**1) A wine seller sells wine of different qualities to a buyer who can have different quality preferences. The buyer may thus of two "types": the “type-2” agent (“gourmet”) who is willing to pay a large sum for a great vintage and the coarse “type-1”agent (“gourmand”) that does not care much about quality.

It is realistic to assume that in reality the seller often cannot perfectly distinguish the two types, although she might have knowledge about the probability of their type. The seller however may keep the two types apart since the type-2 agent is willing to pay more for an increase in quality per unit, compared to the type-1 agent.

The utility of the seller is her profit , namely $\pi = \pi(q,T)$, while the buyer has a utility of $U = U(q,T)$. with

$$\left.\begin{matrix} q_i & \textit{quality} \\ t_i & \textit{transfer} \end{matrix}\right\} \text{ in state } \theta_i \in \{\theta_1,...,\theta_n\}$$

The state θ_i is only observed by the buyer before choosing a bottle (a pair q,T). θ is the type of the buyer , with $\theta = \{\theta_1, \theta_2\}$. We further assume that θ_1 occurs with probability p, while θ_2 with probability $1\text{-}p$. This probability is the seller’s prior beliefs. We furthermore assume that $\theta_2 > \theta_1$.

Under complete contracting the (seller) designs a contract (a “rule” $q(t)$) she offers to the buyer, such that the latter picks the intended bottle. This however requires that the buyer must be able to capture at least the same utility when revealing his type truthfully than when misreporting, namely $U(q(\theta_i,\theta_i)) + T(\theta_i) \geq U(q(\hat{\theta}_i,\theta_i)) + T(\hat{\theta}_i)$.

The buyer has a utility function of $U = \theta q - t$. We assume that for $\theta_2 > \theta_1$ also $u(q,\theta_2) - u(q,\theta_1)$ increases in q. The seller is able to produce wine of any quality $q \in [0,\infty]$, at strictly convex costs of $C = c(q)$. We assume the cost function is twice differentiable. We simplify further $C = cq$ and thus get $\pi = T - cq$.

The seller's maximization problem reads:

$$\max_{T(.),q(.)} p\left[T(\theta_1) - cq(\theta_1)\right] + (1-p)\left[T(\theta_2) - cq(\theta_2)\right]$$
$$s.t.\ \theta_i U(q(\theta_i)) - T(\theta_i) \geq \theta_j U(q(\theta_i)) - T(\theta_i) \qquad \forall i,j \qquad (IC)$$
$$\theta_i U(q(\theta_i)) - T(\theta_i) \geq 0 \qquad \forall i \qquad (IR)$$

(a) Assume the seller can observe the type θ_i of the consumer. Formulate his maximization problem and solve it. Interpret the result. Draw the two first-best contracts into the (q,t) plane.

(b) It is unrealistic to assume that the seller knows exactly the type (it is moreover forbidden by the law to use first-degree price discrimination since sales should be anonymous). The seller thus will use a different method to separate the two possible types of buyers.

Formulate the maximization problem of the principal if the type is unknown. Solve the problem and compare the results. Draw the result into the (q,t) plane.

Solution:

To ensure truthtelling, the so-called Revelation Principle must hold: the agent must be able to capture at least the same utility when reporting his type truthfully than when mis-reporting:

$$U(q(\theta_i,\theta_i)) + T(\theta_i) \geq U(q(\hat{\theta}_i,\theta_i)) + T(\hat{\theta}_i)$$

This is explained in the inequality above: Reporting the true state (left-hand side) leads at least to the same utility than misreporting the state $\hat{\theta} \neq \theta$.

For the two-type model we use a simple utility function for the buyer, given by

$$U = \theta q - t$$

and that for $\theta_2 > \theta_1$ *also* $u(q,\theta_2) - u(q,\theta_1)$ *increases in q.*

We simplify further $C = cq$ *and thus get* $\pi = T - cq$.

$$\max_{T(.),q(.)} p\left[T(\theta_1) - cq(\theta_1)\right] + (1-p)\left[T(\theta_2) - cq(\theta_2)\right]$$
$$s.t.\ \theta_i U(q(\theta_i)) - T(\theta_i) \geq \theta_j U(q(\theta_i)) - T(\theta_i) \qquad \forall i,j \qquad (IC)$$
$$\theta_i U(q(\theta_i)) - T(\theta_i) \geq 0 \qquad \forall i \qquad (IR)$$

<u>First-best:</u>

If the seller knows the type of the buyer, he offers the quality accordingly to the type. This means that only the IR constraint is binding:

$$T(\theta_i) = \theta_i U(q(\theta_i))$$

leading to a non-constrained problem over q and θq:

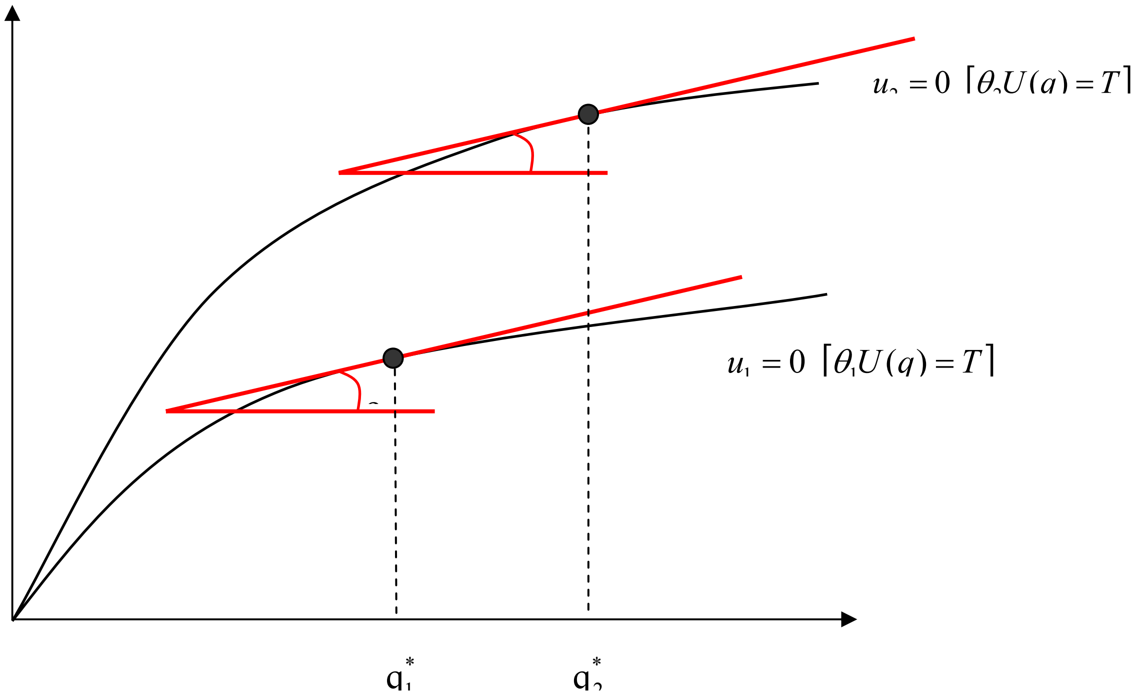
$$\max p\left[\theta_1 U(q(\theta_1)) - cq(\theta_1)\right] + (1-p)\left[\theta_2 U(q(\theta_2)) - cq(\theta_2)\right]$$

The first-order conditions denote

$$\theta_1 U'(q_1) = c$$

$\theta_2 U'(q_2) = c$.

This solution can be illustrated in the q-T space as follows. Buyers' utilities increase toward southeast, while higher isoprofit lines are found toward northwest:



Second-best:

If the seller cannot observe the buyer's type, the problem of adverse selection occurs: the type-2 buyer buys the low-type wine since he gets a positive utility and a strictly pareto-dominant contract mimicking the type-1 buyer: $u_2(q_1^*, T_1^*) > 0$.

The seller thus needs to bribe the type-2 buyer in order to reach truthful revelation of his type, which means that the type-2 buyer receives a strictly positive information rent. As we will see, the seller optimally distorts the type-1 buyer's quality by setting $q^{SB} < q_1^*$.

The IC constraint of the type-2 buyer is binding (you need to pay him to reveal his type), so is the IR constraint of type 1:

$$\theta_2 U(q(\theta_2)) - T(\theta_2) \underbrace{\geq}_{IC_2} \theta_2 U(q(\theta_1)) - T(\theta_1) \underbrace{\geq}_{\text{single-crossing}} \theta_1 U(q(\theta_1)) - T(\theta_1) \underbrace{\geq}_{\text{IR}_1} 0 \qquad (0.1)$$

$$\max_{T(.),q(.)} p\left[T(\theta_1)-cq(\theta_1)\right]+(1-p)\left[T(\theta_2)-cq(\theta_2)\right]$$

$$s.t.\ \theta_2 U(q_2)-T_2=\theta_2 U(q_1)-T_1 \qquad (IC_2)$$

$$\theta_1 U(q_1)-T_1=0 \qquad (IR_1)$$

Using these conditions we can eliminate the transfers as follows:

$$T_1=\theta_1 U(q_1)$$

and

$$T_2=\theta_2\left(U(q_2)-U(q_1)\right)+\underbrace{\theta_1 U(q_1)}_{T_1}$$

This, again, permits us to reduce the problem to an unconstrained one:

