Ch 3: Preferences

Intuition

In Ch. 2 we studied which consumption bundles are **affordable.**

In this chapter, we want to know how consumers rank bundles if faced with choices among bundles.

Any combination in the $x_1 - x_2$ space is a **consumption bundle.**

Preferences: Behavior

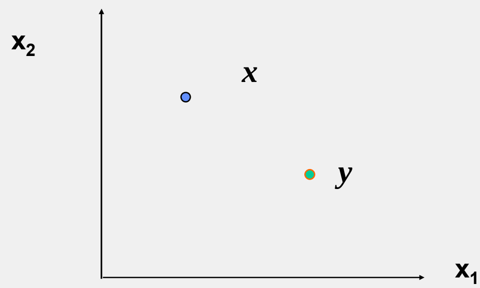
Behavioral Postulate:

A decision maker always chooses its **most preferred** alternative from its set of available alternatives.

=> To model a decision maker's choice we need to know her preferences.

Preference Relations

Why? We want to build a theory of individual values and thus individual choice that puts the least possible restrictions on validity and nevertheless has some "bite" in that it permits predictions.



Comparing two different consumption bundles, x and y:

- **strict preference:** x **is more preferred than** y.
- **weak preference:** x **is as at least as preferred as** y.
- **indifference:** x **is exactly as preferred as** y.

Preference Relations

- **Strict preference, weak preference and indifference are all preference relations.**
- **Particularly, they are ordinal relations; i.e. they state only the order in which bundles are preferred.**

Preference Relations

$\succ$ **denotes strict preference;**

x $\succ$ y means that bundle x is preferred strictly to bundle y.

~ denotes indifference;

x ~ y means x and y are equally preferred.

$\succsim$ **denotes weak preference;**

x $\succsim$ y means x is preferred at least as much as is y.

Assumptions about Preference Relations

1. Completeness:

For any two bundles x and y it is always possible to make the statement that either

$$x \succsim y$$

or

$$y \succsim x.$$

Assumptions about Preference Relations

2. Reflexivity:

A bundle x is always at least as preferred as itself; i.e.

$$x \succsim x.$$

Assumptions about Preference Relations

3. Transitivity:

If x is at least as preferred as y, and
y is at least as preferred as z, then
x is at least as preferred as z; i.e.

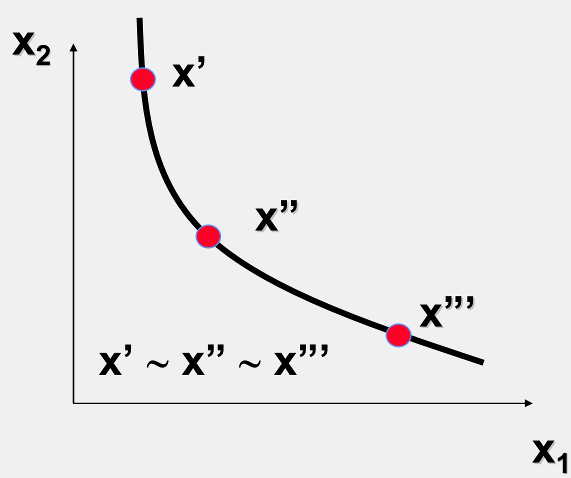
$$x \succsim y \text{ and } y \succsim z \Rightarrow x \succsim z.$$

Indifference Curves

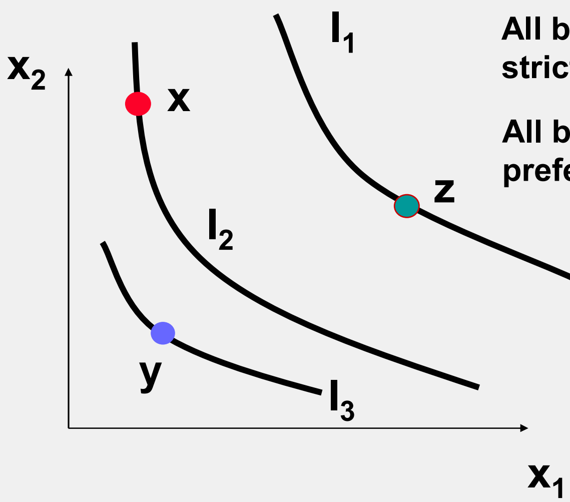
Take a reference bundle x'.

