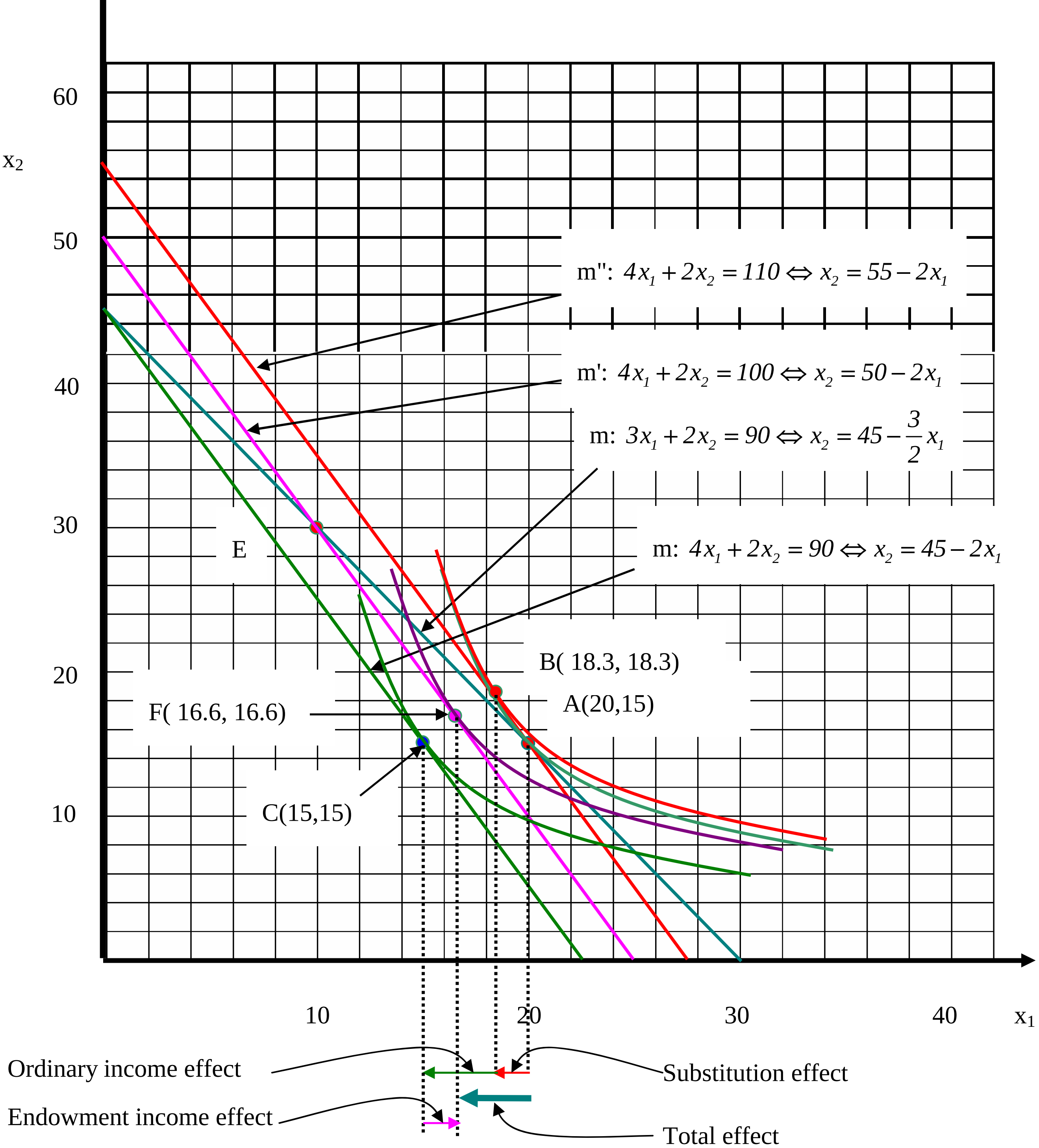
Solutions to "More Midterm 2 Exercises" Aug 7, 07

1. Charles LaSalle is a farmer in Dubuque, IA. He produces corn (good 1) and soybeans (good 2). This year, his crop amounts to 30 bushels of soybeans and 10 bushels of corn.

(a) Assume Charles is a self-supporter Draw his endowment point E into the graph below.