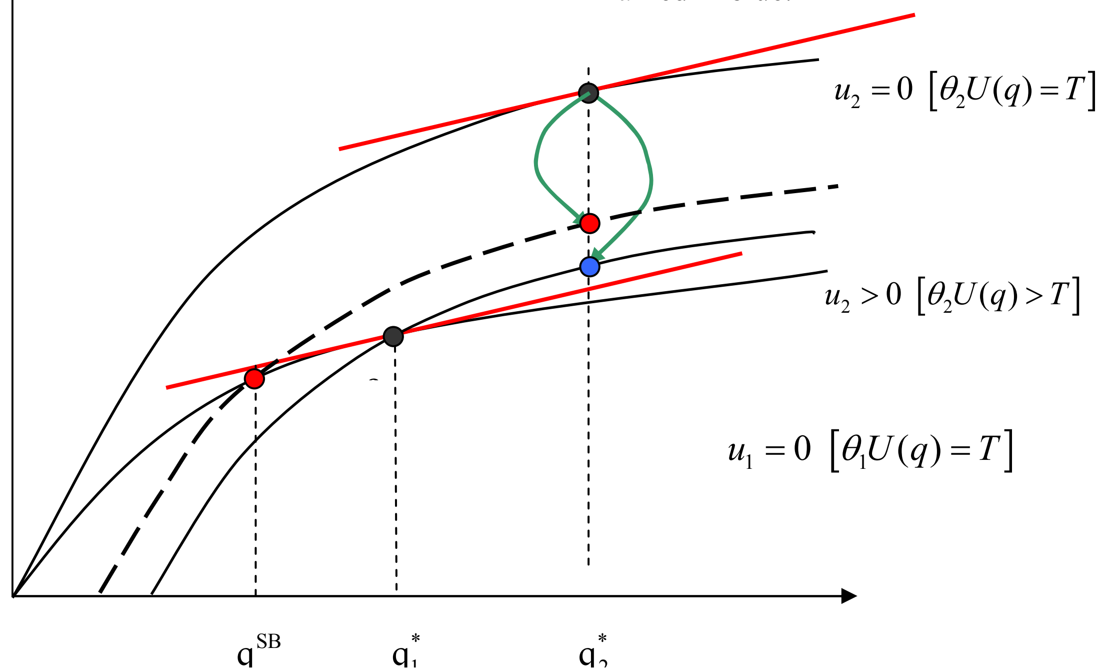
$$\max_{q_1,q_2}\left\{p\left[\theta_1 U(q_1)-cq_1\right]+(1-p)\left[\theta_2\left(U(q_2)-U(q_1)\right)+\theta_1 U(q_1)-cq_2\right]\right\}$$

The first-order conditions change into $\theta_2 U'(q_2)=c \quad \Rightarrow q_2=q_2{}^*$ and

$p(\theta_1 U'(q_1)-c)+(1-p)(\theta_1-\theta_2)U'(q_1)=0$, leading to a downward distortion with

$$\theta_1 U'(q_1)=\frac{c}{1-\left(\frac{1}{1-p}\right)\cdot\left(\frac{\theta_2-\theta_1}{\theta_1}\right)}>c$$

Comments: Isoprofit lines are red, buyer's utility black. First-best bundles are black, second-best bundles are red (with distortion of the low type's quality). The incentive-compatible bundle for the high type is marked in blue.



2) Bill Barriers, CEO of MightySoft software, is contemplating a new marketing strategy: bundling their best-selling wordprocessor and their spreadsheet together and selling the pair of software products for one price.

From the viewpoint of the company, bundling software and selling it at a discounted price has two effects on sales: (1) revenues go up due to additional sales of the bundle; and (2) revenues go down since there is less of a demand for the individual components of the bundle.

The profitability of bundling depends on which of these two effects dominates. Suppose that MightySoft sells the wordprocessor for $200 and the spreadsheet for $250. A marketing survey of 100 people who purchased either of these packages in the last year turned up the following facts:

1) 20 people bought both.

2) 40 people bought only the wordprocessor. They would be willing to spend up to $120 more for the spreadsheet.

3) 40 people bought only the spreadsheet. They would be willing to spend up to $100 more for the wordprocessor.

In answering the following questions you may assume the following:

1) New purchasers of MightySoft products will have the same characteristics as this group.

2) There is a zero marginal cost to producing extra copies of either software package.

3) There is a zero marginal cost to creating a bundle.

(a) Let us assume that MightySoft also offers the products separately as well as bundled. In order to determine how to price the bundle, Bill Barriers asks himself the following questions. In order to sell the bundle to the wordprocessor purchasers, the price would have to be less than:

Solution: The price would need to be less than 320 for word customers (trivial: they are willing to by the word processor for $200 and would be willing to spend $120 more for the spreadsheet).

(b) In order to sell the bundle to the spreadsheet users, the price would have to be less than:

Less than $350 (same argument: they are willing to pay $250 for the spreadsheet and $100 more for the word processor).

(c) What would MightySoft's profits be on a group of 100 users if it priced the bundle at $320?

$32,000.

(d) What would MightySoft's profits be on a group of 100 users if it priced the bundle at $350?

60 times 350 plus 40 times 200 = $29,000. Intuition: Group 1) and 3) go for the bundle, group 2 buys the word processor only (note that the goods are offered bundled and alone).

(e) If MightySoft offers the bundle, what price should it set?

From this follows that the optimal price for the bundle is $320. A bundle price of $350 would lose too much of consumer surplus through Group 2) buying the single good.

(f) What would profits be without offering the bundle?

Multiplying the willingnesses to pay with the number of customers per group:

$$20 \cdot 450 + 40 \cdot 200 + 40 \cdot 250 = 27{,}000.$$

20 people would buy both anyway, 40 people would buy the spreadsheet only and would be willing to buy the bundle, 40 people would buy the word processor, but not the spreadsheet.

(g) What would profits be with the bundle?

$32,000 (trivial, see c)).

h) Comparing this result with f clearly tells that bundling is more profitable.

i) The new belief structure reveals:

Fraction buy-ing both	***Fraction buy-ing word***	***Fraction buying spreadsheed***
t	*(100-t)/2*	*(100-t)/2*

Since the willingnesses to pay without bundling are the same, we have a profit without bundling of:

$$\pi = 450t + \frac{200(100-t)}{2} + \frac{250(100-t)}{2} = 225t + 22{,}500.$$

Setting this equal to the profit of the bundle as in e) we have

$$225t - 22{,}500 = 32{,}000 \Leftrightarrow t = 42.\overline{2}\ .$$

If there are 42 or less customers that buy both products anyway, it pays to bundle. If there are 43 or more, it would become unprofitable to bundle.

3) Megan's inverse demand function for running shoes is still $p(q) = \frac{1}{q^2}$.

(a) Find her price elasticity of demand for price p.

We first write the demand function as a function of p:

$$q(p) = \frac{1}{\sqrt{p}}\ .$$

$$\varepsilon = \frac{dq}{dp}\frac{p}{q} = -\frac{1}{2}p^{-3/2} \cdot p\ \cdot \sqrt{p} = -\frac{p\sqrt{p}}{2\sqrt[2]{p^3}} = -\frac{p\sqrt{p}}{2p\sqrt{p}} = -\frac{1}{2}.$$

(b) If the price of running shoes is \$20, what is the price elasticity of demand?

Elasticity is a constant. It is always the same, for any p. Thus

$$\varepsilon\Big|_{p\,=\,20} = -\frac{1}{2}.$$

(c) Assume that now her demand function changes to $p(q)$= 30-q. Find her price elasticity of demand for price p.

We again rewrite demand as a function of p:

$$q(p) = 30 - p.$$

$$\varepsilon = \frac{dq}{dp}\frac{p}{p} = -1 \cdot \frac{p}{30-p} = -\frac{p}{30-p}.$$

(d) If the price of running shoes is \$20, what is the price elasticity of demand in this new case?

Using p=20 we have $\varepsilon_{p=20} = -\frac{20}{30-20} = -2.$

(e) Assume there is just one monopolist producing the monopoly quantity for her, which she accepts to buy. He knows that demand is $p(q)$= 30-q. The monopolist has constant marginal costs of MC=10. Find the profit-maximizing quantity of the monopolist, price, and compute the Lerner index (Hint: You may as well directly derive the Lerner index from elasticity).

An algebraically nice and easy way (since we know the derivative) to find MR is as follows:

A monopolist maximizes profit by setting MR=MC.

$$30 - 2q = 10 \Leftrightarrow q^* = 10.$$

Plugging into demand we have p=20.*

We derive the Lerner index as follows:

$$\begin{aligned}
&MR(y) = p + p'y = MC(y) \quad \Leftrightarrow \\
&p(1 + \frac{p'y}{p}) = MC(y) \quad \Leftrightarrow \\
&p(1 - \frac{1}{|\varepsilon_{x,p}|}) = MC(y) \quad \Leftrightarrow \\
&\frac{p - MC(y)}{p} = \frac{1}{|\varepsilon_{x,p}|}
\end{aligned}$$

Since we already computed the elasticity ind (d), we have

$$LI = \frac{1}{|\varepsilon|} = \frac{1}{2}.$$

(f) Assume the monopolist has fixed costs of Cf = 100. Find the quantity produced by the monopolist if it is regulated by government according to average cost pricing (case 1), and according to marginal cost pricing (case 2). Find the profit situation in both cases and comment on the result.

Costs are C(q) = 100+10q.

$$AC = \frac{C(q)}{q} = \frac{100 + 10q}{q} = \frac{100}{q} + 10.$$

Case 1: AC = p:

$$\frac{100}{q} + 10 = 30 - q \Leftrightarrow q^2 - 20q + 100 = 0.$$

Solving yields the unique solution of q=10. This is exactly the solution we got in part (e). Profit is zero since TC=TR=\$200.

Case 2: MC = p:

10=30-q.

q=20. This solution involves TC>TR and a loss equal to fixed costs:

Loss = TC – TR = \$300 – \$200 = \$100.