The set of all bundles equally preferred to x' is the indifference curve containing x'; the set of all bundles y ~ x'.

Indifference Curves



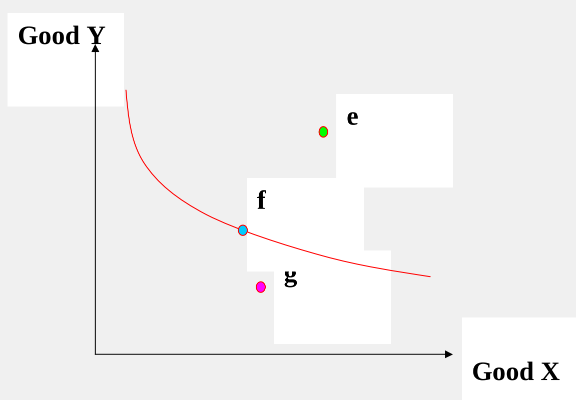
Indifference Curves



All bundles in I_1 are strictly preferred to all in I_2.

All bundles in I_2 are strictly preferred to all in I_3.

Example: What we can tell from the graph



True of false?

$f \succ e$

$f \prec e$

$g \succ e$

$g \prec e$

$g \prec f$

$g \succ f$

$f \preceq e$

$f \succeq e$

$g \succeq e$

$g \preceq e$

$g \preceq f$

$g \succeq f$

Preference Relations are not independent

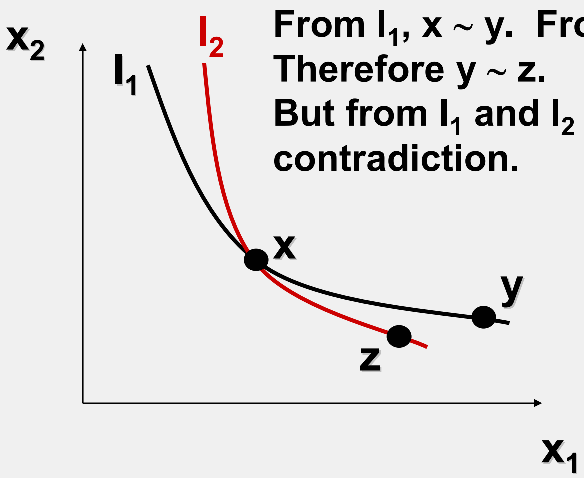
1. $x \succsim y$ and $y \succsim x$ imply $x \sim y$. Or:

$$\neg x \prec y \;\wedge\; \neg y \prec x \Rightarrow x \sim y$$

2. $x \succsim y$ and (not $y \succsim x$) imply $x \succ y$. Or:

$$x \succsim y \;\wedge\; x \succ y \Rightarrow x \succ y$$

Indifference Curves Cannot Intersect



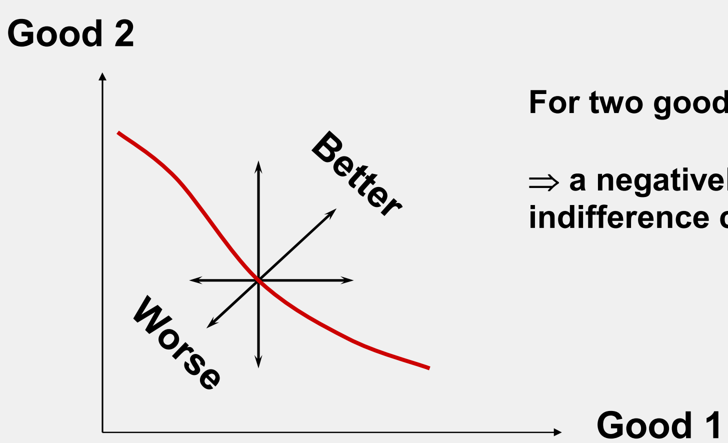
From I_1, $x \sim y$. From I_2, $x \sim z$. Therefore $y \sim z$. But from I_1 and I_2 we see $y \succ z$, a contradiction.

Slopes of Indifference Curves

When more of a commodity is always preferred, the commodity is a good.

If every commodity is a good then indifference curves are negatively sloped.

