model: Interest rates and saving

Intuition: Nonzero interest rate and consumer can borrow and lend money at this interest rate *r*.

- **At any positive interest rate, $c_1 < m_1$ implies that saving permits the consumer to consume in the next period:**

$$c_2 = \underbrace{m_2}_{\text{second-period income}} + \underbrace{(m_1 - c_1)}_{\text{amount saved}} + \underbrace{r(m_1 - c_1)}_{\text{interest payment for amount saved.}}$$

or $c_2 = m_2 + (1+r)(m_1 - c_1)$.

Interest rates and borrowing

What if the consumer is a borrower?

>>> Happens only if $c_1 > m_1$. New budget constraint:

$$c_2 = \underbrace{m_2}_{\substack{\text{second-}\\\text{period}\\\text{income}}} - \underbrace{(c_1 - m_1)}_{\substack{\text{amount}\\\text{borrowed}}} - \underbrace{r(c_1 - m_1)}_{\substack{\text{interest}\\\text{payment}\\\text{for amount}\\\text{borrowed}}}$$

from which $c_2 = m_2 + (1+r)(m_1 - c_1)$.

This is the same budget constraint as before.

Note also: if $c_1 = m_1$, then necessarily $c_2 = m_2$.

More identities

Rewriting the budget constraint:

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

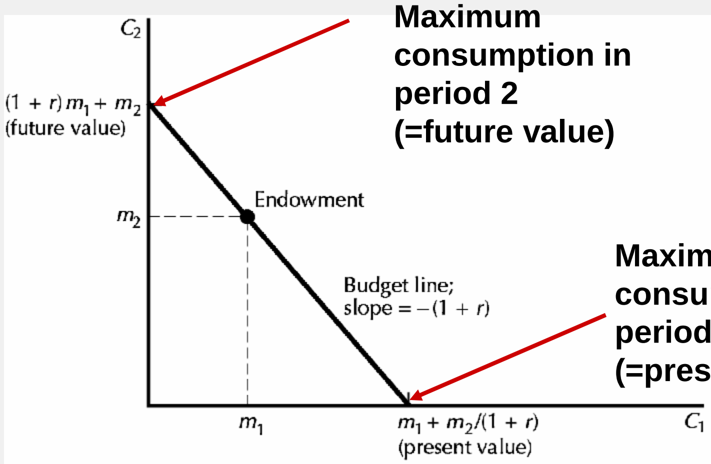
$$c_2 = (1+r)m_1 + m_2 - (1+r)c_1$$

$$\underbrace{(1+r)c_1 + c_2}_{\substack{\text{future value}\\\text{of consumption}}} = \underbrace{(1+r)m_1 + m_2}_{\substack{\text{future value}\\\text{of endowment}}}$$

or, dividing by $(1+r)$:

$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\substack{\text{present value}\\\text{of consumption}}} = \underbrace{m_1 + \frac{m_2}{1+r}}_{\substack{\text{present value}\\\text{of endowment}}}$$

Present and future values



Maximum consumption in period 2 (=future value)

Maximum consumption in period 1 (=present value)

10.2. Preferences for Consumption

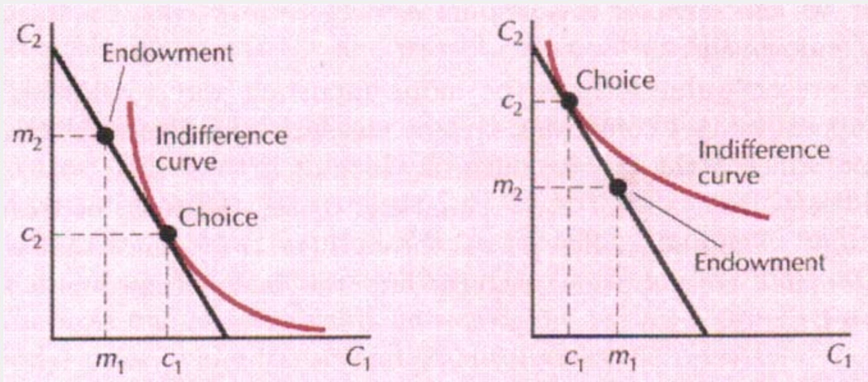
General idea:

- Perfect substitutes

- Perfect complements

-Convexity of preferences: consumers try to "average" between periods than choosing extreme combinations.

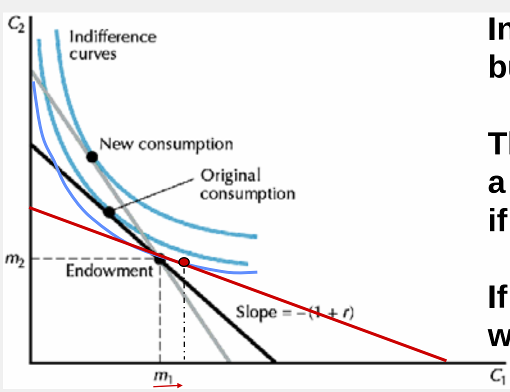
10.3. Comparative statics



Borrower (left) vs. Lender: From $c_2 = m_2 + (1+r)(m_1 - c_1)$ **we see**

1. **Reducing c_1 by a given amount yields a relatively higher c_2.**
2. **Endowment remains affordable: Pivoting around E.**

10.4. Slutsky equation: Person is a lender



Increasing *r* clockwise pivots the budget line around E.

Thus, a person that is initially a lender, will remain a lender if *r* increases.

If *r* decreases, the person may want to switch.

There are 2 questions:

1. **Could it be that if *r* changes, the individual would like to switch from being a lender to become a borrower (or vice versa?**

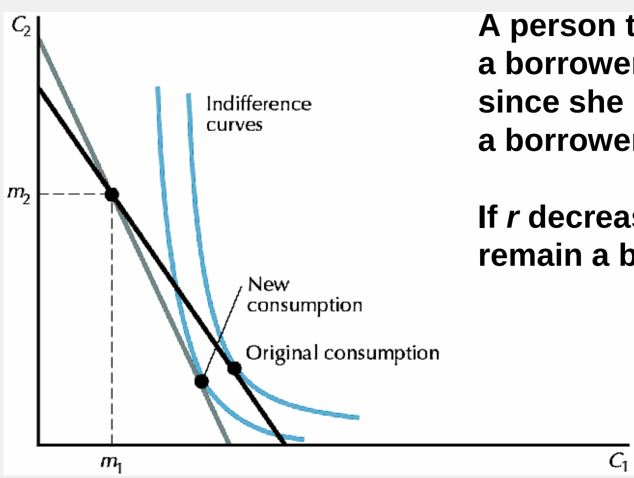
10.4. Slutsky equation of intertemporal choice

$$\underbrace{\frac{\Delta c_1^t}{\Delta p_1}}_{\text{total effect (ambiguous)}} = \underbrace{\frac{\Delta c_1^s}{\Delta p_1}}_{\text{always negative (opposed to the original effect)}} + \underbrace{(m_1 - c_1)}_{\text{opposite signs: positive if lender, negative if borrower.}} \underbrace{\frac{\Delta c_1^m}{\Delta m}}_{\text{always positive}}.$$

2. **Could it be that if *r* changes, the individual would like to decrease or increase present consumption (good 1)?**

Intuition: If term right of "+" is negative, then only sign matters, otherwise we need to know the magnitude.

10.4. Slutsky equation: Person is a borrower



A person that is initially a borrower, may want to switch since she is worse off remaining a borrower if *r* increases.

If *r* decreases, the person will remain a borrower.