

Problem Set #3 Econ21 – Summer 2007

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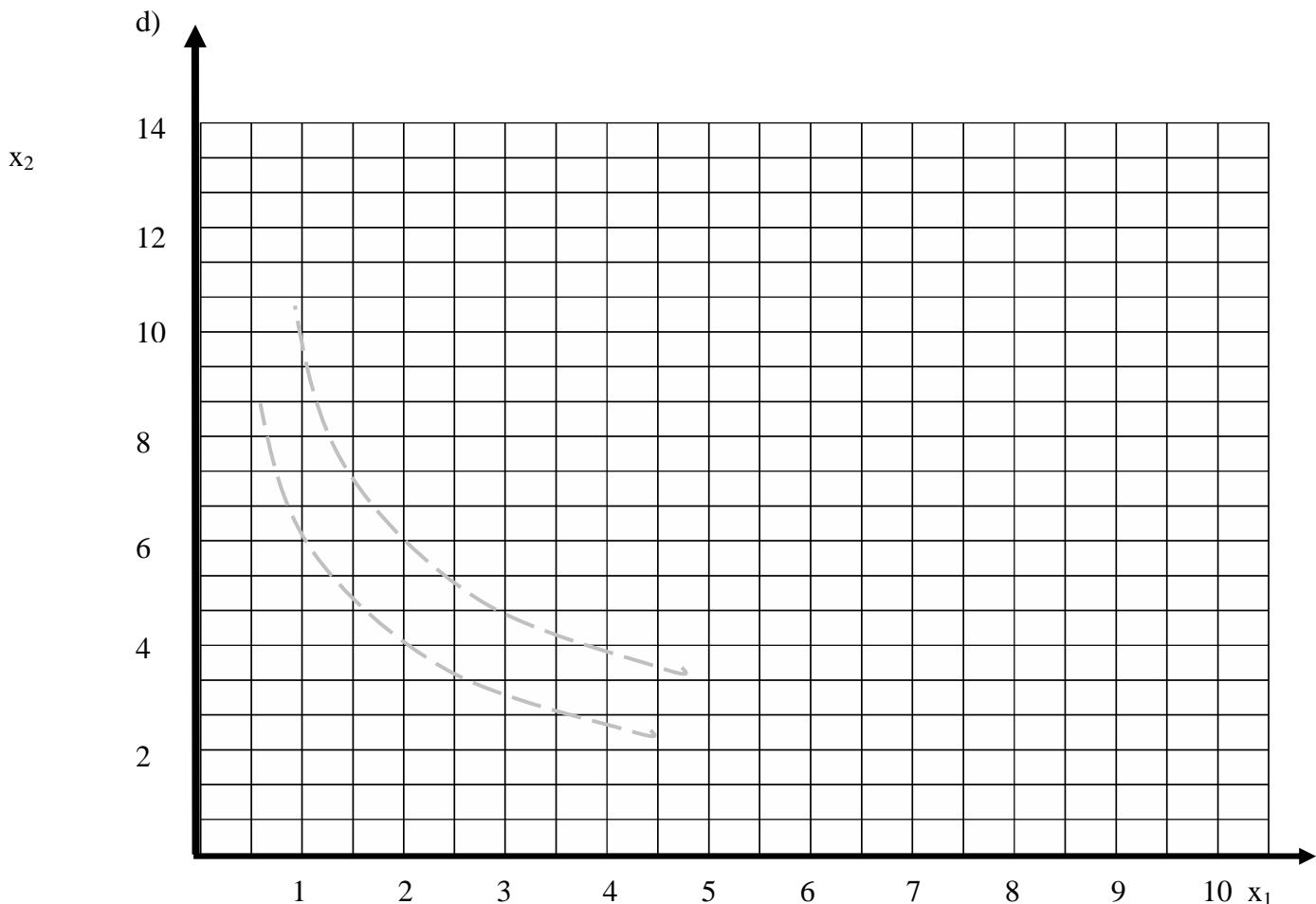
Posted July 25, presentations in class Aug 1 (problems marked \*\* are a bit advanced)

1) Kelly Umpleby is a Cobb-Douglas consumer. Her preferences follow the utility function  $u(x_1, x_2) = x_1^2 x_2^3$ . Good 1 has a positive price of  $p_1$ . Good 2 is consumption, with a price normalized to one. Her budget is  $m$ .