Thus, regulating the monopolist according to average cost pricing (case 1) involves no loss for the firm. This is a second-best situation, since the output is less than under marginal cost pricing and therefore less consumers can benefit. Marginal cost pricing (case 2), although increasing welfare for the consumers would involve a loss for the firm. ■

4) Sal's satellite company broadcasts TV to subscribers in Los Angeles and New York. The demand functions for each of these two groups are

$$Q_{NY} = 60 - 0.25P_{NY}$$
$$Q_{LA} = 100 - 0.50P_{LA}$$

where Q is in thousands of subscriptions per year and P is the subscription price per year. The costs of providing Q units of service is given by

$$C = 1000 + 40Q,$$

where $Q = Q_{LA} + Q_{NY}$.

a) What are the profit-maximizing prices and quantities for the New York and Los Angeles markets?

Sal can third-degree price discriminate. He produces a quantity in each market such that the marginal revenues are equal and equate MC. MC is given by the slope of the cost curve, 40. To find marginal revenues in each markets, we find inverse demand (price as a function of qty):

From the demand curves

$$Q_{NY} = 60 - 0.25P_{NY}$$
$$Q_{LA} = 100 - 0.50P_{LA}$$

we reach inverse demands:

$$P_{NY} = 240 - 4Q_{NY}$$
$$P_{LA} = 200 - 2Q_{LA}.$$

Total revenues are:

$$TR_{NY} = Q_{NY}P_{NY} = Q_{NY}(240 - Q_{NY})$$
$$TR_{LA} = Q_{LA}P_{LA} = Q_{LA}(200 - 2Q_{LA}).$$

The First-order Condition to maximize profits is to set MR=MC:

$$240-8Q_{NY}=40 \Leftrightarrow Q_{NY}=25$$
$$200-4Q_{LA}=40 \Leftrightarrow Q_{LA}=40.$$

We now can answer the question asked in a) by plugging in the quantities into inverse demand:

$$P_{NY}=240-4\cdot 25=\$140$$
$$P_{LA}=200-2\cdot 40=\$120.$$

b) As a consequence of a new satellite that the Pentagon recently deployed, people in Los Angeles receive Sal's New York broadcasts and people in New York receive Sal's Los Angeles broadcast. As a result, anyone in NY or LA can receive Sal's broadcast in either city. Thus Sal can charge only a single price. What price should he charge, and what quantities will he sell in NY and LA?

Suggested answer: The new satellite makes it impossible for Sal to separate the two markets. Therefore, he needs to maximize profits using total demand, which is the horizontal summation of the two markets. Note that above a price of \$200, only New York costumers will demand, and for any price below \$200, we add the two demands:

$$Q_T=60-0.25P+100-0.50P=160-0.75P.$$

Again, MR is set to MC:

$$213.33-2.67Q=40$$
$$Q_T{*}=65, P_T{*}=126.67.$$

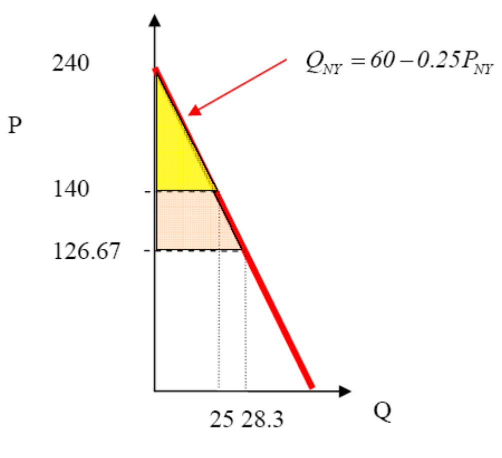
Note first that this price is below the "kink", which means that optimally the price is set such that both consumer groups are willing to consume. Quantities are found by plugging price back into the separate demands (!):

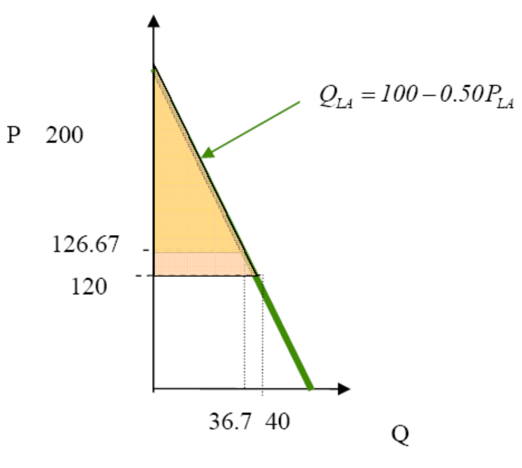
$$Q_{NY}=60-0.25P_{NY}$$
$$Q_{LA}=100-0.50P_{LA}$$

Using $P_T{*}=126.67$*, we get*

$$Q_{NY}=60-0.25P_T=28.3.$$
$$Q_{LA}=100-0.50P_T=36.7.$$

It is easy to see that total quantity is 65.





c) Compute the price elasticity of demand in the two cases with price discrimination and in the case without. Also, compute Sal's profit under price discrimination and without, as well as the consumer surplus in each market. Which situation do people in New York prefer, and which do people in Los Angeles prefer? Why, and how does this relate to price elasticity? Explain.

Elasticities:

$$\varepsilon_{Q_{NY},P_{NY}} = \frac{dQ_{NY}}{dP_{NY}}\frac{P_{NY}^{*}}{Q_{NY}^{*}} = -0.25\cdot\frac{140}{25} = -1.4$$

$$\varepsilon_{Q_{LA},P_{LA}} = \frac{dQ_{LA}}{dP_{LA}}\frac{P_{LA}^{*}}{Q_{LA}^{*}} = -0.5\cdot\frac{120}{40} = -1.5$$

$$\varepsilon_{Q_{TOT},P_{TOT}} = \frac{dQ_{TOT}}{dP_{TOT}}\frac{P_{TOT}^{*}}{Q_{TOT}^{*}} = -0.75\cdot\frac{126.67}{65} = -1.46$$

Profits: Under market conditions in (a), profit is equal to the sum of revenues from each market minus the cost of producing quantity for both markets:

Adding up the two revenues and subtracting the fixed cost (once!) and the variable costs for a qty of 65 yields a profit of $4700.

Under the market conditions in (b), profit is equal to the total revenue minus the cost of producing quantity for both markets: 65x126.67-1000-2600=4633

Sal's profit is higher when the markets are separated (which is what you expected).

We find the consumers' surplus:

Consumer Surpluses:

Under market conditions in (a), the consumer surplus in the NY market is

$$\frac{(240-140)\cdot 25}{2} = 1250.$$

The consumer surplus in the LA market is:

$$\frac{(200-120)\cdot 40}{2} = 1600.$$

Under the market conditions in (b), the consumer surplus in the NY market is

$$\frac{(240-126.67)\cdot 28.\overline{3}}{2} = 1603.62.$$

The consumer surplus in the LA market is:

$$\frac{(200-126.67)\cdot 36.7}{2}=1345.6.$$

Interpretation:

The most elastic demand is in the LA market. This explains the relatively lower price, compared to the NY market. Sal's profits are highest under PD. Consumer surpluses tell us that NY consumers are better off without PD, while LA consumers are better of under the scenario with PD. This relates to our findings on price elasticity: Sal charges a higher price in the less elastic NY market when he can price discriminate, and a lower in the more elastic LA market. Reason is that MRs are set equal under PD in each market. If this holds, the market with the "higher" demand has a lower price elasticity, yielding a higher price at the optimum.

5) A Vermont quilt store can sell its products in two markets: Canada and the U.S. . The demand functions are 120 -1.1p in the U.S., and 90-p in Canada. The store's cost function is C=20q.

a) Find the profit-maximizing prices and quantities for the two markets if the store can third-degree price discriminate. Find the Consumer Surplus in each market and the total profit of the quilt store under 3rd-degree price discrimination.

We first find the inverse demand curve in each market and compute total revenue

$$Q_{US}=120-1.1P_{US}\Leftrightarrow P_{US}=109.1-\frac{10}{11}Q_{US}$$

$$Q_{CA}=90-P_{CA}\Leftrightarrow P_{CA}=90-Q_{CA}.$$

U.S. market:

$$TR=P_{US}Q_{US}=109.1Q_{US}-\frac{10}{11}Q_{US}^{2}$$

$$MR=\frac{dTR}{dQ}=109.1-\frac{20}{11}Q_{US}$$

Setting MR=MC:

$$109.1-\frac{20}{11}Q_{US}=20\Leftrightarrow 89.1=\frac{20}{11}Q_{US}\Leftrightarrow Q_{US}*=49.$$

Plugging into inverse demand:

$$P_{US}*=109.1-\frac{10}{11}(49)=64.\overline{54}$$

Canadian market:

$$TR=P_{CA}Q_{CA}=90Q_{CA}-Q_{CA}^{2}$$

$$MR=\frac{dTR}{dQ}=90-2Q_{CA}$$

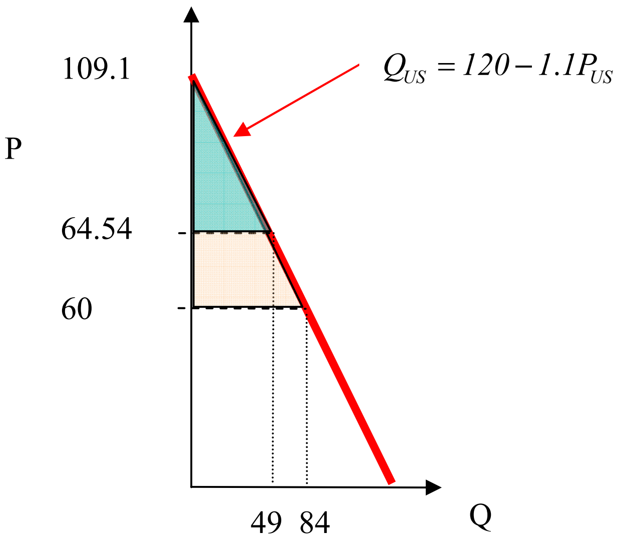
Setting MR=MC:

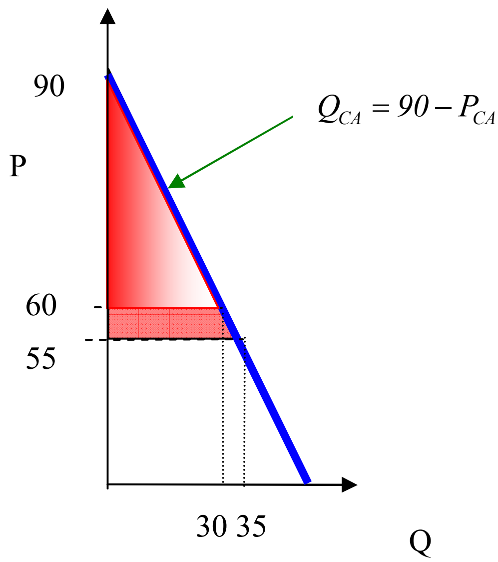
$$90-2Q_{CA}=20 \Leftrightarrow 70=2Q_{CA} \Leftrightarrow Q_{CA}{}^{*}=35.$$

Plugging into inverse demand:

$$P_{CA}{}^{*}=90-35=55.$$

Finding CS and profits:





CS in the U.S. market is the blue triangle with the base of 40 and the height of (109.1-64.54)

CS_{US}=1091.36

CS in the Canadian market is the entire red triangle with the base of 30 and the height of (90-55)

CS_{CA}=612.5

Profit is $TR_{US}+TR_{CA}-TC=$ $64.\overline{54}\cdot 49+55\cdot 35-20(84)$ =3162.72+1925-1680=3407.72

b) Now assume now that 3rd-degree price discrimination is not possible. Find demand, profit maximizing price and quantity in this case, as well as price and quantity in each market. Find the new profit and the new Consumer Surpluses in each market.

Horizontal summation implies that we add up demands, NOT INVERSE DEMANDS:

$$Q_{US} = 120 - 1.1P_{US}$$
$$Q_{CA} = 90 - P_{CA}$$

$$Q = 210 - 2.1P$$

This leads to a total inverse demand curve of P=100 – 0.476Q

We again find total revenue as before: TR=PQ=100Q – 0.476Q^2.

MR=MC and 100-0.952Q=20, from which

Q=84*

P=60.*

Plugging a price of 60 into U.S. demand yields a quantity of 54, and a quantity of 30 in the Canadian market (see graph above).

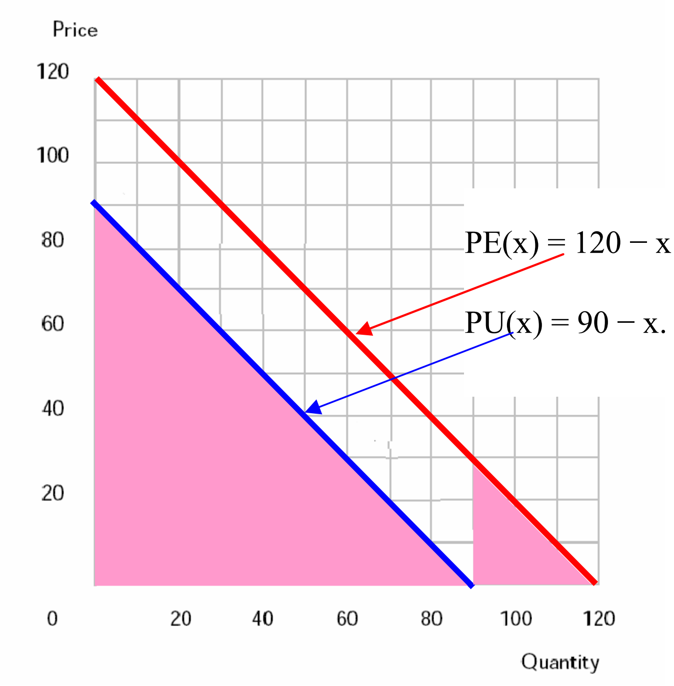
Consumer surplus increases in the U.S. market to (1/2)(109.1-60)(54)=1325.7, and decreases in the Canadian market to (1/2)(90-60)(30)=450. Total profit decreases to (60-20)84=3360 . Thus, profit has decreased, and consumer surplus has increased with the U.S consumers now being better off.

c) Graph the market equilibrium with and without 3rd degree price discrimination in the two diagrams below, mark inverse demand, marginal revenue, marginal cost, and all consumer surpluses. Label the axes.

(See above) .

6) An online publisher (seller hereafter) is considering offering articles to readers by email. Their market research division indicates that there are two types of potential users in the market, students and executives. There are the same number of students and executives in the market. Let x be the number of articles that a user requests per year.

The executives have an inverse demand function PE(x) = 120 − x and the students have an inverse demand function PU(x) = 90 − x. (Prices are measured in cents.) The publisher has a zero marginal cost of sending articles via email. Draw these demand functions in the graph below and label them.



a) Suppose that the seller can perfectly price discriminate. What is the optimal pricing strategy? What quantities are sold at what price?

In this case, the following price-quantity pairs are offered:

	Units	Price
Package for low demand consumer	90	4050
Package for high demand consumer	120	7200

b) Now assume that the seller cannot tell the two consumer groups apart. An executive can always buy the package size designed for the student. Assume the size is not changed compared to part a): what are the prices now for each bundle? Explain why the seller gives a "discount" to one group of buyers.

The discount is the entire area between the two demands and up to a quantity of 90. This is because of the fact that the seller cannot prevent the high demand consumer buying the package designed for the other type. By reducing price to the pink area, the buyer makes sure that the high demand consumer buys the package with 120 units.

	Units	Price
Package for low demand consumer	90	4050
Package for high demand consumer	120	**4500**

c) It is known that the seller can improve its profit by reducing the package size for the students. Find the optimal package size and profits. Explain.

The optimal reduction is found when the marginal discount equals the marginal loss through distorting the package for the low demand consumer. Graphically, this is where *the vertical distance between the red and blue demand equals the vertical distance between blue demand and zero. It is easy to see that this is where **Q=60**.*

	Units	Price
Package for low demand consumer	60	**3600**
Package for high demand consumer	120	**5400**

This yields a total profit of 9000. It can be checked that this is profit maximizing: Imagine the seller would distort further and sell packages with 59 and 120 units. Then, it could charge the low demand consumer a price of 3569.5, while the price of the high demand consumer could increase to 5430. Total profit is 8999.5, which is less than with the 60/120 combination. Trying out the combination 61 and 120 yields the same lower profit.

7) Suppose that BMW can produce any quantity of cars at a constant marginal cost equal to $20,000 and a fixed cost of $10 billion. You are asked to advise the CEO as to what prices and quantities BMW should set for sales in Europe and in the United States. The demand for BMWs in each market is given by

$Q_E = 4,000,000 - 10P_E$,

$Q_U = 1,000,000 - 20P_U$,

where the subscripts indicate the region. All prices and costs are in thousands of dollars. Assume that BMW can restrict U.S. sales to authorized BMW dealers only.

a) What quantity of BMWs should the firm sell in each market, and what should the price be in each market? What should the total profit be?

From demand we derive inverse demand:

$Q_E = 4,000,000 - 10P_E \Leftrightarrow P_E = 400,000 - 0.1Q_E$

$Q_U = 1,000,000 - 20P_U \Leftrightarrow P_U = 50,000 - 0.05Q_U$

Under 3rd-degree PD we find for the European market:

$TR_E = P_E Q_E = (400,000 - 0.1Q_E)Q_E = 400,000Q_E - 0.1Q_E{}^2$

$MR_E = 400,000 - 0.2Q_E$

$MR \doteq MC$:

$400,000 - 0.2Q_E = 20,000$

$Q_E{}^* = 1,900,000$

$P_E{}^* = 400,000 - 0.1 \cdot 1,900,000 = 210,000$

Similarly, we have for the U.S. market:

$TR_U = P_U Q_U = (50,000 - 0.05Q_U)Q_U = 50,000Q_u - 0.05Q_U{}^2$

$MR_E = 50,000 - 0.1Q_U$

$MR \doteq MC$:

$50,000 - 0.1Q_U = 20,000$

$Q_U{}^* = 300,000$

$$P_U* = 50,000 - 0.05Q_U = 35,000$$

$$\pi = TR - TC = 399B + 10.5B - 10B - 44B = 409.5B - 54B = 355.5B.$$

b) If BMW were forced to charge the same price in each market, what would be the quantity sold in each market, the equilibrium price, and the company's profit?