Slopes of Indifference Curves

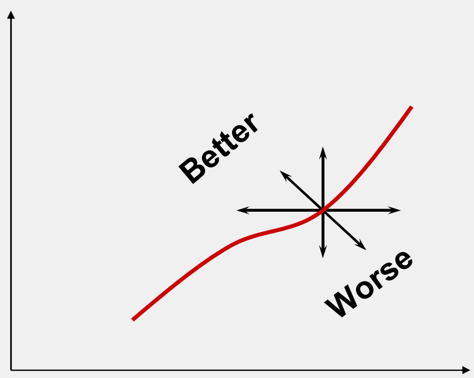


For two goods:

⇒ a negatively sloped indifference curve.

Slopes of Indifference Curves

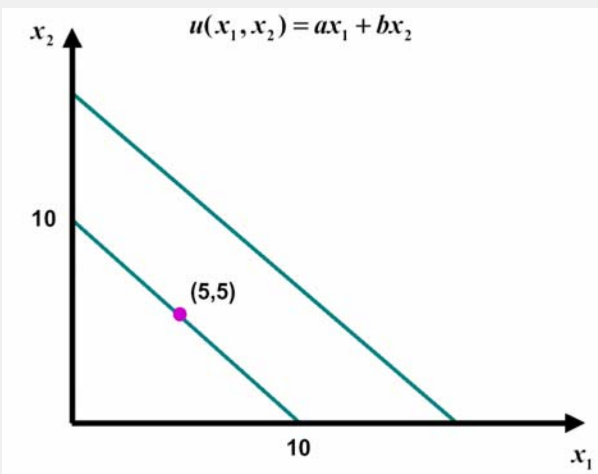
If less of a commodity is always preferred then the commodity is a **bad**.



One good and one bad:

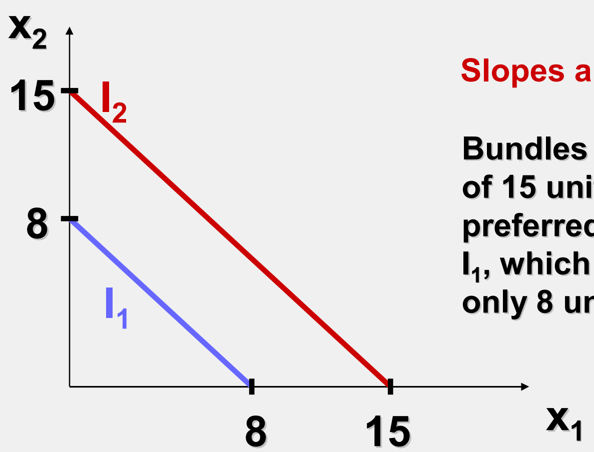
$\Rightarrow$ a positively sloped indifference curve.

Extreme Cases: Perfect Substitutes



*If a consumer always regards units of commodities 1 and 2 as equivalent, then the commodities are **perfect substitutes** and only the total amount of the two commodities in bundles determines their preference rank-order.*

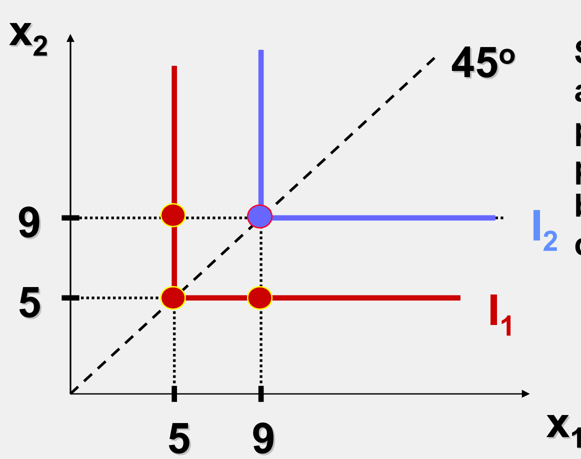
Perfect Substitutes: Example



Slopes are constant at - 1.

Bundles in I_2 all have a total of 15 units and are strictly preferred to all bundles in I_1, which have a total of only 8 units in them.