a) Find Kelly's individual demands. Explain verbally why the utility function  $\tilde{u}(x_1, x_2) = x_1^{2/5} x_2^{3/5}$  leads to the same individual demands. Show that this is the case, either by using the Lagrangian anew and maximizing  $\tilde{u}$ , or by monotonically transforming it using a valid monotonic transformation for  $u$ .

b) Find Kelly's price elasticity of demand.

c) Now use the following initial numeric values: price of Good 1 is  $p_1 = 2$ , budget is  $m = 10$ . Compute demands and utility for this initial bundle. Graphically illustrate the solution by drawing the budget line and the original allocation (Point A) into the chart below. Find Kelly's utility in Point A.

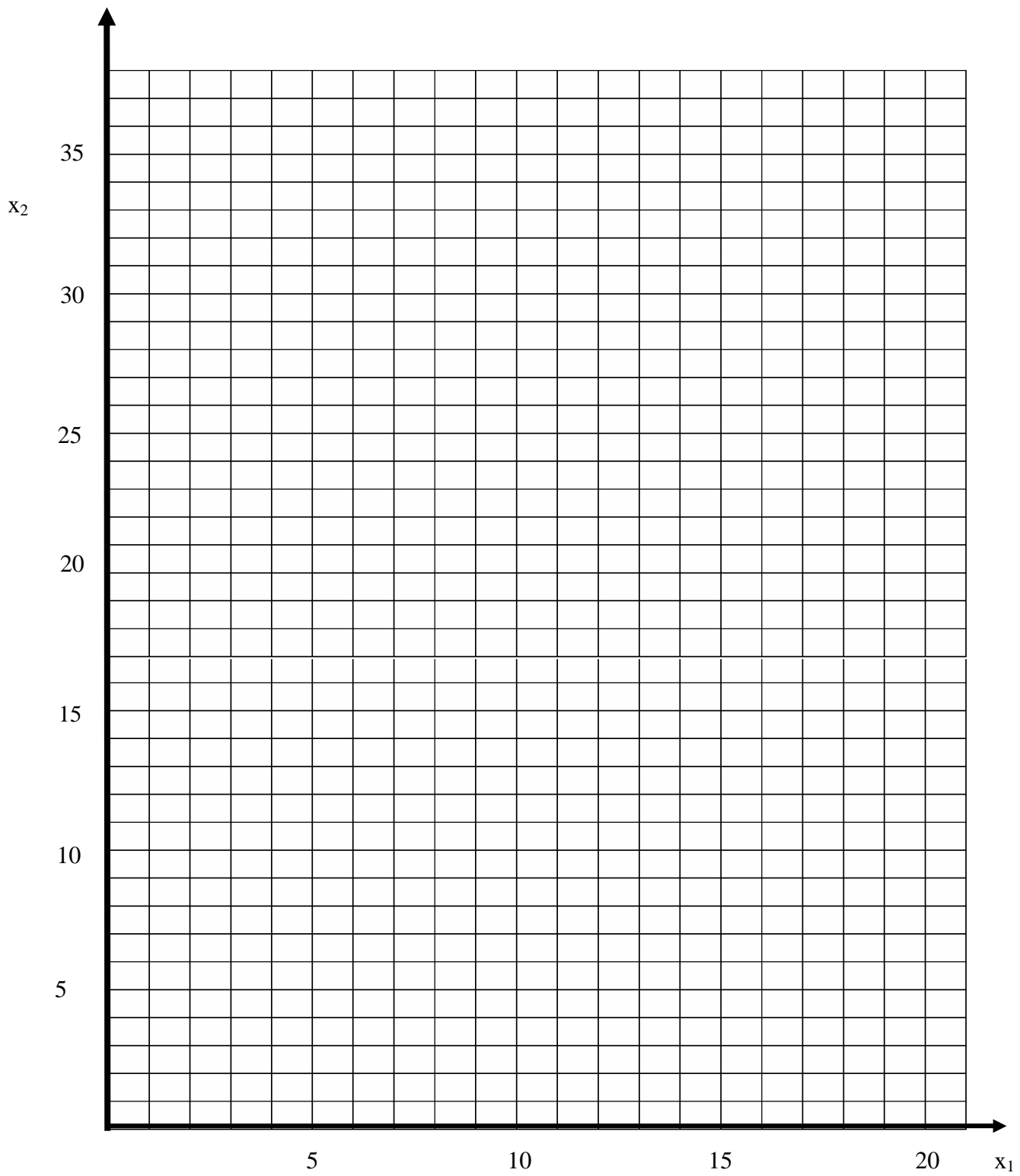


- e) Government is now preparing a quantity tax on Good 1, which increases the price to  $p_1' = 4$ . Compute Kelly's new equilibrium and illustrate this change in the diagram above by marking the new equilibrium point (label it with any letter you like) and the budget line that runs through it. Find Kelly's utility in this point.
- f) Find the Slutsky substitution and income effect mathematically and illustrate graphically. You need to draw all the budget lines. As as in d) you are free to use any letter to mark the new point in the diagram.
- g) Graphically mark the Hicks substitution effects by finding  $B^{CV}$  and  $B^{EV}$  in the diagram above. Make sure you draw the right budget line through these two points. Mark the CV and the EV in the diagram. The gray lines mark the indifference curves in their (roughly) correct position to help you graphing the steps.
- h) Now compute the CV and the EV. You may either use (i) Hicks demand and find the real coordinates to  $B^{CV}$  and  $B^{EV}$  with their income, respectively, or (ii) use the "Varian method" by plugging individual demands into utility and solve for  $m$ .