We find total demand:

$$Q_E = 4,000,000 - 10P_E$$
$$Q_U = 1,000,000 - 20P_U$$

$$Q = 5,000,000 - 30P$$

Inverse demand is *P=166,666.67-0.03Q.*

TR=PQ=166,666.67Q-1/30Q²

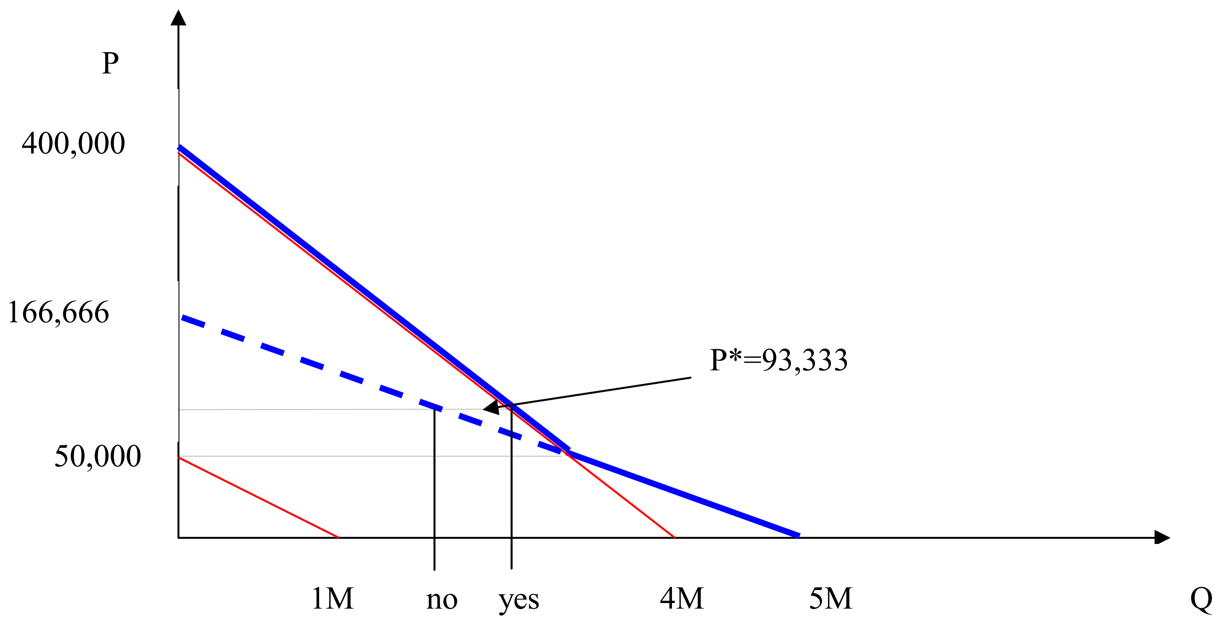
MR=166,666.67-1/15Q

MR=MC:

166,666.67-1/15Q=20,000

Q=2,200,000*

P=166,666.67-1/30Q*=93,333*



Note that this is above the prohibitive price in the U.S. market, which does not have demand above $50,000. Thus, a price of 93,333 only creates demand in the European market. This still make sense because the problem assumes that " BMW were forced to charge the same price in each market". In other words, the total demand Q=5,000,000-3P function is only defined for a price P between zero and 50,000. Above that, it is equal to the higher demand (it's a demand with a kink).

We plug this quantity into the European demand only. $Q_E = 4,000,000 - 10P_E =$ 4,000,000 – 933,333=3,066,666, *while no quantity were to be sold in the U.S. . Total revenue is 286.22 B and profit is 214.92B. This situation leads to a lower profit for BMW than under (a) for the European market.*

8) Elizabeth Airlines (EA) flies only one route: Chicago-Honolulu. The demand for each flight is Q=500-P. EA's cost of running each flight is $30,000 plus $100 per passenger.

a) What is the profit-maximizing price that EA will charge? How many people will be on each flight? What is EA's profit for each flight?

We first compute the inverse demand curve:

P=500-Q

TR=P(Q)Q=(500-Q)Q=500Q-Q^2*. MR=500-2Q*

MC=100

Profit is maximized at a quantity where MR=MC:

500-2Q=100

400=2Q

Q=200.*

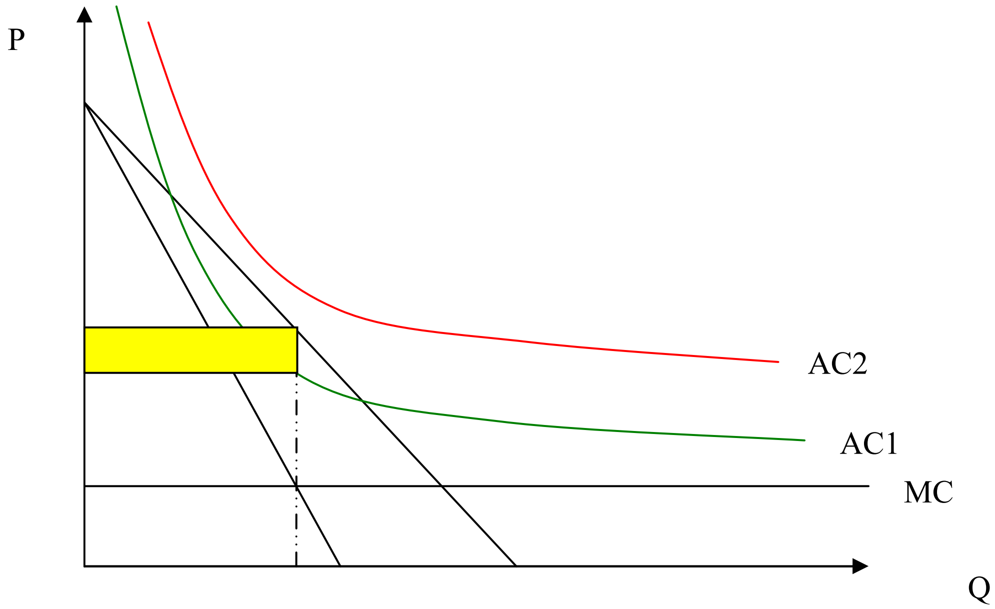
P=500-200=300.*

$\pi = TR - TC = 60,000 - 30,000 - 20,000 = 10,000.$

b) EA learns that the fixed costs per flight are in fact $41,000 instead of $30,000. Will the airline stay in business for long? Illustrate your answer using a graph of the demand curve that EA faces and the average cost curve with $41,000 and with $30,000.

With higher fixed costs, the firm earns a loss:

$\pi = TR - TC = 60,000 - 41,000 - 20,000 = -1,000.$ *This is illustrated in the graph below.*



The average cost curve with higher fixed costs (red) has no intersection anymore with demand. Thus, the firm suffers a loss.

c) Now EA finds out that two different types of people fly to Honolulu. Type A consists of business people with a demand of $QA=260-0.4P$, and Type B consists of students with a total demand of QB=240-0.6P. Since students are easy to spot, EA decides to charge them different prices.

Graph each of these demand curves and their horizontal sum. Mathematically find the profit maximizing quantity and prices in each market and illustrate.

This involves third-degree price-discrimination. Inverse demands are:

$PA=650-2.5Q$

$PB=400-1.66Q$

<u>Market A:</u> $TR=650Q-2.5Q^2$

$MR=650-5Q$

MR=MC when

$650-5Q=100$ *and* $QA^*=110$.

$P^*=650-2.5Q=375$ *Total revenue is 41,250, and total cost is 30,000+11000=41,000.*

Market B:

From $PB=400-1.66Q$ *we get*

$TR=400Q-1.66Q^2$

MR=400-3.33Q

MR=MC, that is

400-3.33Q =100 and QB=90.*

P=400-1.66Q =250.   Total revenue is 22,500, and total cost is 30,000+9000=39000.*

<u>Without 3rd degree PD:</u>

Q=QA+QB= 260-0.4P+240-0.6P=500-P. That is, we have the same situation as in A.

d) What would EA's profit be for each flight? Would the airline stay in business? Calculate the consumer surplus of each consumer group. What is the total CS?

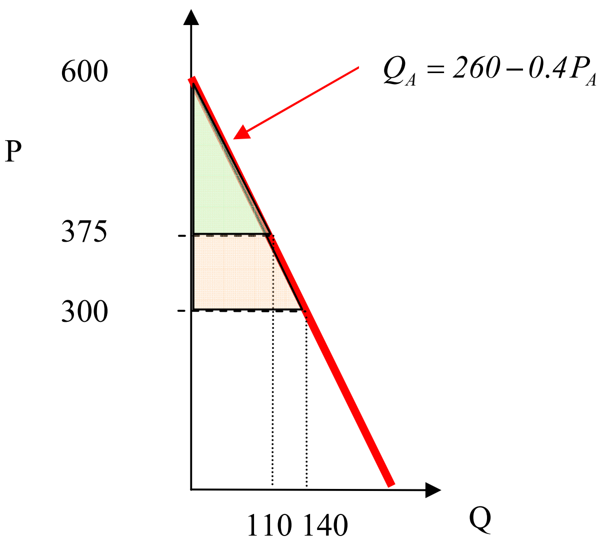
Calculating profits:

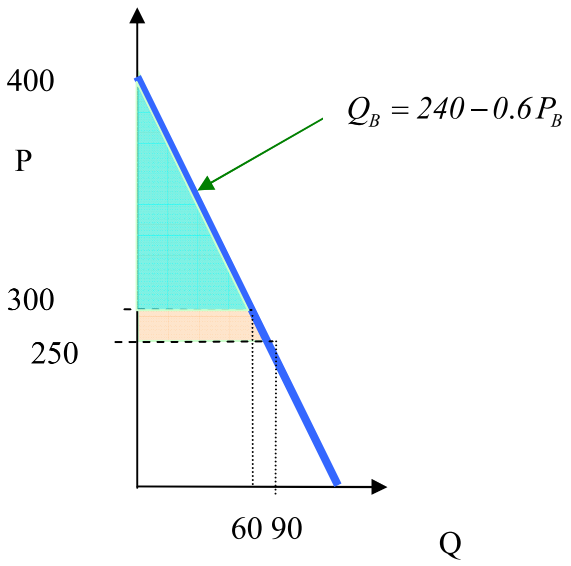
$\pi = TR - TC = QA \cdot PA + QB \cdot TB - TC = 110 \cdot 375 + 90 \cdot 250 - 30,000 - 20000 = 13.750.$

This is a positive profit under PD. The airline stays in business.

e) Before EA started price discriminating, how much consumer surplus was Type A demand getting from air travel to Honolulu? Type B? Why did total consumer surplus decline with price discrimination, even though total quantity sold remained unchanged?

