Ch 5: Choice

Consider the continuous utility function

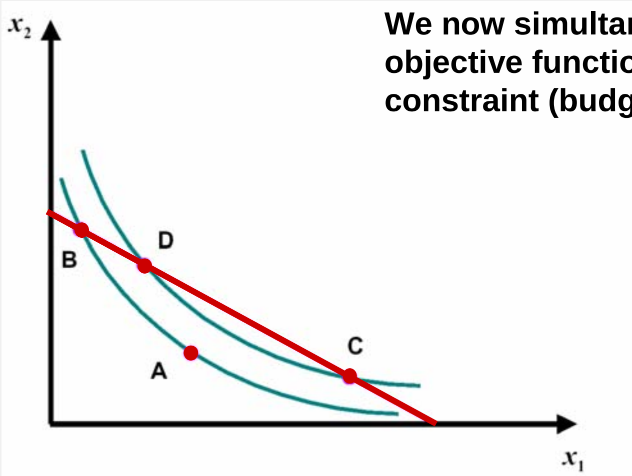
$$u(x_1, x_2(x_1)) = \overline{u}$$

By using the concept of indifference curves we express consumption of Good 2 as function of Good 1. Thus:

$$\frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2}\frac{dx_2}{dx_1} = 0 \quad \Leftrightarrow$$

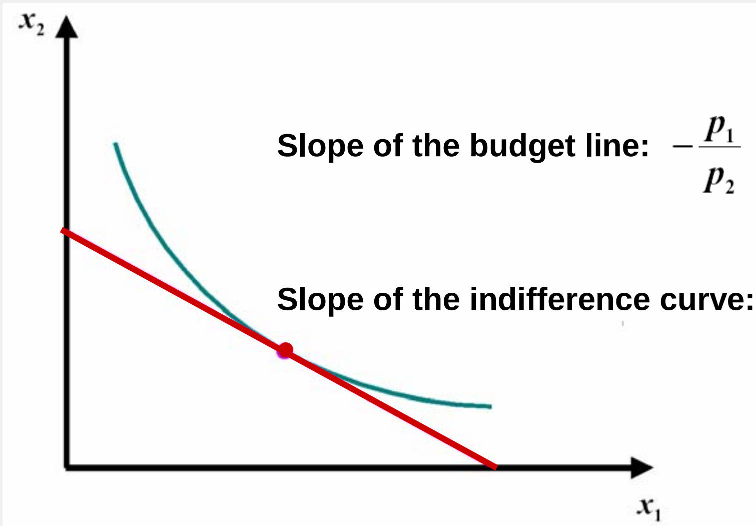
$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2} = -\frac{MU_1}{MU_2}$$

Ch 5: Choice



We now simultaneously consider the objective function (utility) plus the constraint (budget).

