

Posted August 16, presentations in class August 22

** indicates an advanced problem.

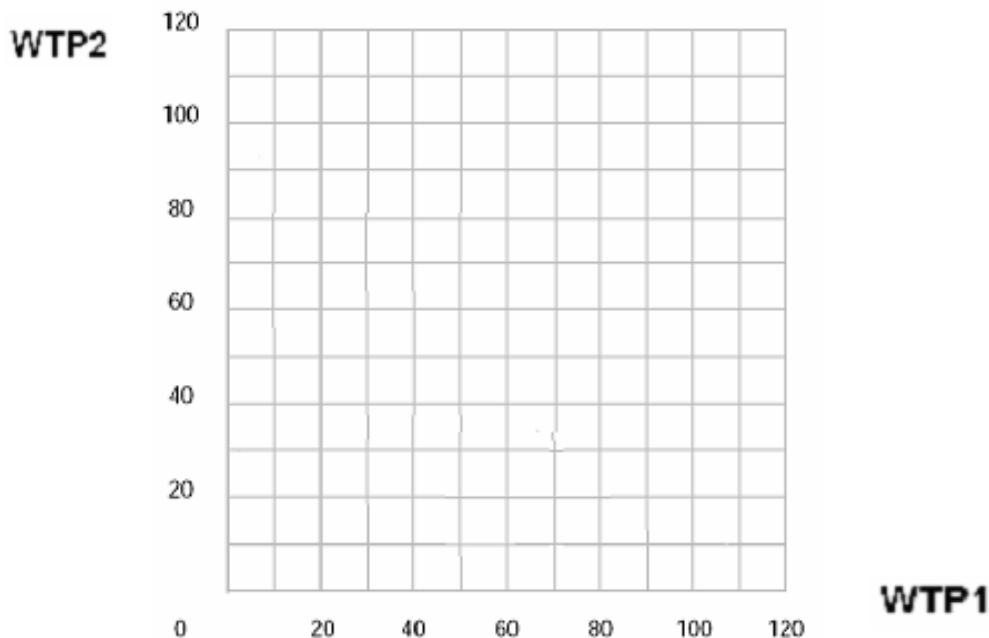
1) A producer sells two goods and knows the willingnesses to pay of five of his consumers for each good. These are independent and given in the table below.

Consumer	Good 1 (\$)	Good 2 (\$)
A	10	80
B	40	70
C	60	60
D	70	40
E	80	10

Consider two scenarios to be analyzed:

- (1) the existence of no marginal cost,
- (2) the marginal cost for producing one unit of each good being 20.

Solve mathematically the optimal strategy for selling separately, pure bundling and mixed bundling. Which pricing strategy is best in each case (1) or (2)? Use the graph below to illustrate. Draw the willingness-to-pay lines for the bundle. Explain your result.



2) As the owner of the only tennis club in an isolated wealthy community, you must decide on membership dues and fees for court time. There are two types of tennis players.

"Serious" players have demand $Q_1 = 10 - p$, where Q_1 is court hours per week and p is the fee per hour for each individual player. There are also "occasional" players with demand $Q_2 = 4 - p$. Assume that there are 1000 players of each type. Because you have plenty of courts, the marginal cost of court time is zero. You have fixed costs of \$10,000 per week. Serious and occasional players look alike, so you must charge them the same prices.

a) Suppose that to maintain a "professional" atmosphere, you want to limit membership to serious players. How should you set the *annual* membership dues and court fees (assume 52 weeks per year) to maximize profits, keeping in mind the constraint that only serious players choose to join? What would the profits be (per week)?

b) A friend tells you that you could make greater profit by encouraging both types of players to join and charging them a two-part tariff. Is your friend right? What annual dues and court fees would maximize your weekly profits? What would these profits be? What would the entrance/membership fee be per week and per year? Use the graph below to illustrate. Label the axes.



c) Suppose that over the years, more occasional players move to your community. There are now 1000 serious players and 4000 occasional players. Find the profit-maximizing entrance fees and court fees. What would profits be per week? Compare the result to (b) and explain.

d) Last, imagine you want to charge a membership fee only and use second-degree price-discrimination. To make this possible we assume that you indeed could observe the amount of round played by each type and stop them when they want to play more. You now offer two packages according to each type's maximum willingness to play. Find profit-maximizing price (membership due) and total profit in this situation. For this part, assume again 1000 players of each type. Use the graph below to illustrate the two different entrance fees. Label the axes.



