Problem Set #4 Econ21 – Summer 2007 *W. Gick*

Posted August 9, presentations in class August 15

All problems except # 3 are open for presentation. ** indicates an advanced problem.

** 1) A wine seller sells wine of different qualities to a buyer who can have different quality preferences. The buyer may thus of two "types": the “type-2” agent (“gourmet”) who is willing to pay a large sum for a great vintage and the coarse “type-1”agent (“gourmand”) that does not care much about quality.

It is realistic to assume that in reality the seller cannot *perfectly* distinguish the two types, but might have knowledge about the probability of their type. The seller may keep the two types apart since the type-2 agent is willing to pay more for an increase in quality per unit, compared to the type-1 agent.

The utility of the seller is her profit , namely $\pi = \pi(q,T)$, while the buyer has a utility of $U = U(q,T)$. with

$$\left.\begin{array}{ll} q_i & \textit{quality} \\ t_i & \textit{transfer} \end{array}\right\} \text{ in state } \theta_i \in \{\theta_1,...,\theta_n\}$$

The state θ_i is only observed by the buyer before choosing a bottle (a pair q,T). θ nothing but the type of the buyer, with $\theta = \{\theta_1,\theta_2\}$. We further assume that θ_1 occurs with probability *p*, while θ_2 with probability *1-p* . This probability is the seller’s prior beliefs. We furthermore assume that $\theta_2 > \theta_1$.

The buyer has a utility function of $U = \theta q - t$. We assume that for $\theta_2 > \theta_1$ also $u(q,\theta_2) - u(q,\theta_1)$ increases in q. The seller is able to produce wine of any quality $q \in [0,\infty]$, at strictly convex costs of $C = c(q)$. We assume the cost function is twice differentiable. We simplify further $C = cq$ and thus get $\pi = T - cq$.

(a) Assume the seller can observe the type θ_i of the consumer. Set up the seller's maximization problem and solve it. Interpret the conditions. Draw the two first-best contracts into the (q,t) plane.

(b) It is unrealistic to assume that the seller knows exactly the type. The seller will instead use a different method to separate the two possible types of buyers. Formulate the maximization problem of the principal if the type is unknown. Solve the problem and compare the results. Draw the result into the (q,t) plane.

2) Bill Barriers, CEO of MightySoft software, is contemplating a new marketing strategy: bundling their best-selling wordprocessor and their spreadsheet together and selling the pair of software products for one price.
From the viewpoint of the company, bundling software and selling it at a discounted price has two effects on sales: (1) revenues go up due to additional sales of the bundle; and (2)

revenues go down since there is less of a demand for the individual components of the bundle.
The profitability of bundling depends on which of these two effects dominates. Suppose that MightySoft sells the wordprocessor for $200 and the spreadsheet for $250. A marketing survey of 100 people who purchased either of these packages in the last year turned up the following facts:

1) 20 people bought both.
2) 40 people bought only the wordprocessor. They would be willing to spend up to $120 more for the spreadsheet.
3) 40 people bought only the spreadsheet. They would be willing to spend up to $100 more for the wordprocessor.

In answering the following questions you may assume the following:
1) New purchasers of MightySoft products will have the same characteristics as this group.
2) There is a zero marginal cost to producing extra copies of either software package.
3) There is a zero marginal cost to creating a bundle.

(a) Let us assume that MightySoft also offers the products separately as well as bundled. In order to determine how to price the bundle, Bill Barriers asks himself the following questions. In order to sell the bundle to the wordprocessor purchasers, the price would have to be less than:

(b) In order to sell the bundle to the spreadsheet users, the price would have to be less than:

(c) What would MightySoft's profits be on a group of 100 users if it priced the bundle at $320?

(d) What would MightySoft's profits be on a group of 100 users if it priced the bundle at $350?

(e) If MightySoft offers the bundle, what price should it set?

(f) What would profits be without offering the bundle?

(g) What would profits be with the bundle?

(h) Is it more profitable to bundle or not bundle?

(i) Suppose that MightySoft worries about the reliability of their market survey and decides that they believe that without bundling t of the 100 people will buy both products, and $(100-t)/2$ will buy the wordprocessor only and $(100-t)/2$ will buy the spreadsheet only. Assume that the willingnesses to pay of these groups are the same as before. Calculate profits as a function of t if there is no bundling.
At what values of t would it be unprofitable to offer the bundle? Assume that the optimum price to bundle is the one as in (e). Would t increase, would it be better to bundle or not to bundle? Explain why.