- i) Find Kelly's Net Consumer Surplus by using the integral over demand function  $x_1(p_1)$ .

2) Castaway survives on Coconut Island by eating fish (good 1) and coconuts (good 2). In one week, he is able to catch 10 baskets of fish and harvest 5 tons of coconuts. He is a self supporter since he lives alone on his island.

(a) Castaway discovers that he lives on an island surrounded by other islands with a population of really friendly aborigines (henceforth called his trade partners). They actually discover him on their usual jetski joyrides. They think of him as of a romantic dropout that likes living in this nice part of the world. They believe that he likes trading fish and coconuts as much as they do, so they decide to visit him regularly. After trying for weeks, Castaway actually has given up explaining that he is stranded and wants to be rescued, but makes the best out of his situation by playing their trading game. Castaway has the following preferences for the two goods:  $u(x_1, x_2) = x_1^4 x_2^6$ . His trade partners pay him 1 Lira for each coconut and 2 Lire for each fish.

Mark his endowment point  $E$ . Use the chart on the next page. Then, both algebraically and graphically find his choice  $A$ . Is Castaway a net seller or net buyer of fish?



(b) Now assume that the next week his trade partners become more interested in fish and pay him 3 Lire per fish. Algebraically and graphically find his individual demand for fish

after substitution effect, after his ordinary income effect, and after his endowment income effect.

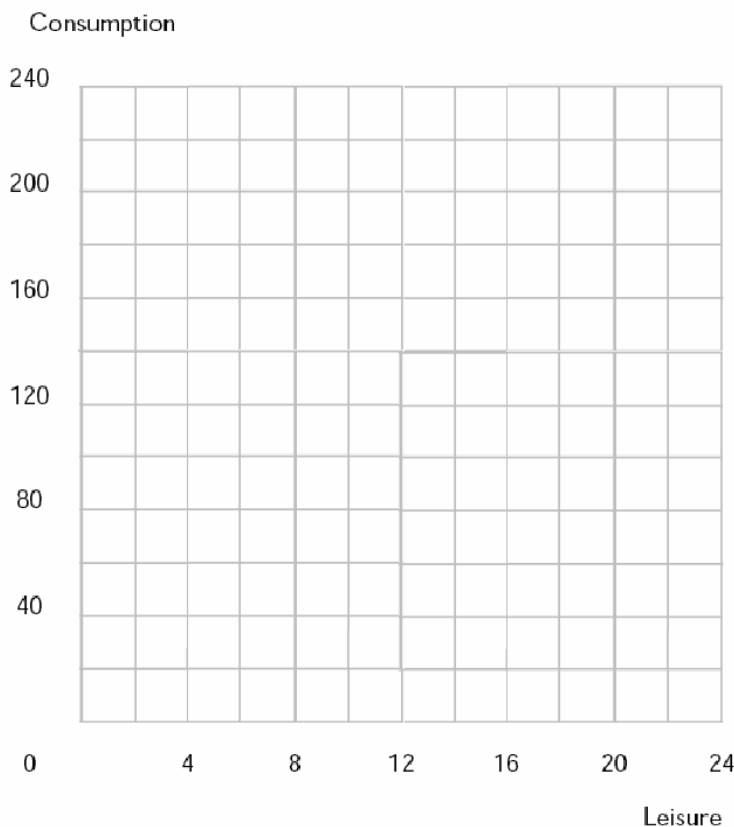
(c) Mark all points that represent the change according to the substitution effect and the two income effects in the chart above. Use arrows to mark the effects along the  $x_1$ -axis.