Particular preferences: Perfect Complements



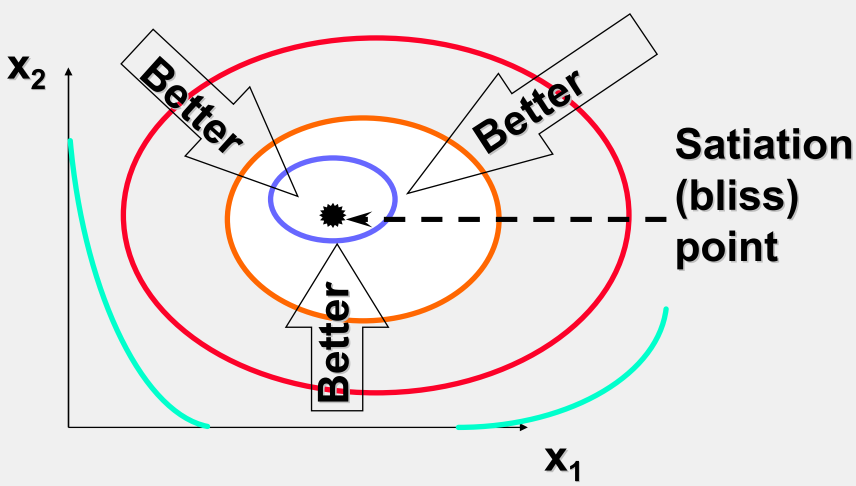
Since each of (5,5), (5,9) and (9,5) contains 5 pairs, each is less preferred than the bundle (9,9) which contains 9 pairs.

Preferences Exhibiting Satiation

A bundle strictly preferred to any other is a satiation point or a bliss point.

What do indifference curves look like for preferences exhibiting satiation?

Satiated Preferences



Digression: Deriving indifference curves

Indifference curves can be seen as a special graph of a utility function:

An indifference curve is the locus of bundles that give the consumer under consideration the same "amount" of utility:

$$u(x_1, x_2) = \bar{u} = const.$$

$$\{(x_1, x_2) | u(x_1, x_2) = \bar{u}\}$$

Indifference curves: example

Assume the utility function

$$u(x_1, x_2) = x_1 x_2$$

Formally, we reach the indifference curve

$$u(x_1, x_2) = \bar{u} = x_1 x_2$$

$$\Rightarrow x_2 = \frac{\bar{u}}{x_1}$$

"Well-Behaved" Preferences

A preference relation is "well-behaved" if it is monotonic and convex.

1 Monotonicity: "More is always better"….

More of any commodity is always preferred (i.e. no satiation and every commodity is a good).

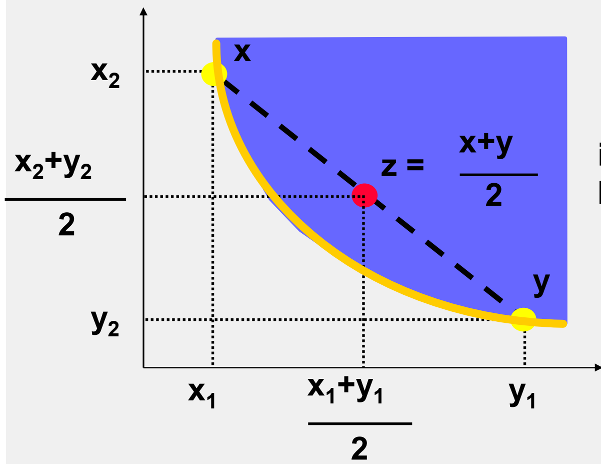
Well-Behaved Preferences

2 Convexity: Mixtures of bundles are (at least weakly) preferred to the bundles themselves.

Take e.g. the 50-50 mixture of the bundles x and y
$z = (0.5)x + (0.5)y$.

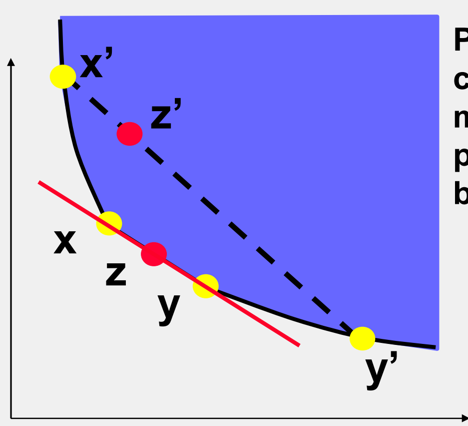
z is at least as preferred as x or y.