(b) The solution below uses Lagrange:

$$L(x_1, x_2, \lambda) = u(x_1, x_2) = 4x_1^4 x_2^2 + \lambda(m - p_1 x_1 - p_2 x_2)$$

with

$$\frac{\partial L}{\partial x_1} = 16x_1^3 x_2^2 - \lambda p_1 = 0 \quad (1)$$

$$\frac{\partial L}{\partial x_2} = 8x_1^4 x_2 - \lambda p_2 = 0 \quad (2)$$

$$\frac{\partial L}{\partial \lambda} = m - p_1 x_1 - p_2 x_2 = 0 \quad (3)$$

We proceed as usual and divide (1) by (2) , which yields the tangency condition

$$\frac{p_1}{p_2} = 2\frac{x_2}{x_1}$$

Solving e.g. for x_2 yields $x_2 = \frac{p_1 x_1}{2p_2}$

and inserting into the budget constraint (3) yields individual demand:

$$p_1 x_1 + p_2\left(\frac{p_1 x_1}{2p_2}\right) = m \Leftrightarrow x_1 = \frac{2m}{3p_1}.$$

and, since $x_2 = \frac{p_1 x_1}{2p_2} \Leftrightarrow x_2 = \frac{p_1 \frac{2m}{3p_1}}{2p_2} = \frac{m}{3p_2}$. .

c) The income he derives from the endowment is: $p_1\omega_1 + p_2\omega_1 = 3\cdot 10 + 2\cdot 30 = 90.$

We use our individual demands to determine his optimal allocation A:

$$x_1^A = \frac{2m}{3p_1} = \frac{2\cdot 90}{3\cdot 3} = 20,\; x_2^A = \frac{90}{3\cdot 2} = 15.$$

We have A(20,15). Since A is right of E, he is a net buyer of corn. He buys $x_1 - \omega_1 = 20 - 10 = 10$ bushels of corn and sells $\omega_2 - x_2 = 30 - 15 = 15$ bushels of soy-beans.

(d) The price of corn increases to $p_1' = 4$. According to Slutsky, his budget line pivots through A. To reach A, he is compensated such that he can afford the bundle A at new prices, namely $p_1' = 4$. We call his new income m":

$$m'' = p_1' x_1^A + p_2 x_2^A = 4x_1 + 2x_2 = 4 \cdot 20 + 2 \cdot 15 = 110.$$

Plugging into demand using m" and new prices we find the coordinates B($(18.\overline{3}, 18.\overline{3})$.

- The substitution effect is the change in corn demand when moving from A to B

$$\frac{\Delta x_1^s}{\Delta p_1} = x_1(m, p_1, p_2) - x_1(m'', p_1', p_2) = x_1^B - x_1^a = \frac{2m''}{3p_1'} - \frac{2m}{3p_1}.$$

Using numeric values:

$$\frac{\Delta x_1^s}{\Delta p_1} = x_1^B - x_1^A = \frac{2m''}{3p_1'} - \frac{2m}{3p_1} = \frac{220}{12} - \frac{180}{9} = 18.\overline{3} - 20 = -1.\overline{6}.$$

We find Point C by plugging into demand using original income and new prices: C(15,15).

- The ordinary income effect represents the change in corn demand when moving from B to C:

$$-x_1 \frac{\Delta x_1^m}{\Delta p_1} = x_1(m, p_1', p_2) - x_1(m'', p_1', p_2) =$$

$$x_1^C - x_1^B = \frac{2m}{3p_1'} - \frac{2m''}{3p_1'} = \frac{180}{12} - \frac{220}{12} = 15 - 18.\overline{3} = -3.\overline{3} \ .$$

- His final choice is income compensated with respect to E. Thus, he has the same purchasing power as in E but at new prices. To guarantee E at new prices, the price increase from p_1 to p_1' must be compensated at the given old bundle of a quantity $\omega_1 = 10$ apples. We find out his *new* (pink) budget line through E

$$m' = p_1' x_1^E + p_2 x_2^E = 4x_1 + 2x_2 = 4 \cdot 10 + 2 \cdot 30 = 100.$$

Using m', p_1', and p_2 we have $x_1^F = \frac{2m'}{3p_1'} = \frac{2 \cdot 100}{3 \cdot 4} = 16.\overline{6}$, $x_2^F = \frac{100}{3 \cdot 2} = 16.\overline{6}$.