(d) Write down the Slutsky equation in the form including substitution effect and the two income effects. Mark the signs of the derivatives and explain. By using only your information about the position of  $E$  and  $A$ , can you predict the outcome (reduction/increase of consumption of good 1) from the Slutsky equation? Explain.

(e) Find Castaway's price elasticity of demand.

3) Michael Smart works in a bookstore. He can work as many hours per day as he wishes at a wage rate of  $w$ . Let  $C$  be the number of dollars he spends on consumer goods and let  $R$  be the number of hours of leisure that he chooses.

(a) Michael is paid \$11 an hour and has 16 hours per day to devote to labor or leisure. He furthermore receives a daily nonlabor income of \$ 24 from his mother. Price of Consumption is \$1. Write down the equation for his budget between consumption and leisure and draw this line into the chart below. Mark his endowment point.



b) Assume that Michael has Cobb-Douglas preferences for leisure and consumption of the style  $U(R,C)=R^2C^2$ . How many hours will he work (you may round the result)? What will be his amount of Consumption worth in \$?

c) Write down Michael's Slutsky equation for labor and leisure and mark the signs of each term. Please use the correct notation for the variables. Can you predict without knowing the magnitude of each effect whether he will reduce or increase labor supply? Why or why not? Explain.

d) Michael is lucky. His wage increases to  $w=12$ . Write down the equation for the new budget line and draw this new budget line into the graph on page 1. Mathematically find his new allocation. Does he increase or decrease labor supply? Thus, which effect has dominated, the substitution or the (total) income effect?

4) Solve the workbook problem #10.3

5) Will Smith is a proud citizen of a Wrightsville, a town with far-reaching democratic achievements. One of them is that no tax (and we assume the town has the right to introduce quantity taxes on any kind of goods that Will consumes) can be imposed on a citizen without having his written consent.

Will is also a proud consumer of the *Cobb-Douglas* type, consuming blue woodberries (a local specialty) as good 1 and a bundle of all goods that we call good 2. His preferences are expressed by  $u(x_1, x_2) = 2 \ln x_1 + \ln x_2$ . Price of good 2 is normalized to 1, and price of good 1 is 4. His income is  $m=120$ .

The mayor of the Wrightsville, in constant need of money, now thinks about introducing a  $t = \$1$  quantity tax on woodberries. Solve algebraically:

(a) What is the amount of money that Will would lose in terms of lost utility should such a tax be imposed, compared to his original situation?

(b) Assume Will's position is less favorable and the mayor has the right to introduce any tax without consent. Assume he already introduced the 20% quantity tax on woodberries and now the two bargain over the "damage" in lost utilities, without a chance that the tax could be revoked. How much money would he need to be compensated compared to the old utility level?

(c) Compare the result with his Net Consumer Surplus.

6) Solve the workbook problems #14.3, #14.6, #14.8.

\*\*7) Algebraically show that for any quasilinear utility function the income effect is zero.

\*\*8) Consider *Cobb-Douglas* preferences of  $u(x_1, x_2) = x_1^a x_2^{1-a}$ . Assume an increase in the price of good 1 from  $p_1$  to  $p'_1$ . Algebraically derive the Equivalent Variation (EV) and the Compensating Variation (CV). Algebraically show that  $CV > EV$ .

Hints (for EV): Since B and C (for CV: A and B) are on the same indifference curve, you may e.g. solve the utility function for good 2 and setting it equal to your demand value of good 2. Since you have two different income levels, namely  $m$  and  $m'$ , by comparing the

two utility levels you will reach a relation of  $\frac{m'}{m} = \left(\frac{p_1'}{p_1}\right)^a$  which you can solve for  $m'$ .

From this you can define the EV as income differences, and you know that  $p_1' > p_1$ . The definition for CV follows a similar procedure. Comparison EV-CV is trivial). ■

9) Solve the workbook problems #15.1, #15.2, #15.3, #15.4