3) Megan's inverse demand function for running shoes is still $p(q) = \frac{1}{q^2}$.

(a) Find her price elasticity of demand for price *p*.

(b) If the price of running shoes is $20, what is the price elasticity of demand?

(c) Assume that now her demand function changes to $p(q)= 30-q$. Find her price elasticity of demand for price *p*.

(d) If the price of running shoes is $20, what is the price elasticity of demand in this new case?

(e) Assume there is just one monopolist producing the monopoly quantity for her, which she accepts to buy. He knows that demand is $p(q)= 30-q$. The monopolist has constant marginal costs of MC=10. Find the profit-maximizing quantity of the monopolist, price, and compute the Lerner index (Hint: You may as well directly derive the Lerner index from elasticity).

(f) Assume the monopolist has fixed costs of Cf = 100. Find the quantity produced by the monopolist if it is regulated by government according to average cost pricing (case 1), and according to marginal cost pricing (case 2). Find the profit situation in both cases and comment on the result.

4) Sal's satellite company broadcasts TV to subscribers in Los Angeles and New York. The demand functions for each of these two groups are

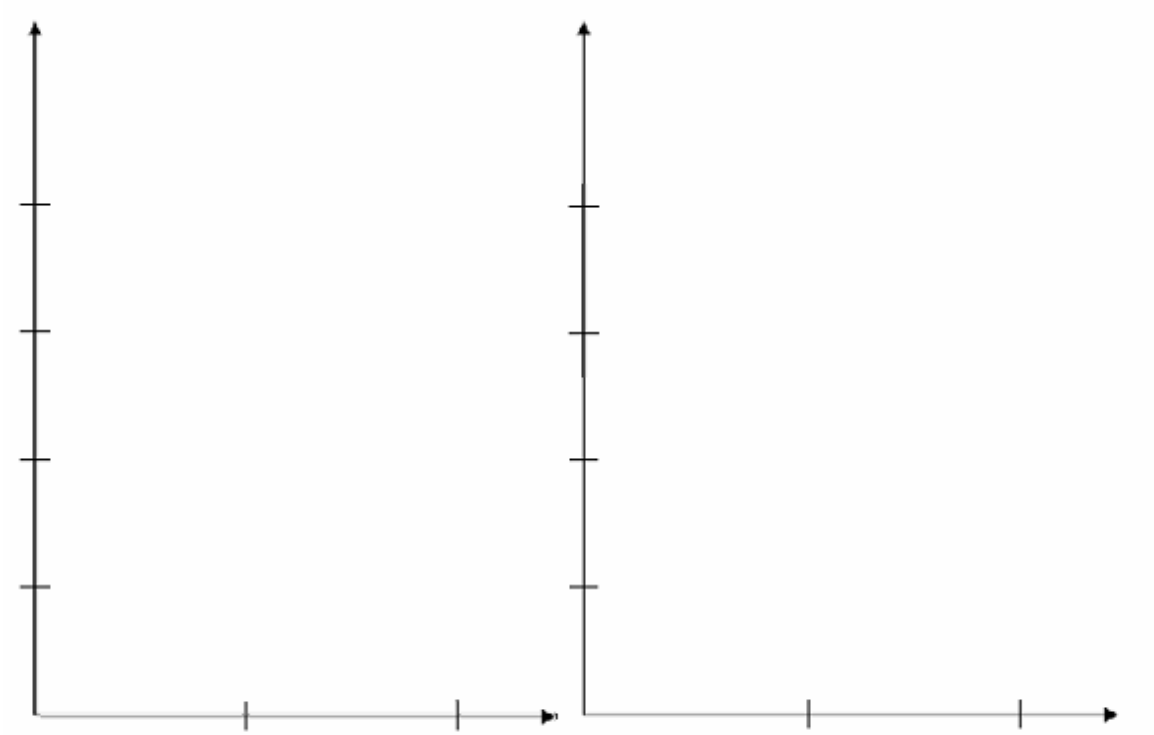
$$Q_{NY} = 60 - 0.25P_{NY}$$
$$Q_{LA} = 100 - 0.50P_{LA}$$

where *Q* is in thousands of subscriptions per year and *P* is the subscription price per year. The costs of providing *Q* units of service is given by

$C = 1000 + 40Q$,

where $Q = Q_{LA} + Q_{NY}$.

a) What are the profit-maximizing prices and quantities for the New York and Los Angeles markets under 3rd degree price discrimination? Sketch the two markets in the space below. Label the axes and intercepts.



b) As a consequence of a new satellite that the Pentagon recently deployed, people in Los Angeles receive Sal's New York broadcasts and people in New York receive Sal's Los Angeles broadcast. As a result, anyone in NY or LA can receive Sal's broadcast in either city. Thus Sal can charge only a single price. What price should he charge, and what quantities will he sell in NY and LA?

