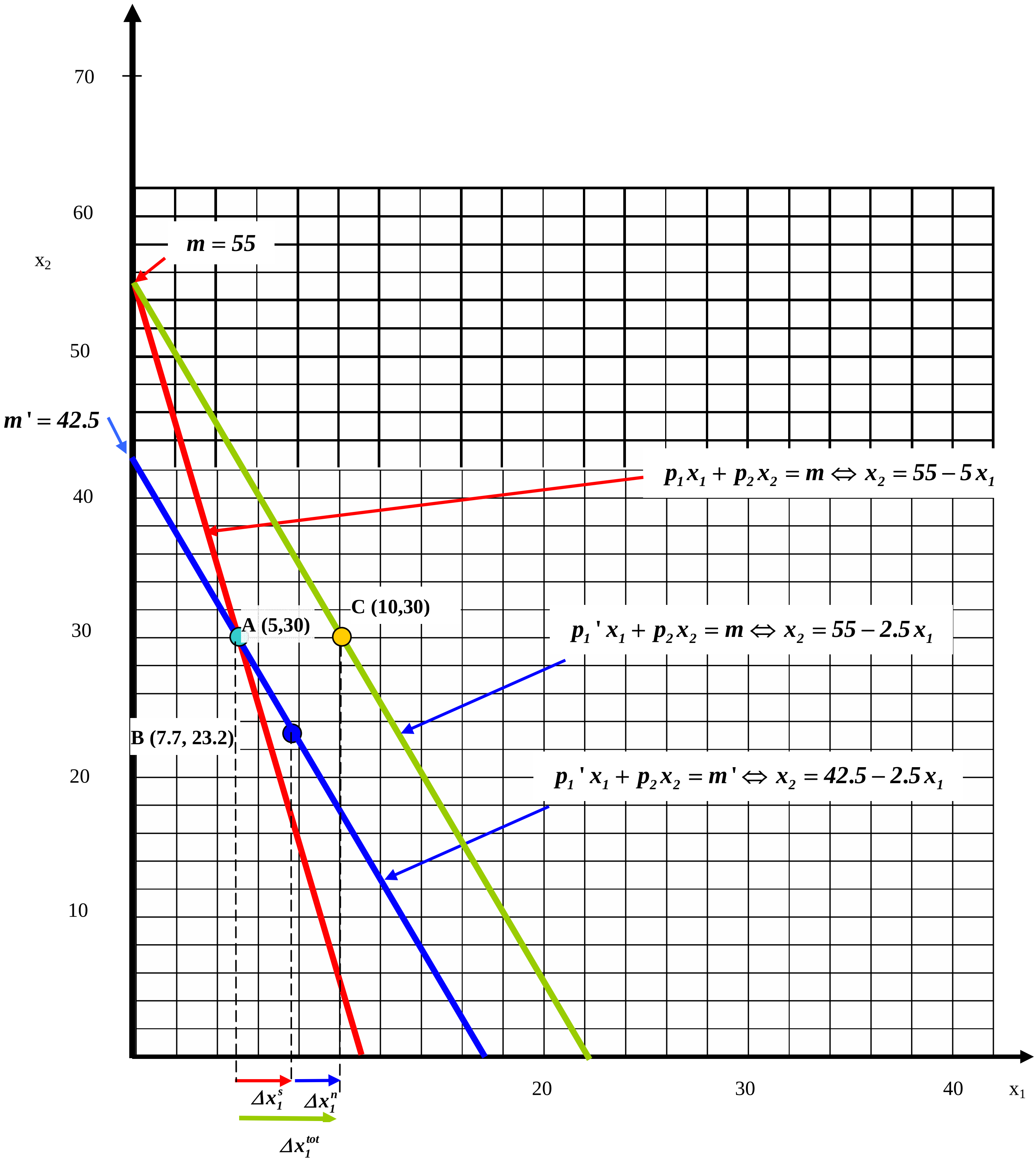
More exercises – July 17, 07

1 a) Francesco has Cobb-Douglas preferences of $u(x_1, x_2) = 7x_1^5 x_2^6$. By using the Lagrange method, find his individual demands for the two goods.

b) Find his initial consumption bundle for the two goods, assuming $m=55$, $p_1 = 5$ and $p_2 = 1$. Draw his original budget line and his bundle into the graph below. Mark the bundle with an "A" and the budget line using the budget equation with the values for m and the prices.



c) Assume that the price for good 1 decreases to $p_1' = 2.5$. Find his income compensation in A and compute the new compensated equilibrium, and mark it with a "B." Draw the new budget line into the above diagram and label it with the right expression using new prices and new income. Mathematically find his substitution effect and mark it in the diagram above as well.

d) Find his final consumption bundle at new prices and mark it with a "C." Draw the final budget line into the above diagram and label it with the right expression using the right values for income and prices. Mathematically find his income effect and mark it in the diagram above as well.