Well-Behaved Preferences -- Convexity.



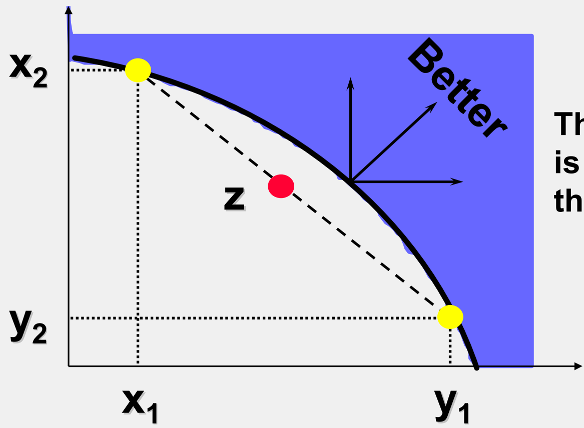
is strictly preferred to both x and y.

Well-Behaved Preferences -- Weak Convexity.



Preferences are weakly convex if at least one mixture z is equally preferred to a component bundle.

Non-Convex Preferences



The mixture z is less preferred than x or y.

Q: Why would we want to exclude non-convex preferences?

Formal definition: Monotonicity

Recall:

More of any commodity is always preferred (i.e. no satiation and every commodity is a good).

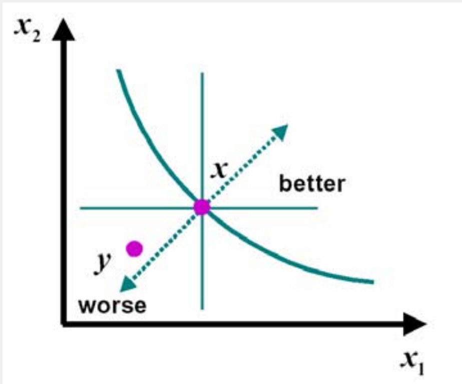
$$(x_1, x_2) \geq (y_1, y_2) \Rightarrow (x_1, x_2) \succsim (y_1, y_2)$$

Hint to understand the L.H.S.: At least one good must be "more" in the bundle..

What follows?

From "more is better" follows:

Indifference curves are downward sloped.



Or: For any given bundle you can draw a vertical and horizontal line. The indifference curve satisfying monotonicity cannot go through the lower left or upper right area

Formal Definition: Convexity of Preferences:

$$(x_1, x_2) \sim (y_1, y_2) \Rightarrow (tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succsim (x_1, x_2),$$
$$for\ all\ t \in [0,1]$$

Intuition: Mixture between the two goods preferred.

Consequence: Indifference curves are "inward bowed".

Motivation: Consumers usually don't make "extreme choices", [and if so, this is not an interesting case for the economist]:

If your choice is between Pizza and Coke, you usually prefer 1 Pizza plus 1 Coke over 0 Pizzas and 2 Cokes or 2 Pizzas and 0 Coke.

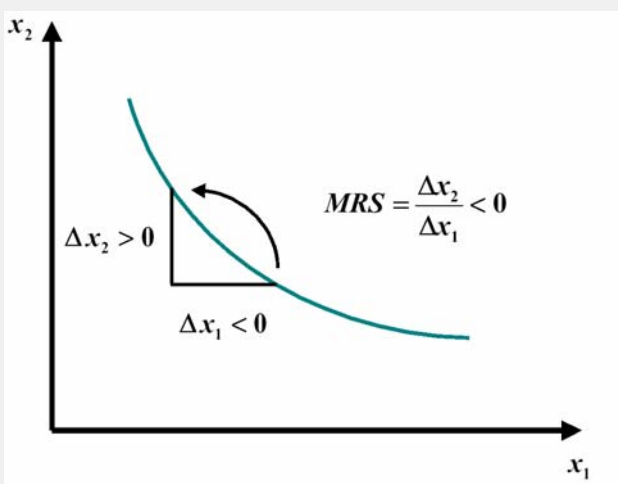
3.6 Marginal Rate of Substitution

The slope of an indifference curve is its **marginal rate-of-substitution** (MRS).