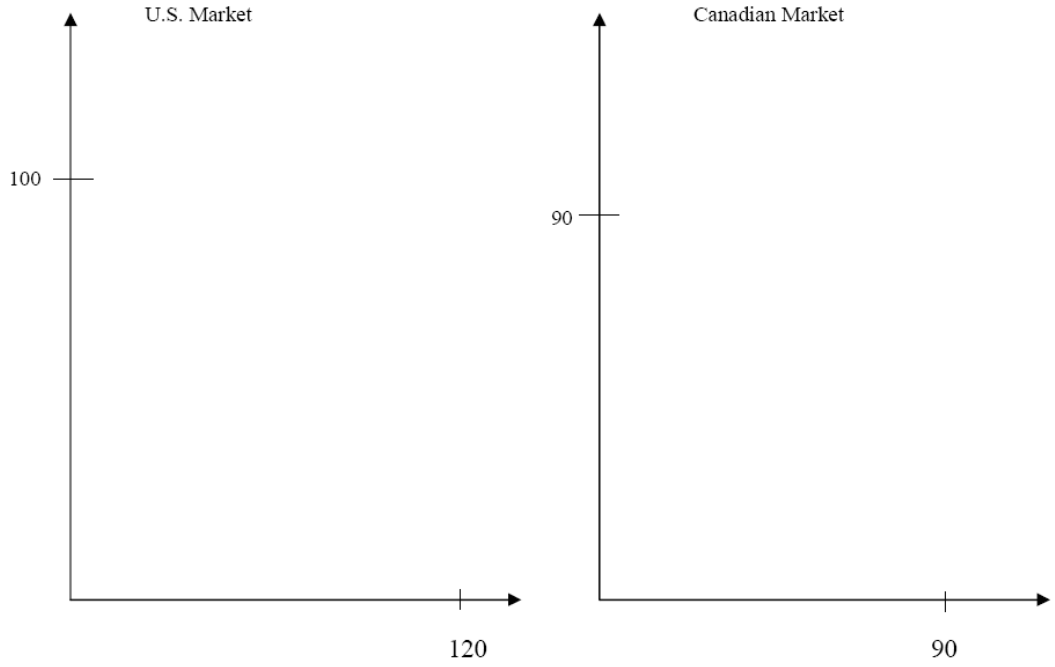
c) Compute the price elasticity of demand in the two cases with price discrimination and in the case without. Also, compute Sal's profit under price discrimination and without, as well as the consumer surplus in each market. Which situation do the consumers in New York prefer, and which do the consumers in Los Angeles prefer? How does this relate to price elasticity? Explain verbally.

5) A Vermont quilt store can sell its products in two markets: Canada and the U.S. . The demand functions are 120 -1.1p in the U.S., and 90-p in Canada. The store's cost function is C=20q.

a) Find the profit-maximizing prices and quantities for the two markets if the store can third-degree price discriminate. Find the Consumer Surplus in each market and the total profit of the quilt store under 3rd-degree price discrimination.

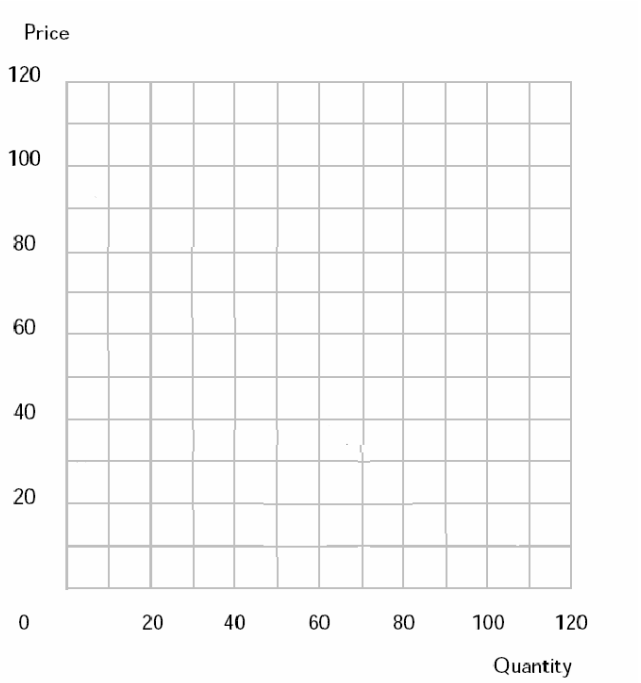
b) Now assume now that 3rd-degree price discrimination is not possible. Find demand, profit maximizing price and quantity in this case, as well as price and quantity in each market. Find the new profit and the new Consumer Surpluses in each market.