Ch 5: Choice

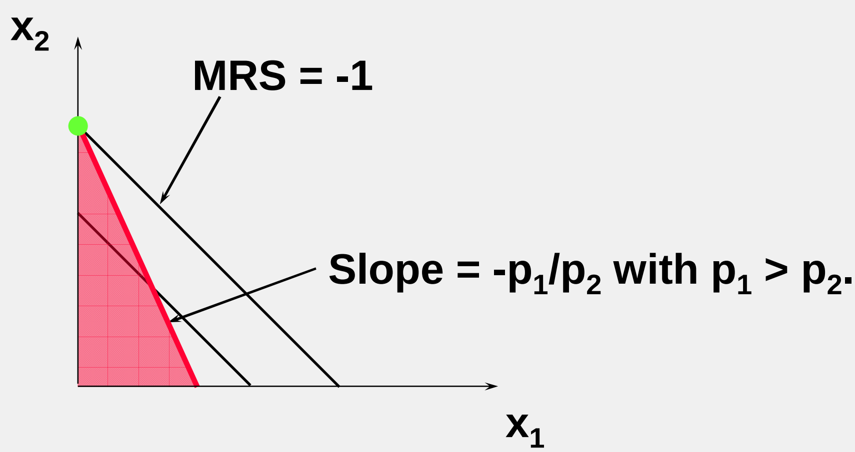


Slope of the budget line: $-\frac{p_1}{p_2}$

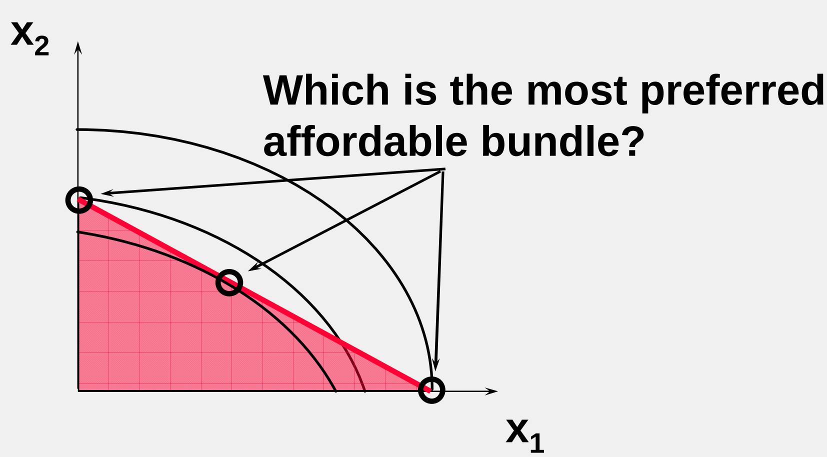
Slope of the indifference curve: $-\frac{MU_1}{MU_2} = MRS$

Perfect complements

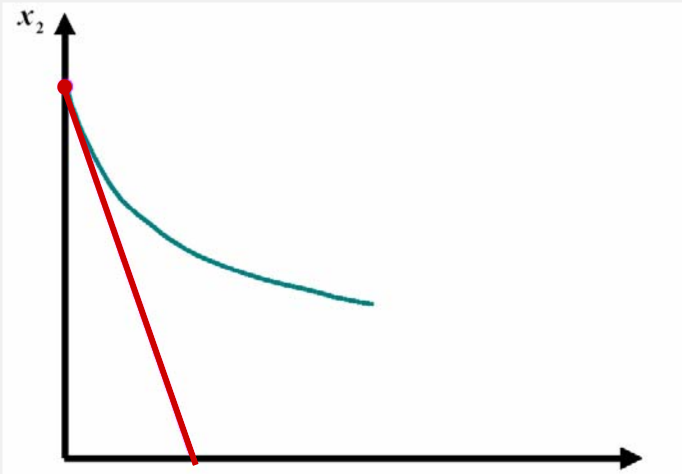
Corner solutions



Corner solutions under concave preferences



Quasilinear preferences



Conditions...

Assume an everywhere differentiable utility function (no kinks) and an internal solution (no corner solution):

Then, a tangent solution is a necessary condition for the optimum.

Since, in addition, preferences are convex,

the tangency condition is both necessary and sufficient.

Why Rational People Think At The Margin

What does this condition imply?

$$MRS = -\frac{p_1}{p_2} \left(= \frac{\text{quantity of good 2}}{\text{quantity of good 1}} \right)$$

Willingness <u>of the consumer</u> to trade good 1 for good 2

Willingness <u>of the market</u> to trade good 1 for good 2

Why Rational People Think At The Margin...

Assume $|MRS| > \frac{p_1}{p_2}$.

This implies

$$\frac{\Delta x_2^i}{\Delta x_1^i} > \frac{\Delta x_2^m}{\Delta x_1^m} \qquad \left(\frac{dx_2^i}{dx_1^i} > \frac{dx_2^m}{dx_1^m} \right)$$

⇒ Indifference curve and budget line intersect. At this point, the indifference curve is steeper.

Why Rational People Think At The Margin...

Consequence of:

$$MRS = -\frac{p_1}{p_2}\left(= \frac{\text{quantity of good 2}}{\text{quantity of good 1}}\right)$$

If all consumers face the same prices, they reach the same MRS in their optimum.