3) A monopolist faces a linear demand of $Q=20-P$. He has a cost function of $15+5Q$.

a) Find his revenue function and his marginal revenue function $TR(Q)$ and $MR(Q)$. Find his profit maximizing quantity, price P^* and monopoly profit, and the Lerner index with using P^* .

b) Draw demand, marginal revenue, marginal cost and average cost. Mark the areas that illustrate total cost, profit, consumer surplus, and deadweight loss in the chart. You need to have 4-5 value pairs to draw the average cost function. Label the axes.

c) The monopolist's R&D department finds a new process innovation that cuts marginal cost in half. Find the new situation (profit maximizing quantity, price and monopoly profit, Lerner index in the profit maximum, and draw demand, marginal revenue, marginal cost and average cost – again with some value pairs - into the graph below). Explain the changes compared to the situation in b). Graph the result in the next graph, including the new average cost function.

d) Assume the original cost function. Now assume that government wants to reduce the deadweight loss to zero and regulates the monopolist. What quantity would it ask the

monopolist to produce in this case? Find quantity, price, Lerner index and illustrate the monopolist's loss in the graph below. You need to again graph the average cost curve to answer the question.

4) Your firm produces two products, the demands for which are independent. Both products are produced at zero marginal cost. You face four consumers (or groups of consumers) with the following reservation prices:

Consumer	Good 1 (\$)	Good 2 (\$)
A	10	90
B	70	50
C	80	40
D	90	10

a) Consider three alternative pricing strategies:

- selling the goods separately
- pure bundling
- mixed bundling.

For each strategy, determine the optimal prices to be charged and the resulting profits. Which strategy is the best?

5) Chandler Bing, a former local tennis star, starts a new tennis club and needs to decide on membership dues and fees for court time. There are two types of tennis players. "Serious" players have a demand $Q_1 = 10 - p$, where Q_1 is court hours per week and p is the fee per hour for each individual player. There are also "occasional" players with demand $Q_2 = 5 - \frac{2}{3}p$. Assume that there are 100 players of each type. Marginal cost of court time is zero. Chandler has fixed costs of \$ 5,000 per week.

a) Chandler wants to try out a few pricing schemes. He first thinks that charging membership dues only sounds easy. Assume Chandler can perfectly price discriminate. What would he optimally charge to each type of consumer? What is his total profit using this pricing scheme?

b) Chandler finds out quickly that he cannot perfectly price discriminate. However, he still wants to charge a membership fee only, applying second-degree price-discrimination and using the original demand size (to make this possible we assume that he indeed could observe the amount of round played by each type and stop them when they want to play more...). Find optimal membership dues and total profit in this situation.

c) After a few months, Chandler finally decides to use a two-part tariff, realizing that he is incapable of telling the serious from the occasional players. They look alike, so he must charge them the same prices. Find the optimal court fee and the resulting optimal mem-

bership due for each player. Find total profit and compare the result with the outcome in a) and b). What do you observe? Graph demand, membership due and court fees for each type of player into the two charts below. Mark the intercepts, quantities and price.

6) A monopolist knows that it has four consumers, A, B, C, and D, each interested in buying two goods, 1 and 2. MC of good 1 is \$100, and MC of good 2 is \$ 150. The reservation prices of each consumer are given in the table below.

Consumer	Good 1 (\$)	Good 2 (\$)
A	50	450
B	250	275
C	300	220
D	450	50

- Are the willingnesses to pay perfectly negatively correlated? Why or why not?
- Find out the profit-maximizing prices under separate selling.
- What is the best strategy for pure bundling? Is pure bundling preferable to selling separately?
- Can a mixed bundling strategy do better? Find the best strategy that the monopolist can adopt under mixed bundling.

7) A cable company has two services. One service is the Basic Service channel. The other is the Walt Disney Movie channel. The potential subscribers for the services are listed below, together with their reservation prices. The two goods are not seen as complementary products and thus demand is completely unrelated for each and every consumer.

The marginal cost of each service is \$3. Assume there are equal numbers of consumers in each category.