c) Graph the market equilibrium with and without 3rd degree price discrimination in the two diagrams below, mark inverse demand, marginal revenue, marginal cost, and all consumer surpluses. Label the axes.



6) An online publisher (seller hereafter) is considering offering articles to readers by email. Their market research division indicates that there are two types of potential users in the market, students and executives. There are the same number of students and executives in the market. Let x be the number of articles that a user requests per year.

The executives have an inverse demand function $PE(x) = 120 - x$ and the students have an inverse demand function $PU(x) = 90 - x$. (Prices are measured in cents.) The publisher has a zero marginal cost of sending articles via email. Draw these demand functions in the graph below and label them.



a) Suppose that the seller can perfectly price discriminate. What is the optimal pricing strategy? What quantities are sold at what price?

b) Now assume that the seller cannot tell the two consumer groups apart. An executive can always buy the package size designed for the student. Assume the size is not changed compared to part a): what are the prices now for each bundle? Explain why the seller gives a "discount" to one group of buyers.

c) It is known that the seller can improve its profit by reducing the package size for the students. Find the optimal package size and profits. Explain.

7) Suppose that BMW can produce any quantity of cars at a constant marginal cost equal to $20,000 and a fixed cost of $10 billion. You are asked to advise the CEO as to what prices and quantities BMW should set for sales in Europe and in the United States. The demand for BMWs in each market is given by

$Q_E = 4,000,000 - 10P_E$,

$Q_U = 1,000,000 - 20P_U$,

where the subscripts indicate the region. All prices and costs are in thousands of dollars. Assume that BMW can restrict U.S. sales to authorized BMW dealers only.

a) What quantity of BMWs should the firm sell in each market, and what should the price be in each market? What should the total profit be?

b) If BMW were forced to charge the same price in each market, what would be the quantity sold in each market, the equilibrium price, and the company's profit?

8) Elizabeth Airlines (EA) flies only one route: Chicago-Honolulu. The demand for each flight is Q=500-P. EA's cost of running each flight is $30,000 plus $100 per passenger.

a) What is the profit-maximizing price that EA will charge? How many people will be on each flight? What is EA's profit for each flight?

b) EA learns that the fixed costs per flight are in fact $41,000 instead of $30,000. Will the airline stay in business for long? Illustrate your answer using a graph of the demand curve that EA faces and the average cost curve with $41,000 and with $30,000.

c) Now EA finds out that two different types of people fly to Honolulu. Type A consists of business people with a demand of QA=260-0.4P, and Type B consists of students with a total demand of QB=240-0.6P. Since students are easy to spot, EA decides to charge them different prices.

Graph each of these demand curves and their horizontal sum. Mathematically find the profit maximizing quantity and prices in each market and illustrate.

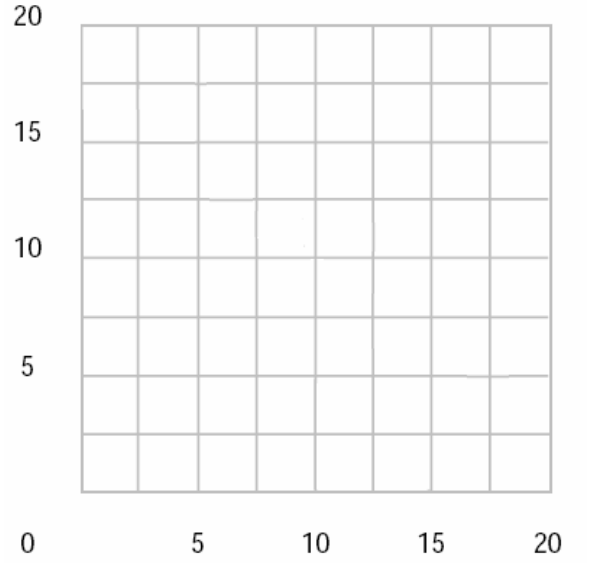
d) What would EA's profit be for each flight? Would the airline stay in business? Calculate the consumer surplus of each consumer group. What is the total CS?

e) Before EA started price discriminating, how much consumer surplus was Type A demand getting from air travel to Honolulu? Type B? Why did total consumer surplus decline with price discrimination, even though total quantity sold remained unchanged?