Finding the optimum I: Plugging in relative prices

$$\max_{x_1, x_2} \quad u(x_1, x_2) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

$$\Leftrightarrow \max_{x_1, x_2} \quad u(x_1, x_2(x_1)) \;=\; \max_{x_1, x_2} \quad u\left(x_1, \frac{m}{p_2} - \frac{p_1}{p_2} x_1\right)$$

$$\Rightarrow \frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2}\frac{dx_2}{dx_1} = \frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2}\left(-\frac{p_1}{p_2}\right) \overset{!}{=} 0$$

$$\Leftrightarrow \frac{\partial u / \partial x_1}{\partial u / \partial x_2} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

Continued..

$$\max_{x_1, x_2} \quad x_1 x_2 \quad \text{s.t.} \quad 3x_1 + 2x_2 = 60$$

$$\Leftrightarrow \quad \max_{x_1, x_2} \quad x_1 x_2(x_1) \quad = \quad \max_{x_1, x_2} \quad x_1\left(\frac{60}{2} - \frac{3}{2}x_1\right)$$

$$\Rightarrow \quad \frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2}\frac{dx_2}{dx_1} = x_2 + x_1\left(-\frac{3}{2}\right) \overset{!}{=} 0$$

$$\Leftrightarrow \quad \frac{x_2}{x_1} = \frac{3}{2} \quad \left(\Leftrightarrow \quad \frac{MU_1}{MU_2} = \frac{3}{2}\right)$$

Optimum and MRS

$$x_2 = \frac{3}{2}x_1 \quad \Rightarrow \quad 3x_1 + 2\frac{3}{2}x_1 = 60$$

$$\Rightarrow \quad x_1(3, 2, 60) = 10, \quad x_2(3, 2, 60) = 15$$

$$u(10,15) = 150 \quad \Rightarrow \quad x_2 = \frac{150}{x_1} \quad \Rightarrow \quad MRS(10,15) = -\frac{150}{x_1^2} = -\frac{150}{100} = -\frac{3}{2}$$

$$MU_1(10,15) = 15, \quad MU_2(10,15) = 10 \quad \Rightarrow \quad \frac{MU_1}{MU_2} = \frac{3}{2}$$

Lagrange method - constrained maximization

$$\max_{x_1, x_2} \quad x_1 x_2 \qquad \text{s.t.} \qquad 3x_1 + 2x_2 = 60$$

$$L(x_1, x_2, x_3) = x_1 x_2 - \lambda(3x_1 + 2x_2 - 60)$$

$$\Rightarrow \quad \frac{\partial L}{\partial x_1} = x_2 - 3\lambda \overset{!}{=} 0, \qquad \frac{\partial L}{\partial x_2} = x_1 - 2\lambda \overset{!}{=} 0, \qquad \frac{\partial L}{\partial \lambda} = 3x_1 + 2x_2 - 60 \overset{!}{=} 0$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \frac{3}{2} \quad \left(\Leftrightarrow \quad \frac{MU_1}{MU_2} = \frac{3}{2} \right)$$

$$\Rightarrow \quad x_1(3, 2, 60) = 10, \quad x_2(3, 2, 60) = 15$$

Lagrange method – why it is the same

$$\max_{x_1, x_2} \quad u(x_1, x_2) \qquad \text{s.t.} \qquad p_1 x_1 + p_2 x_2 = m$$

$$\Rightarrow \quad L = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

$$\Rightarrow \quad \frac{\partial L}{\partial x_1} = \frac{\partial u}{\partial x_1} - \lambda p_1 \overset{!}{=} 0 \qquad \frac{\partial L}{\partial x_2} = \frac{\partial u}{\partial x_2} - \lambda p_2 \overset{!}{=} 0$$

$$\Leftrightarrow \quad \frac{\partial u / \partial x_1}{\partial u / \partial x_2} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

Digression: The Cobb-Douglas Consumer

$$x_1 = \frac{c}{c+d}\frac{m}{p_1},\ x_2 = \frac{c}{c+d}\frac{m}{p_2}$$

Property of Cobb-Douglas preferences: What fraction of the income does a consumer spend on good 1?

Easy: $\frac{p_1 x_1}{m}$.

What fraction does the Cobb-Douglas consumer spend?

$$\frac{p_1 x_1}{m} = \frac{p_1}{m} x_1 \Leftrightarrow x_1 = \frac{p_1}{m}\frac{c}{c+d}\frac{m}{p_1} = \frac{c}{c+d}, \text{ for good 2:} \frac{p_2 x_2}{m} = \frac{d}{c+d}.$$

Size of this <u>fixed fraction of income</u> spent per good depends on the exponents in the Cobb-Douglas utility function. Note:

$x_1^c x_2^d \Leftrightarrow$ can be transformed into $x_1^a x_2^{1-a}$ for $a = \frac{c}{c+d}$