Calculating CS:





With Price Discrimination:

$$CS_A = \frac{1}{2}(110)(600-375) = 15,125 .$$

$$CS_B = \frac{1}{2}(90)(400-250) = 6750.$$

$$CS = 21,875 \quad .$$

Without Price Discrimination:

$$CS_A = \frac{1}{2}(140)(650 - 300) = 24,500\ .$$

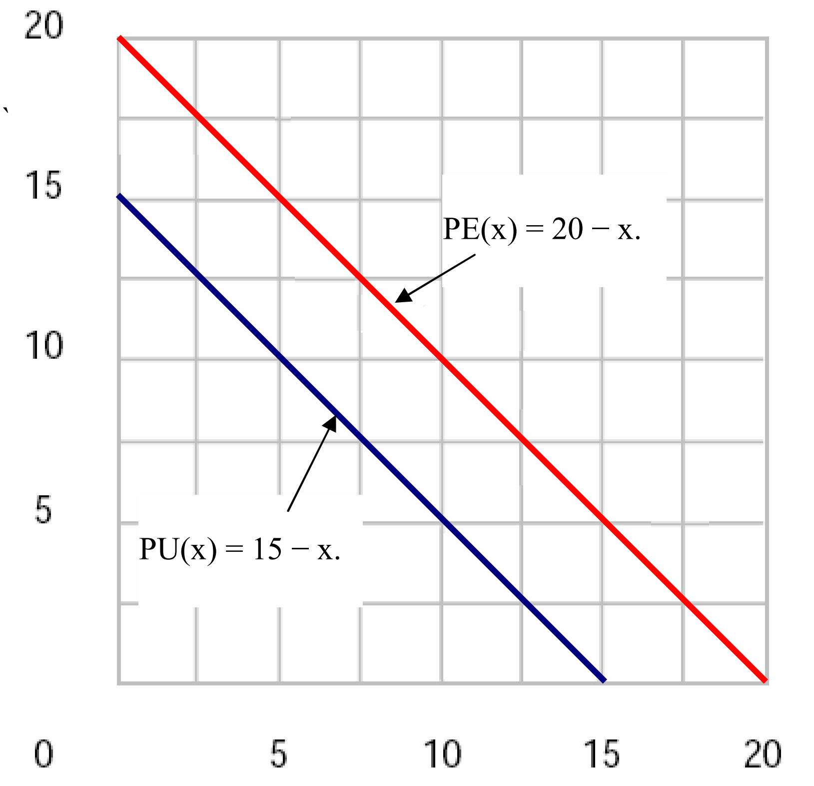
$$CS_B = \frac{1}{2}(60)(400 - 300) = 3,000\ .$$

$$CS = 27,500.$$

9) The Ball Street Journal is considering offering a new service which will send news articles to readers by email. Their market research indicates that there are two types of potential users, impecunious students and high-level executives.

Let x be the number of articles that a user requests per week. The executives have an inverse demand function PE(x) = 20 − x and the students have an inverse demand function PU(x) = 15 − x. The Journal has a zero marginal cost of sending articles via email. There is an equal number of executives and students in the market.

a) Assume first that BSJ can tell the consumer groups apart. Graph the demand curves into the chart below and find the profit-maximizing quantity and price for each group, assuming that BSJ can price discriminate. Label the axes. Find the profit (equal to total revenue) assuming there is 1 consumer in each group.



Solutions: When BSJ can price discriminate, it maximizes revenue (=profit) by setting the bundle to the maximum demand that each group is willing to buy at zero price, and charges the entire triangle:

<u>*Students:*</u> *x=15, p=*$\frac{15 \cdot 15}{2} = 112.5 = \pi$

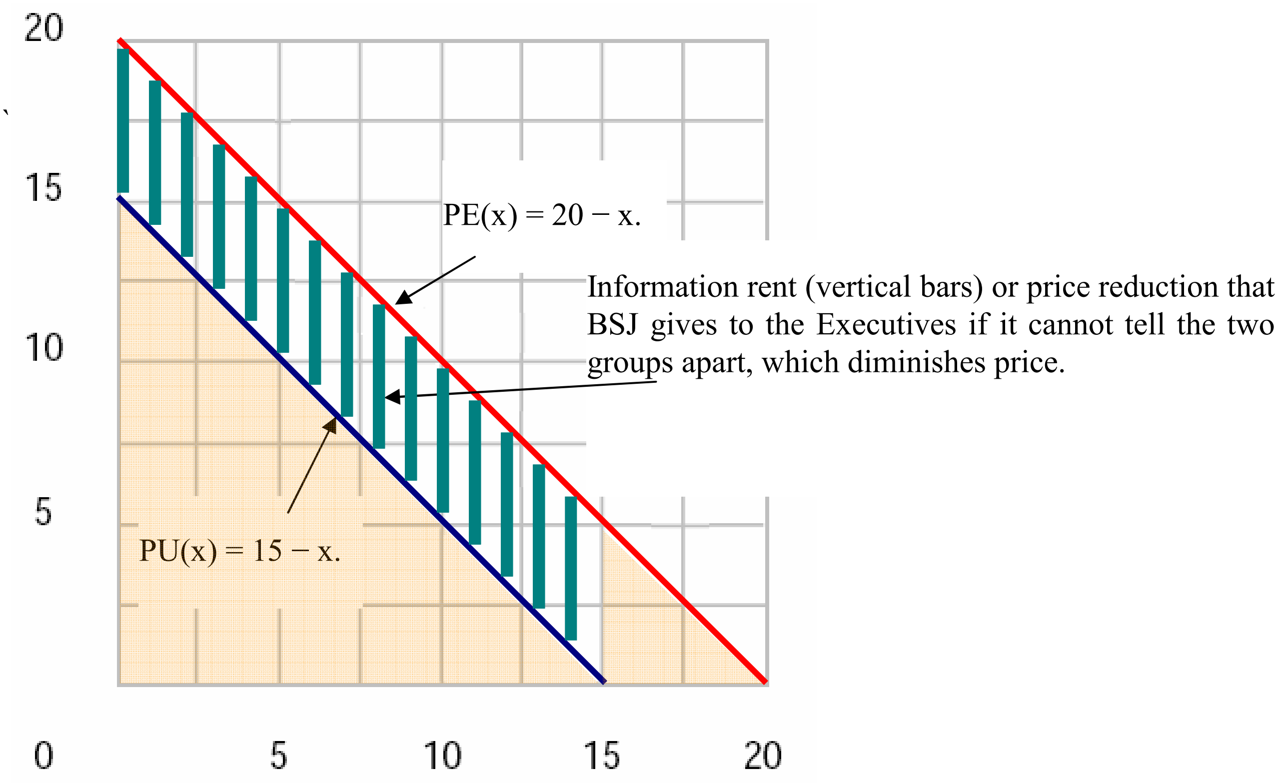
Executives: $x=20,\ p=\frac{20\cdot 20}{2}=200=\pi$

Total Profit: $\pi=312.5$

b) Now assume that BSJ can still price discriminate but cannot tell the consumers apart. That is, by offering the two packages sizes that are optimal in part a), it will need to change the price of one package. Find the optimal price for this particular package as well as the consumer surplus (sometimes called information rent) that is left to one group. Find the profit (equal to total revenue) assuming there is 1 consumer in each group. Graph the result into the chart below. Label the axes.

Solution: The executives can require a cheaper price, otherwise they would not be willing to reveal their type and BSJ would only sell the student bundle of 15 if it offers both sizes. Everything is as if the executives could require to firs consume the bundle designed for the students, and later on buy the rest along their true (red) demand.

That is, the executives require an information rent equivalent to the area between the two demand curves when q=15:



The numbers now change as follows:

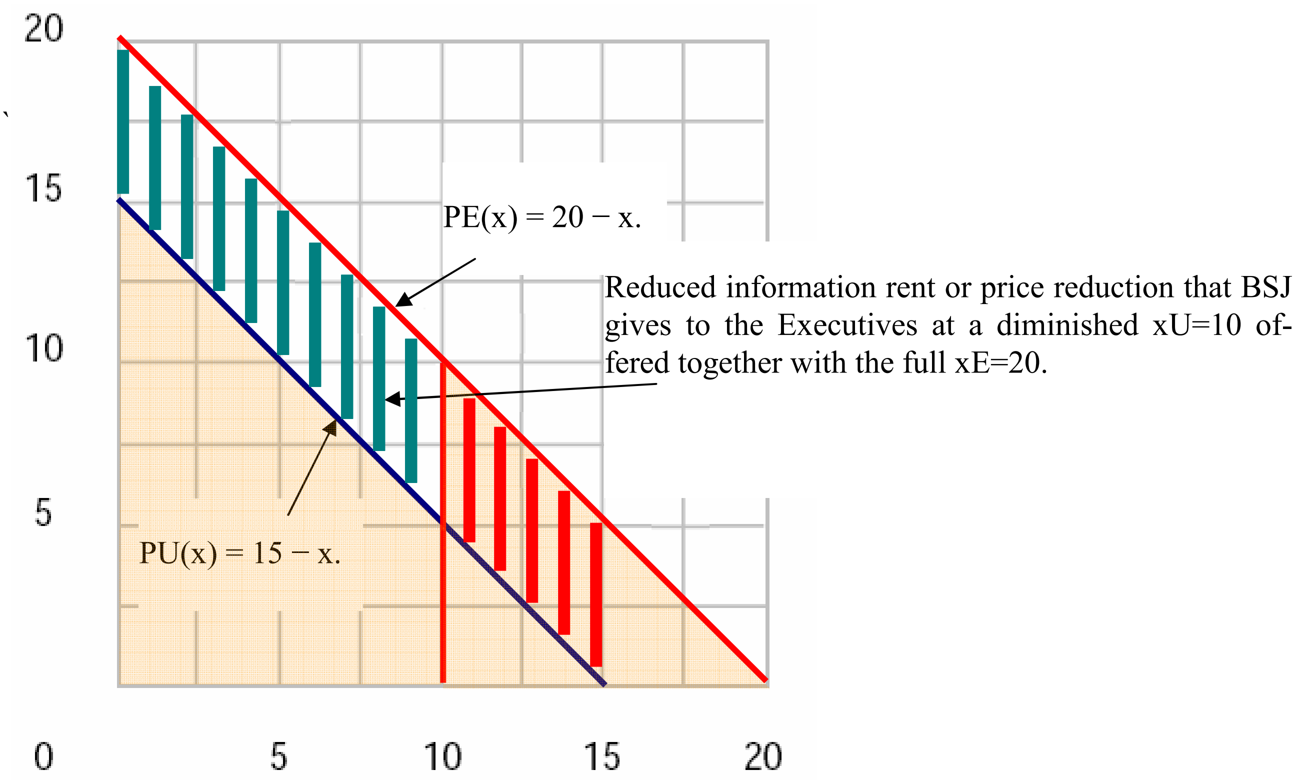
Students: $x=15,\ p=\frac{15\cdot 15}{2}=112.5=\pi$ *(no change)*

Executives: *x=20, price is diminished by the green area, thus equal to the two colored triangles:*

$$p=\frac{15\cdot 15}{2}+\frac{5\cdot 5}{2}=112.5+12.5=125=\pi$$

Total Profit: $\pi=237.5$

c) Now assume the setting in b) but that BSJ sells the same quantity to the executives as before, but reduces the size of the package designed to the students to 10. Find the price that the students are willing to pay, and find the new consumer surplus (or information rent if you want to call it this way) that needs to be paid to the executives in this new case. Again find the profit (equal to total revenue) assuming there is 1 consumer in each group. Graph the result into the chart below. Label the axes.



In this case, the two package quantities are 20 and 10.

Students: $x=10,\ p=\frac{10\cdot 10}{2}+10\cdot 5=100=\pi$ *(triangle below blue demand and right of x=10 is lost)*

Executives: *x=20, price is diminished, but less than in b) and equal to the entire beige shaded area:*

$$p=\frac{10\cdot 10}{2}+10\cdot 5+\frac{10\cdot 10}{2}=150=\pi$$

Total Profit: $\pi=250.$

d) Last, assume that package sizes as in c) and drop the initial assumption of an equal proportion of students to executives in the market.

- For which proportion of students to executives would this package size be revenue-maximizing?

The package size of 10 for students and 20 for executives is profit (revenue) maximizing when the amount of students equals the amount of executives.

Imagine that BSJ would offer a package size of 11 for students and 20 for executives. Then, the profit would be $ 11(4)+11(11)/2=104.5 per student, and the same amount plus plus 40.5=144.5 per executive, yielding at a 1:1 ratio a total profit of $ 249. Compared to the solution in C ($250) this is one $ less. That is, it pays at this ratio to reduce the 11 pack size down to 10.

Imagine now that BSJ would reduce the student package by one more unit, thus offering a size of 9 to students and a size of 20 to executives. Then, the profit would be $ 9(6)+9(9)/2=94.5 per student, and the same amount plus plus now 60.5=154.5 per executive, yielding at a 1:1 ratio again a total profit of $ 249. That is, offering student pack sizes of 9 or 11 is suboptimal, and solution C is the optimal solution for a 1:1 ratio.

- For which proportion of students to executives would BSJ be equally well off to not offer the package with 10 anymore but only cater to executives? Show your work.

We now compare the outcome under c) with the outcome when only the x=20 package is offered (executives only). The "executives only" outcome yields a profit of 200 (no distortion necessary, each executive picks the 200 bundle, no student does.

The c) outcome yields a profit of 250, or 100 per student and 150 per executive.

One way to solve is to call p the percentage of students, and 1-p the percentage of executives, both summing up to one. Then the following p solves our problem:

p(Payoff in state L)+(1-p)(Payoff in state H)=(1-p)(Payoff when package offered to H only)

Using the values in (c) this is:

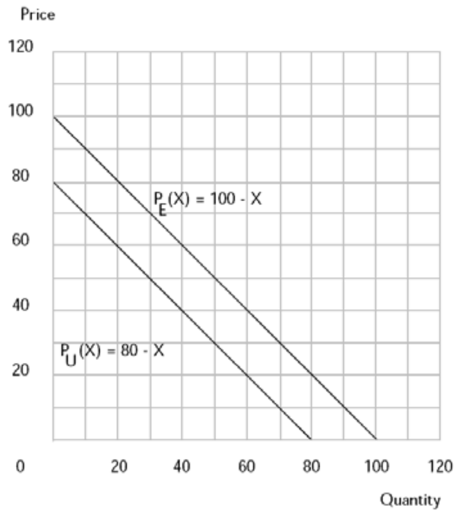
p(100)+(1-p)150=(1-p)200
100p+150-150p=200-200p
150p=50
p=1/3

In other words, whenever the number of executives in the market doubles the number of students, it does not pay anymore to offer two packages, but to only offer the 20 unit bundle and only executives would buy it.

WB 25.6

The Mall Street Journal is considering offering a new service which will send news articles to readers by email. Their market research indicates that there are two types of potential users, impecunious students and high-level executives. Let x be the number of articles that a user requests per year. The executives have an inverse demand function

PE(x) = 100 − x and the students have an inverse demand function PU(x) = 80 − x. (Prices are measured in cents.) The Journal has a zero marginal cost of sending articles via email. Draw these demand functions in the graph below and label them.



(a) Suppose that the *Journal* can identify which users are students and which are executives. It offers each type of user a different all or nothing deal. A student can either buy access to 80 articles per year or to none at all. What is the maximum price a student will be willing to pay for access to 80 articles?

In this case, the student pays the entire area under the demand curve. This is

$$\frac{80 \cdot 80}{2} = 3,200 = \$32.$$

An executive can either buy access to 100 articles per year or to none at all. What is the maximum price an executive would be willing to pay for access to 100 articles?

Similarly, the executive pays the entire area under the demand curve. This is

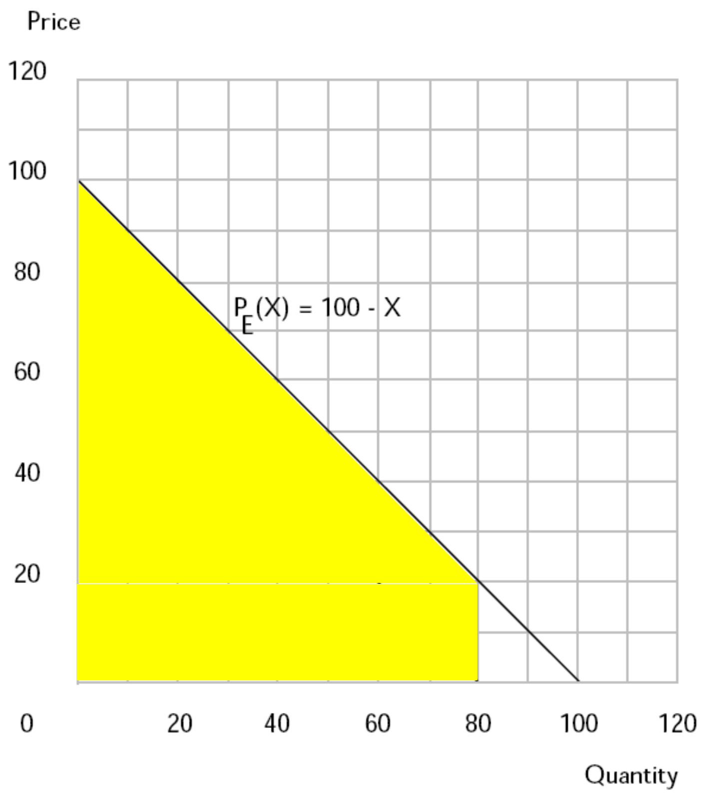
$$\frac{100 \cdot 100}{2} = 5,000 = \$50.$$

(b) Suppose that the *Journal* can't tell which users are executives and which are undergraduates. Thus it can't be sure that executives wouldn't buy the student package if they found it to be a better deal for them.

In this case, the *Journal* can still offer two packages, but it will have to let the users self-select the one that is optimal for them. Suppose that it offers two packages: one that allows up to 80 articles per year the other that allows up to 100 articles per year. What's the highest price that the undergraduates will pay for the 80-article subscription?

As before, the students are willing to pay up to \$32.

(c) What is the total value to the executives of reading 80 articles per year?