Reservation Prices for Each Cable Service by Type of Subscriber		
	Basic Service (\$)	The Disney Channel (\$)
Students	5	15
Families	11	9
Hotels	14	6
Schools	4	16
Young Adults	0	17
Pensioners	17	0

- If the services are sold separately and not offered as a bundle, what price should the cable operator set for each service? What profits will it earn? Which consumers will subscribe to which service?
- Suppose that the operator decides to pursue a mixed bundling strategy. What price should be set for the bundled service? What price should be set for each service if pur-

chased individually? Which consumers buy which options, and what are the cable operator's profits?

c) How would your answers to the first two questions change if the marginal cost of producing each service had been \$10 instead of \$3?

**8) Assume that a monopolist knows that customers are of two types, low-demand customers whose inverse demand is $P_L = 12 - Q_L$, and high-demand customers with a demand of $P_H = 16 - Q_H$. However, the monopolist does not know which type is which. Production costs are \$4 per unit.

a) Complete the following table for this example. Explain why the monopolist does not produce more than 12 units.

LOW-DEMAND CUSTOMERS			HIGH-DEMAND CUSTOMERS			
Number of Units in the Package	Charge for the Package*	Profit per Package	Consumer Surplus from Low-Demand Package	Maximum Willingness to Pay for 12 Units	Charge for Package of 12 Units	Profit from Each Package of 12 Units
0	0	0	0	\$120.00		\$72.00
1	\$11.50		\$4.00	\$120.00	\$116.00	
2		\$14.00	\$8.00	\$120.00		\$64.00
3						
4	\$40.00	\$24.00		\$120.00		
5	\$47.50	\$27.50	\$20.00	\$120.00	\$100.00	\$52.00
6	\$54.00			\$120.00		\$48.00
7	\$59.50	\$31.50	\$28.00	\$120.00	\$92.00	\$44.00
8	\$64.00	\$32.00	\$32.00	\$120.00	\$88.00	\$40.00
9						
10	\$70.00	\$30.00	\$40.00	\$120.00		
11						
12	\$72.00		\$48.00	\$120.00	\$72.00	

* This is the low-demand customer's maximum willingness to pay for the number of units in the package.

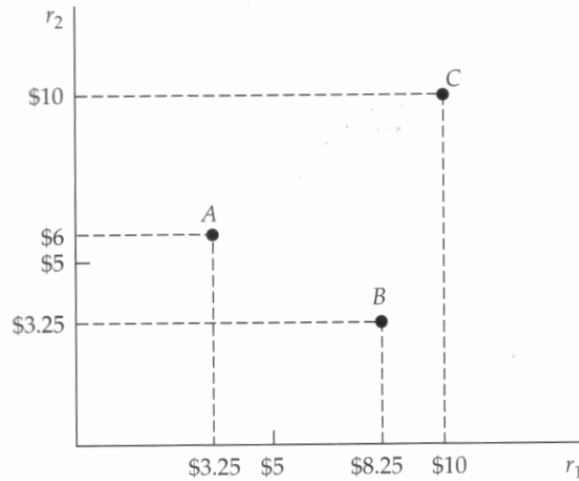
b) Assume that there are the same numbers of high-demand customers and low-demand customers. What is the profit-maximizing number of units that should be offered in the package aimed at the low-demand customers?

c) Now assume that there are twice as many low-demand customers as high-demand customers. What is the profit-maximizing pair of packages for the monopolist?

d) The monopolist is considering offering two packages, one containing six units and the other twelve units. What are the charges at which these packages will be offered? What is

the ratio of high-demand customers to low-demand customers above which it will be better for the monopolist to supply only to the high-demand customers?

**9) In the following graph, the reservation prices r_1 and r_2 for two goods are shown for three consumers, labeled A, B, and C. Consumer A is willing to pay up to \$3.25 for good 1 and up to \$6 for good 2.



Assume that marginal production cost is zero for both goods. Can the producer make the most money by a) selling the goods separately, b) using pure bundling, and c) by using mixed bundling? Find the prices that should be charged.

(Hint: Write down a table with the three consumers, the reservation price for 1, 2, and total, and then a second table with price 1, 2, bundled and the profit, for each selling strategy).

**10) WB #24. 5