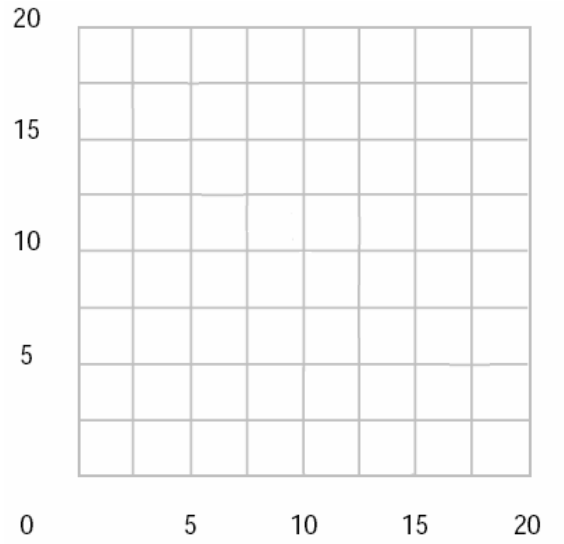
9) The Ball Street Journal is considering offering a new service which will send news articles to readers by email. Their market research indicates that there are two types of potential users, impecunious students and high-level executives.

Let x be the number of articles that a user requests per week. The executives have an inverse demand function $PE(x) = 20 - x$ and the students have an inverse demand function $PU(x) = 15 - x$. The Journal has a zero marginal cost of sending articles via email. There is an equal number of executives and students in the market.

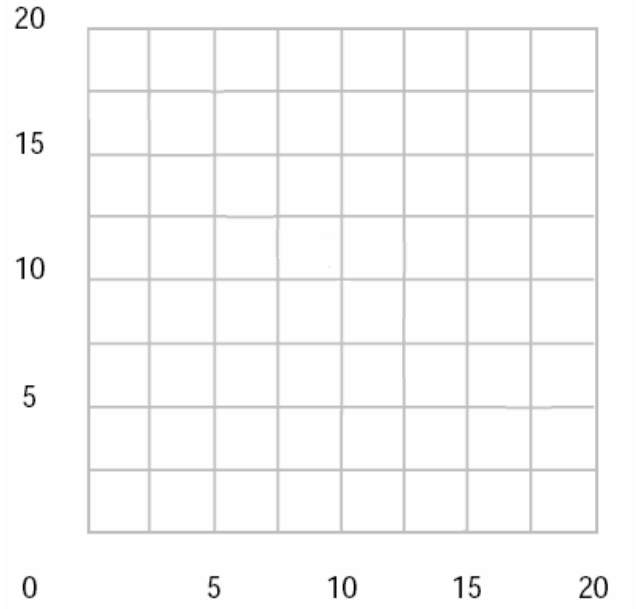
a) Assume first that BSJ can tell the consumer groups apart. Graph the demand curves into the chart below and find the profit-maximizing quantity and price for each group, assuming that BSJ can price discriminate. Label the axes. Find the profit (equal to total revenue) assuming there is 1 consumer in each group.



b) Now assume that BSJ can still price discriminate but cannot tell the consumers apart. That is, by offering the two packages sizes that are optimal in part a), it will need to change the price of one package. Find the optimal price for this particular package as well as the consumer surplus (sometimes called information rent) that is left to one group. Find the profit (equal to total revenue) assuming there is 1 consumer in each group. Graph the result into the chart below. Label the axes.



c) Now assume the setting in b) but that BSJ sells the same quantity to the executives as before, but reduces the size of the package designed to the students to 10. Find the price that the students are willing to pay, and find the new consumer surplus (or information rent if you want to call it this way) that needs to be paid to the executives in this new case. Again find the profit (equal to total revenue) assuming there is 1 consumer in each group. Graph the result into the chart below. Label the axes.



d) Last, assume that package sizes as in c) and drop the initial assumption of an equal proportion of students to executives in the market.

- For which proportion of students to executives would this package size be revenue-maximizing?

- For which proportion of students to executives would BSJ be equally well off to not offer the package with 10 anymore but only cater to executives? Show your work.

10) Solve the workbook problems #25.6, #25.7, #25.8