

## Deriving the extended Slutsky equation for "Buying and Selling" using a numeric example (price increase)

We start with the following example. A consumer has Cobb-Douglas utilities of  $u(x_1, x_2) = x_1 x_2$  and faces positive prices.

Her individual demands are (use Lagrangian)  $x_1 = \frac{m}{2p_1}$  and  $x_2 = \frac{m}{2p_2}$ .

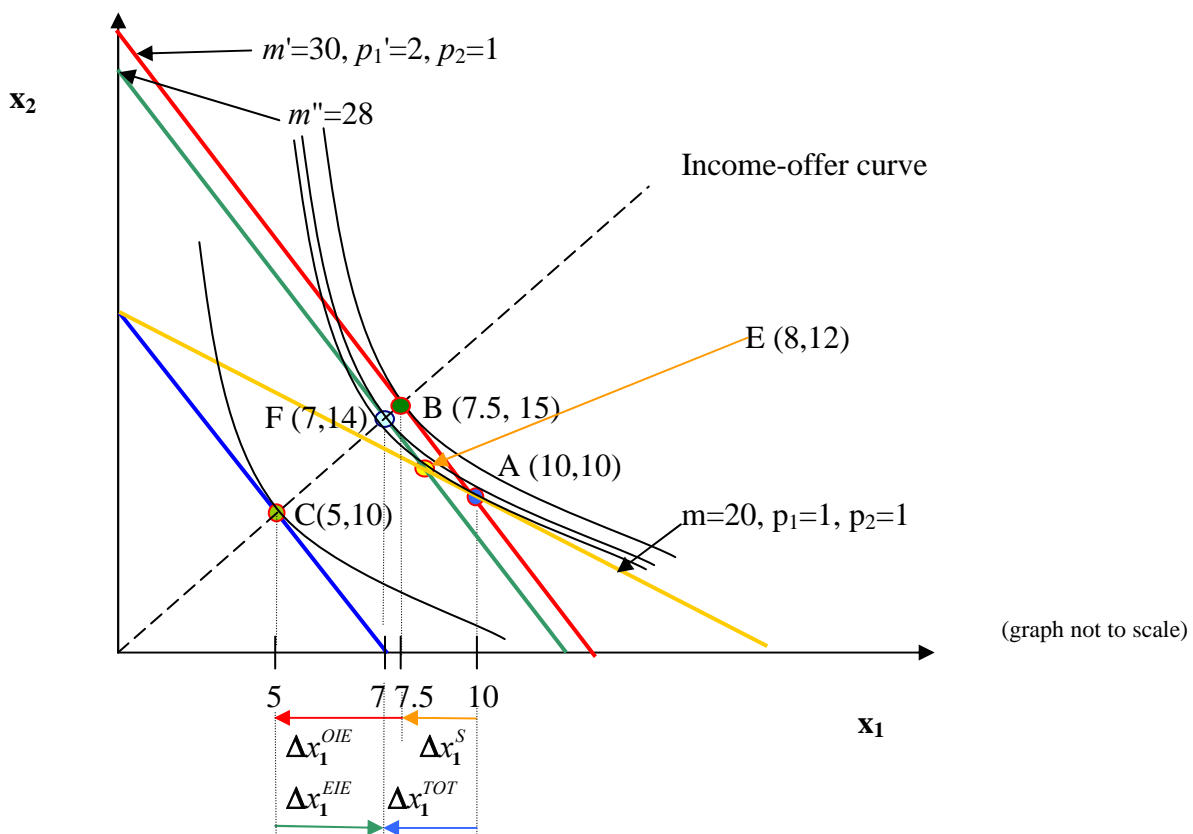
She is endowed with 8 units of good 1, and 12 units of good 2. Prices are  $p_1=1, p_2=1$ .

Her original income generated through the endowment at given prices is

$p_1\omega_1 + p_2\omega_2 = m = 20$ . We find her individual demands:

$x_1^A = \frac{m}{2p_1} = \frac{20}{2} = 10$ , and  $x_2^A = \frac{m}{2p_2} = \frac{20}{2} = 10$ , thus her original bundle is A(10,10) (see

graph below).



Now assume that price increases to  $p_1'=2$ . Slutsky income compensation through A is  $\Delta p_1 x_1 = (2-1)10=10$ . New compensated income is  $m'=30$ . The coordinates for the new compensated bundle are  $x_1^B = \frac{m'}{2p_1'} = \frac{30}{4} = 7.5$ , and  $x_2^B = \frac{m'}{2p_2} = \frac{30}{2} = 15$ .