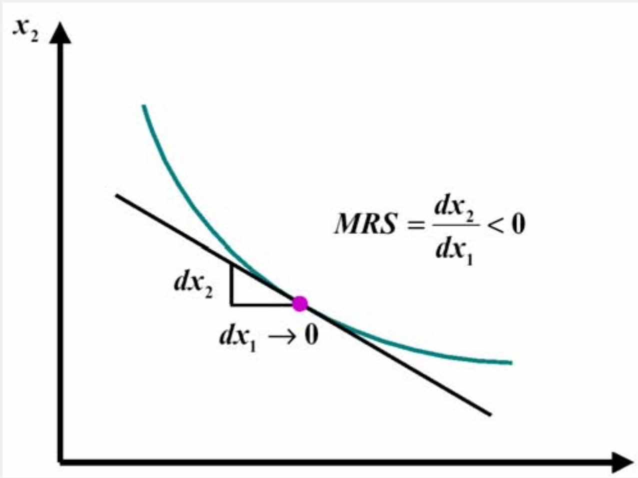
Intuition: How much of Good 2 do we need to offer to a consumer <u>to compensate her for giving up one unit</u> of Good 1, such that she is only just willing to accept (i.e. remaining on her initial level of utility)?

If we consider this rate of exchange for marginal changes, we speak of the **marginal rate-of-substitution** (MRS).

Marginal Rate of Substitution (discrete)



Marginal Rate of Substitution (continuous)



MRS numerically: fix a u and find the slope

$$u(x_1, x_2) = ax_1 + x_2 \quad \Rightarrow \quad \bar{u} = ax_1 + x_2$$
$$\Leftrightarrow \quad x_2 = \bar{u} - ax_1$$
$$\Rightarrow \quad \frac{dx_2}{dx_1} = -a$$

Must be perfect substitutes

$$u(x_1, x_2) = x_1 x_2 \quad \Rightarrow \quad \bar{u} = x_1 x_2$$
$$\Leftrightarrow \quad x_2 = \frac{\bar{u}}{x_1}$$
$$\Rightarrow \quad \frac{dx_2}{dx_1} = -\frac{\bar{u}}{x_1^2}$$

Strictly convex

MRS: Interpretation

♦ MRS is the slope of an indifference curve.

♦ Assuming the indifference curve to be a function $x_1(x_2)$ implies that MRS is the first derivative of this function.

♦ MRS indicates the marginal rate of exchange, for which the consumer is indifferent between exchanging and not.

♦ MRS measures the marginal willingness to pay of the consumer: it indicates how much he is willing to pay in units of Good 2 in order to reach one additional unit of Good 1.

This consideration is particularly important if Good 2 represents "all other goods" with $p_2 = 1$.

MRS: Trading at a given exchange rate

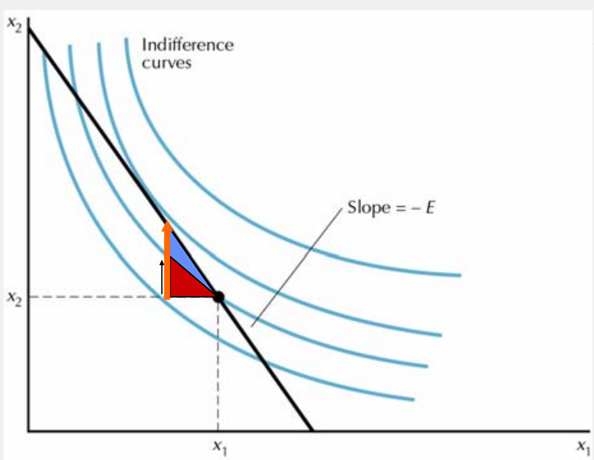


Figure 3.12 Trading at an exchange rate

Assume the rate of exchange is *E.*

If the consumer gives up Δx_1 units of good 1, she can get $E\Delta x_2$ units of good 2 in exchange.

Conversely, if the consumer gives up Δx_2 units of good 2, she can get $\Delta x_1/E$ units of good 1.

When does the consumer want to stay put at (x_1, x_2)?

⇒ When the exchange line is tangent to the indifference curve.

MRS: decreasing for strictly convex preferences

As a consequence following our two assumptions concerning monotonicity and (strong) convexity of preferences: MRS is decreasing with the units of the good consumed.

Interpretation: The more units of a good have already been consumed, the less the good is preferred.