Solution:

a) Before starting, we find that a simple monotonic transformation of v=u/7 reduces complecity. We henceforth use $v(x_1, x_2) = x_1^5 x_2^6$

$$L(x_1, x_2, \lambda) = x_1^6 x_2^5 + \lambda(m - p_1 x_1 - p_2 x_2)$$

$$\frac{\partial L}{\partial x_1} = 6x_1^5 x_2^5 - \lambda p_1 = 0 \qquad (1)$$

$$\frac{\partial L}{\partial x_2} = 5x_1^6 x_2^4 - \lambda p_2 = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = m - p_1 x_1 - p_2 x_2 = 0. \qquad (3)$$

Dividing (1) by (2) gives the tangency condition $\frac{6x_2}{5x_1} = \frac{p_1}{p_2}$, from which $x_2 = \frac{5}{6}\frac{p_1}{p_2}x_1$.

Using the budget line we find demand for good 1: $p_1 x_1 + p_2\left(\frac{5}{6}\frac{p_1}{p_2}x_1\right) = m$. This leads to

$x_1 = \frac{5}{11}\frac{m}{p_1}$, plugged into tangency condition again yields $x_2 = \frac{6}{11}\frac{m}{p_2}$.

b) Using the values given above, namely $m=55$, $p_1 = 5$ and $p_2 = 1$ we find the coordinates of Point A:

$$x_1^A = x_1(p1 = 5, m = 55) = \frac{5}{11}\frac{55}{5} = 5\ .$$

$$x_2^A = x_2(p2 = 1, m = 55) = \frac{6}{11}\frac{55}{1} = 30.$$

Point A has the coordinates A(5,30) (see graph).

Now, as given above, price for good 1 falls to $p_1' = 2.5$

We first find his compensated income that permits him to exactly consume bundle A at the new price of 2.5 .

Version 1: Check first the intuition behind: He initially buys 5 units and pays \$5 each. Now he saves half of this money, which is \$12.5

Version 2: Now we are ready to compute this compensation:
$\Delta m = \Delta p_1 x_1{}^A = (2.5-5)5 = 12.5$.

We then reduce his original income m into $m' = m + \Delta m = 55 - 12.5 = 42.5$. (see graph).

Finding Point B:

$$x_1^B = x_1(p_1' = 2.5, m' = 42.5) = \frac{5}{11}\frac{42.5}{2.5} = 7.72 \approx 7.7$$

$$x_2^B = x_2(p_2 = 1, m' = 42.5) = \frac{6}{11}\frac{42.5}{1} = 23.18 \approx 23.2 \ .$$

Finding the Substitution Effect $\Delta x_1{}^s$:

$\Delta x_1{}^s = x_1^B - x_1^A = 7.7 - 5 = 2.7$ (see graph).

Finding Point C:

$$x_1^C = x_1(p_1' = 2.5, m = 55) = \frac{5}{11}\frac{55}{2.5} = 10.$$

$$x_2^C = x_2(p_2 = 1, m = 55) = \frac{6}{11}\frac{55}{1} = 30.$$

Finding the Substitution Effect $\Delta x_1{}^s$:

$\Delta x_1{}^n = x_1^C - x_1^B = 10 - 7.7 = 2.3$ (see graph).

See also the total effect graphed in the diagram above: $\Delta x_1{}^{tot} = \Delta x_1{}^s + \Delta x_1{}^n = 5$.

(2) A consumer has preferences following the utility function $u(x_1, x_2) = x_1^7 x_2^4$.

a) Derive the marginal utilities of the consumer and the MRS.

<u>Solution:</u>

$$MU_1 = \frac{\partial u}{\partial x_1} = 7x_1^6 x_2^4$$

$$MU_2 = \frac{\partial u}{\partial x_2} = 4x_1^7 x_2^3$$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{7x_1^6 x_2^4}{4x_1^7 x_2^3} = -\frac{7x_2}{4x_1}.$$

b) Transform this utility function into any function $v = \ln u$. Algebraically show that the MUs now change but that the MRS is the same as in part a).

Solution:

Using $v = \ln u$ leads to $v = \ln u = \ln(x_1^7 x_2^4) = 7 \ln x_1 + 4 \ln x_2$.

$$\frac{\partial v}{\partial x_1} = \frac{7}{x_1}$$

$$\frac{\partial v}{\partial x_2} = \frac{4}{x_2}$$

The MUs indeed change.

$$MRS = -\frac{\frac{7}{x_1}}{\frac{4}{x_2}} = -\frac{7x_2}{4x_1}$$

This is the same result for the MRS as before.