We label this bundle B (7.5, 15). Slutsky Substitution effect is:

$$\Delta x_1^S = x_1(m', p_1') - x_1(m, p_1) = x_1^B - x_1^A = 7.5 - 10 = -2.5.$$

Before discussing the extended Slutsky equation, we compute the two income effects in our example:

**Ordinary Income Effect:**

We first compute her bundle with old income and new prices:

$$x_1^C = x_1(m, p_1') = \frac{m}{2p_1'} = \frac{20}{4} = 5, \text{ and } x_2^C = x_2^A = x_2(m, p_2) = \frac{m}{2p_2} = \frac{20}{2} = 10 .$$

C (5,10).

We know that the ordinary income effect is

$$\Delta x_1^n = x_1(m, p_1') - x_1(m', p_1') = x_1^C - x_1^B = 5 - 7.5 = -2.5 .$$

On page 145, Varian already restates the total effect, namely

$$\Delta x_1^{tot} = \Delta x_1^s + \Delta x_1^n$$

into

$$\Delta x_1^{tot} = \Delta x_1^s - \Delta x_1^m$$

That is,  $\Delta x_1^m$  is simply defined as negative of the income effect  $\Delta x_1^n$ .

After this transformation, we can define the **rates of change** for the total effect, using equation (8.3) for the ordinary income effect:

$$\frac{\Delta x_1^m}{\Delta m} x_1 = \frac{x_1(p_1', m') - x_1(p_1', m)}{m' - m} x_1 = \underbrace{\frac{x_1^B - x_1^C}{30 - 20}}_{\Delta m} x_1^A = \frac{7.5 - 5}{10} \cdot 10 = 2.5$$

The extended Slutsky equation on p. 170 reads

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - x_1 \frac{\Delta x_1^m}{\Delta m} + \text{endowment income effect}$$

and using  $-x_1 \frac{\Delta x_1^m}{\Delta m} = -10 \frac{2.5}{10} = -2.5$ . As always, the value for  $x_1$  is the original demand for good 1 in Point A.

**Endowment Income Effect:**

We continue with to equation (9.2). Note that  $\Delta x_1^m$  stands for the demand change due to the ordinary income effect, while  $\Delta m = m'' - m$  is used to compensate for the increase in  $p_1$ , yielding

$$p_1' \omega_1 + p_2 \omega_2 = m''.$$

Similar to the ordinary income effect, where  $x_1$  defines the effect multiplied with the income derivative,  $\omega_1$  **now defines the change in income when price changes:**

$$\frac{\Delta m}{\Delta p_1} = \frac{28-20}{1} = \omega_1 = 8. \quad (1)$$

The consumer has 8 units of good 1 to sell in E, and the price increase is 1, thus her income increases by \$8.

Second step: we still need to convert money back into demand: income effect answers the question how demand changes when income changes.

This expression is  $\frac{\Delta x_1^m}{\Delta m}$ ; it is the income derivative. Its sign relates to the slope of the Income Offer Curve. In our Cobb-Douglas case, this derivative is positive. It tells here: doubling the income doubles demand of good 1.

Bundle F has the coordinates

$$x_1^F = x_1(m'', p_1') = \frac{m''}{2p_1'} = \frac{28}{4} = 7, \text{ and } x_2^F = x_2(m'', p_2) = \frac{m''}{2p_2} = \frac{28}{2} = 14.$$

We compute the Endowment Income Effect (which is the demand change from C to F):

$$\Delta x_1^{EIE} = x_1^F - x_1^C = 7 - 5 = 2.$$

Equation (9.3) uses  $\omega_1$  as defined in (1) times the income derivative known from the ordinary income effect to state that

$$\frac{\Delta x_1^m}{\Delta m} \frac{\Delta m}{\Delta p_1} = \frac{\Delta x_1^m}{\Delta m} \omega_1 = \frac{2.5}{10} \cdot 8 = 2$$

We are now ready to analyze the extended Slutsky equation:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - x_1 \frac{\Delta x_1^m}{\Delta m} + \omega_1 \frac{\Delta x_1^m}{\Delta m} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} + \underbrace{(\omega_1 - x_1)}_{\substack{\text{negative} \\ \text{if net buyer}}} \underbrace{\frac{\Delta x_1^m}{\Delta m}}_{+}.$$

Essentially, the equation tells that whenever the individual is a net buyer of good 1 ("A is right of E"), any price increase leads to the predictable result that demand of good 1 will totally **decrease**, (and will increase if the individual faces a price decrease). This can be seen by just looking at the signs of the single effects.

Why? Going back to the graph, we observe that

- B is left of A because of the negativity of the substitution effect.
- F is left of B because of negativity of the total income effect: the positive income derivative (income-offer curve for normal goods is positively sloped, so here in the Cobb-Douglas case) together with the fact that she is a net buyer and A is right of E.

That is,