The endowment income effect represent the change in corn demand when moving from C to F:

$$\omega_1 \frac{\Delta x_1^m}{\Delta p_1} = x_1(m', p_1', p_2) - x_1(m, p_1', p_2) =$$

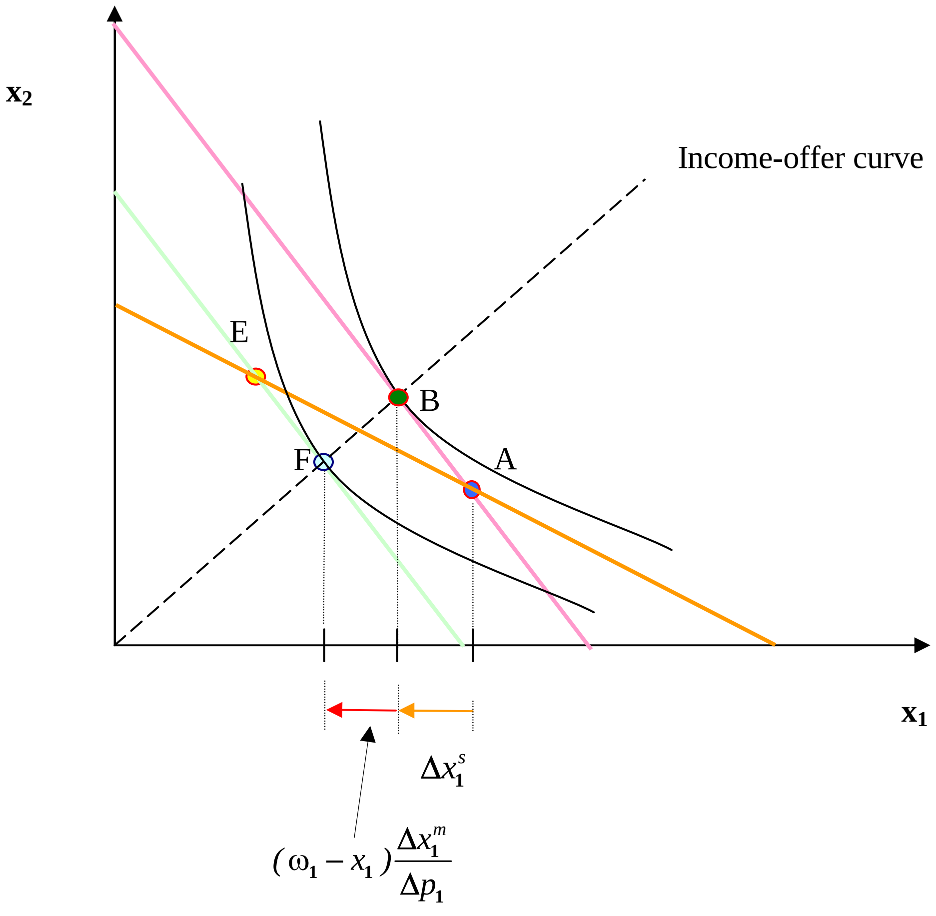
$$x_1^F - x_1^C = \frac{2m'}{3p_1'} - \frac{2m}{3p_1'} = \frac{200}{12} - \frac{180}{12} = 16.\overline{6} - 15 = 1.\overline{6} \ .$$

e) Slutsky equation for buying and selling refers to the rates of change following a price change. It is

$$\frac{\Delta x_1}{\Delta p_1} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} + \underbrace{(\omega_1 - x_1)}_{\substack{(-) \\ \text{if net buyer}}} \underbrace{\frac{\Delta x_1^m}{\Delta p_1}}_{(+)}$$

The first term is the substitution effect. It is always negative as opposed to the price change. Since we have a price increase, B must be left of A, and the substitution effect is negative.

The right-hand term in the sum is the combined income effect. It refers to the remaining net effect of the two income effects. Since E is left of A, x_1 exceeds ω_1, and the term in parenthesis is negative. Since the income derivative (last term) is positive (if we derive $\frac{2m}{3p_1}$ w.r.t. to m we get a positive term since price is positive) , the right-hand term in the sum is negative, and F must be left of B. Thus, we add two arrows pointing to the left: substitution effect (moving from A to B) and combined income effect (B to F). Graph:



2. Peter Morgan sells pigeon pies from a pushcart in Central Park. Morgan's cost function is $C(y)=0.5y^2$. He is the only seller of pigeon pies in Central Park. The inverse demand curve for this delicacy is $p(y)=120-y$, where the price p is measured in cents and y measures the number of pies sold.

a) Find Peter's supply function.

b) Find the price and quantity that maximize Morgan's profit. Find Morgan's profit.

Solution:

a) Morgan is a monopolist. A monopolist has no supply function but determines profit maximizing quantity according to MR=MC.

b) We find the profit maximizing quantity y^* by using the first-order condition:

$$MR = MC$$
$$120 - 2y = y \Leftrightarrow y = 40.$$

Price is found by plugging into inverse demand:

$$p = 120 - y = 80.$$
$$\pi = 40 \cdot 80 + 0.5 \cdot 40^2 = \$2400.$$

Profit is revenue minus cost:

$$\pi = 40 \cdot 80 + 0.5 \cdot 40^2 = \$2400.$$ ■