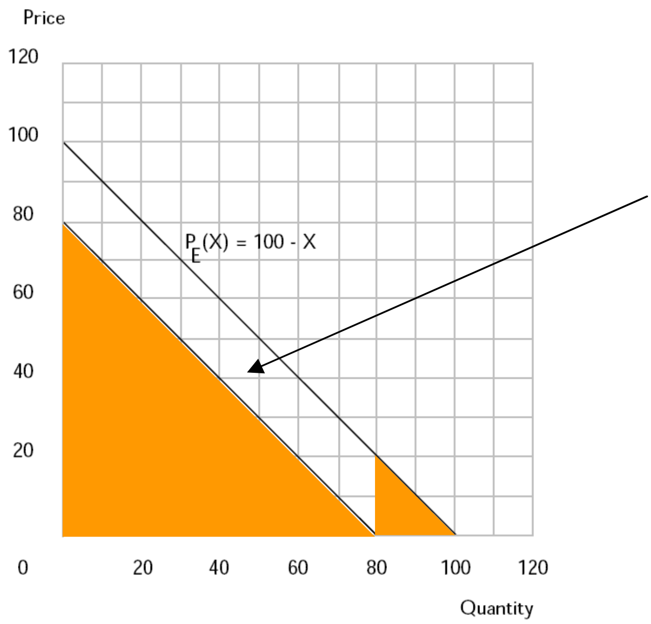


This is the yellow area, which is $\frac{100 \cdot 100}{2} - \frac{20 \cdot 20}{2} = 4,800 = \$48.$

(d) What is the the maximum price that the *Journal* can charge for 100 articles per year if it wants executives to prefer this deal to buying 80 articles a year at the highest price the undergraduates are willing to pay for 80 articles?

So, for 80 articles, the WTP is 48 *for executives and* 32 *for students. What can an executive buy for the* \$16 *he would pay to the journal when revealing his true type? It would be worth the additional quantity of 20 articles. What price would be still be accepted by the executive for 100 articles: The executive has two possible actions: Grab the 80 for \$32, and save \$16.*

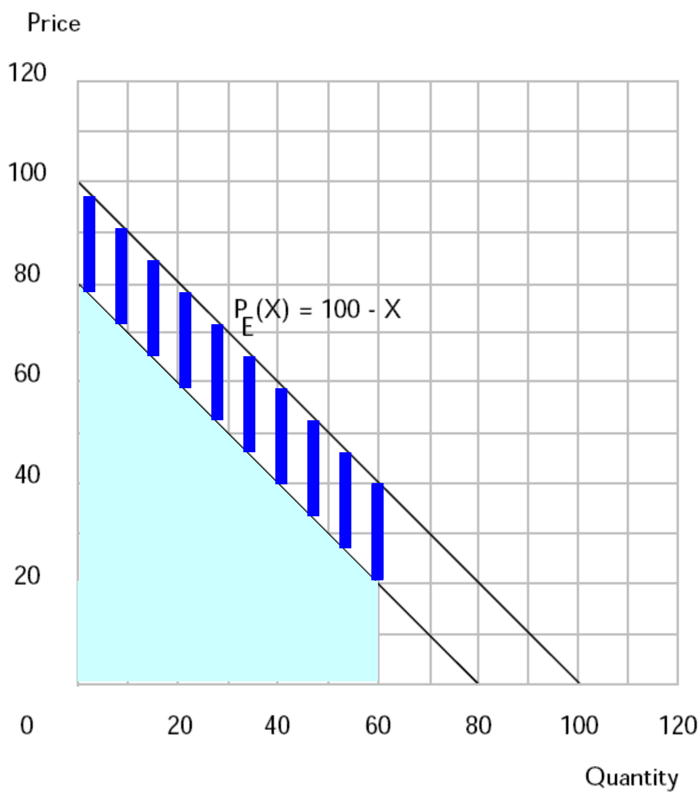
Solution: It is 50-p=16 or p = \$34. *Intuition: Executive is given the \$16, but we take this off your total price if you reveal, so compared to a situation in which you cannot mimick the student, you're by \$16 better off. This is called an <u>information rent</u>, equivalent to a "truth-telling discount" of \$16. Thus, the journal charges \$34. This follows the idea of <u>incentive compatibility.</u>*



Price for executives:

The non-colored area under the demand curve is \$16.

(e) Suppose that the *Mall Street Journal* decides to include only 60 articles in the student package. What is the most it could charge and still get student to buy this package?



The area under the student demand curve with a q-intercept of 60 *is the blue area, which is the triangle* $\frac{60 \cdot 60}{2} = \$18$ *and the rectangle 20x60*=\$12. Adding yields \$30.

(f) If the *Mall Street Journal* offers a "student package" of 60 articles at this price, how much *net* consumer surplus would executives get from buying the student package?

The net consumer surplus is the parallelogram (hatched) between the two demand curves, from zero to 60 units demanded. This is equivalent to the light blue rectangle, namely $12.

(g) What is the most that the *Mall Street Journal* could charge for a 100-article package and expect executives to buy this package rather than the student package?

Same deal as before: Producer agrees that by misrepresenting now $12 can be saved, and charges $38 *instead of $50.*

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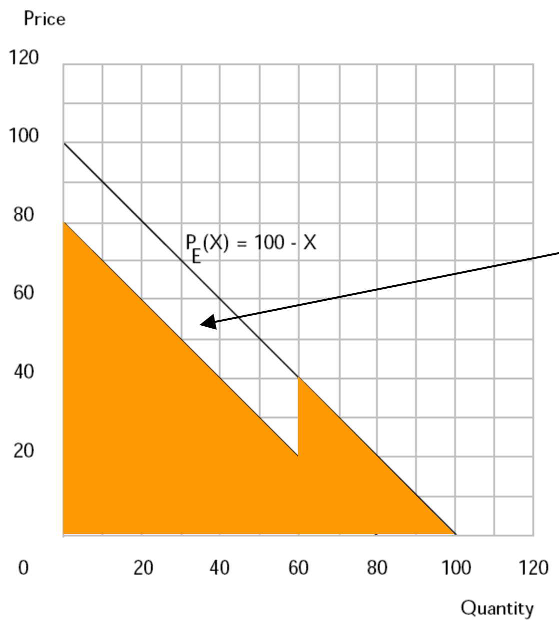
(h) If the number of executives in the population equals the number of students, would the *Mall Street Journal* make higher profits by offering a student package of 80 articles or a student package of 60 articles?

Assume there is one student and one executive (This comes without loss of generality: you can always scale it up).

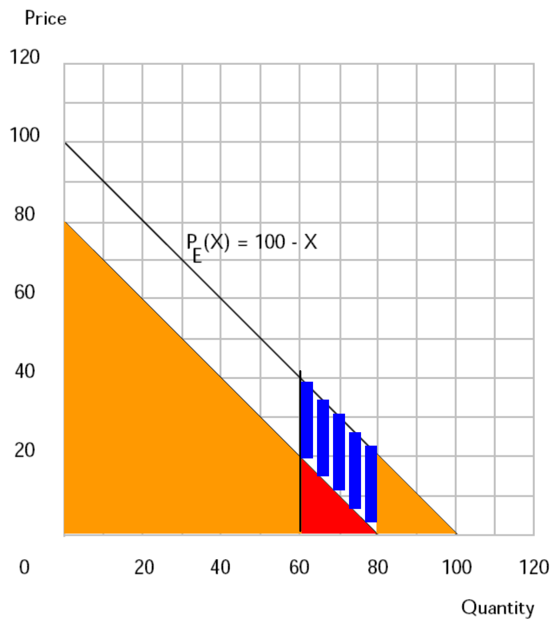


Graphing the new price for executives: The non-colored area under the demand curve reduces from $16 to $12 if the two bundles offered are no longer 80 and 100 but 60 and 100.

a) When offering the 80 package, the revenue is $32+$34=$66.

b) When offering the 60 package, the revenue is higher, since incentive compatibility is reached by paying a lower information rent: $30+$38=$68. Thus, the journal chooses to offer a lower package for students, namely 60 instead of 100 (distortion of the low-type consumer).

Compare to the 'How to solve the problem' slide in the coffee shop example:



Solutions to WB #25.7: see key answers Varian (pdf file)

WB 25.8

2) Colonel Tom Barker is about to open his newest amusement park, Elvis World. Elvis World features a number of exciting attractions: you can ride the rapids in the Blue Suede Chutes, climb the Jailhouse Rock and eat dinner in the Heartburn Hotel. Colonel Tom figures that Elvis World will attract 1,000 people per day, and each person will take $x = 50 - 50p$ rides, where p is the price of a ride. Everyone who visits Elvis World is pretty much the same and negative rides are not allowed. The marginal cost of a ride is essentially zero.

(a) What is each person's inverse demand function for rides?

We solve for p: $p(x) = 1 - x/50$.

(b) If Colonel Tom sets the price to maximize profit, how many rides will be taken per dayb y a typical visitor?

We set MR=0.

Finding MR: $TR = x - \frac{x^2}{50}$, *and* $MR = 1 - x/25$.

$$1 - \frac{x}{25} = 0 \Leftrightarrow 1 = \frac{x}{25} \Leftrightarrow x = 25.$$

(c) What will the price of a ride be?

Plugging into inverse demand yields $p = 1 - \frac{x}{25} = 1 - \frac{25}{50} = \$0.5$.

(d) What will Colonel Tom's profits be per person?

Profits are equal to TR, thus px=\$12.50

(e) What is the Pareto efficient price of a ride?
The Pareto-efficient price is reached where quantity is equal to competitive quantity, or p=MC. Since MC=0, p=0.

(f) If Colonel Tom charged the Pareto efficient price for a ride, how many rides would be purchased?

Plugging into demand yields 50 units.

(g) How much consumers' surplus would be generated at this price and quantity?

This is the entire area under the demand, which is the area of a triangle with the base 50 and the height 1, which is \$25 .

(h) If Colonel Tom decided to use a two-part tariff, he would set an admission fee of ? and charge a price per ride of ?

Obviously, he is exactly best off by asking the entire surplus as admission fee, which is \$25. He does not ask any markup above marginal cost, thus a price of zero. ■