Numeric example (used in class Wed, July 11)

See: Slutsky Substitution and Income Effect (last slides set on ch. 8)

The example on the slides uses a price decrease.

We assume: $m=24$, $p_1 = 2$, $p_2 = 1$

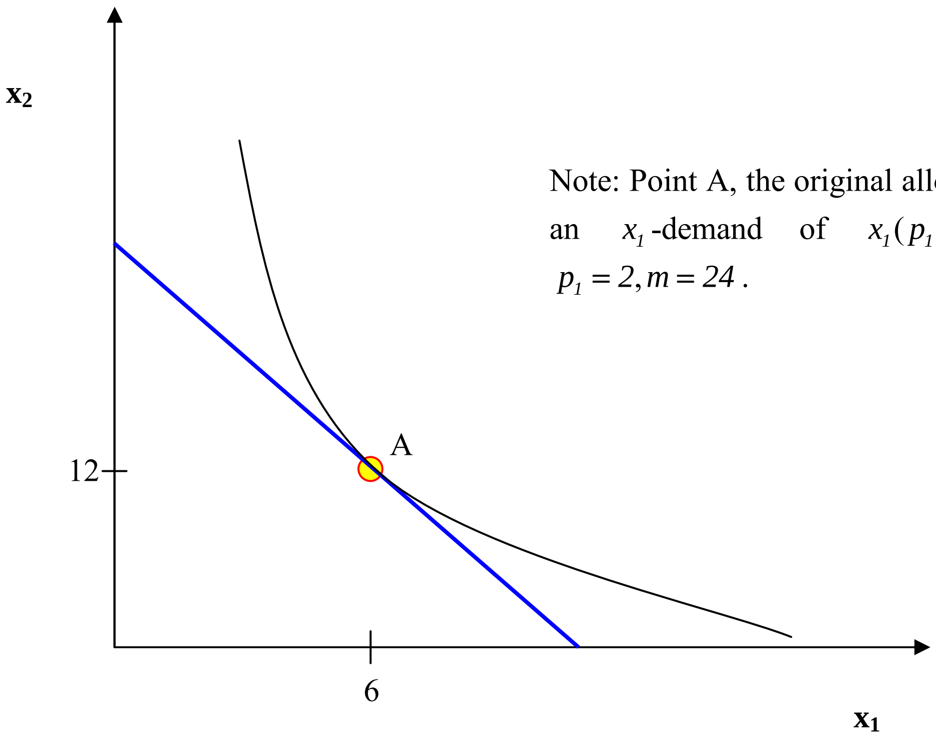
We furthermore assume that the consumer has Cobb-Douglas preferences following $u(x_1, x_2) = x_1 x_2$.

Using Lagrangian it is easy to find out that with a budget constraint of $p_1 x_1 + p_2 x_2 = m$, her individual demands read

$x_1 = \frac{m}{2p_1}$ and $x_2 = \frac{m}{2p_2}$.

At given prices and income, her initial bundle A (6,12) is found by plugging into demands:

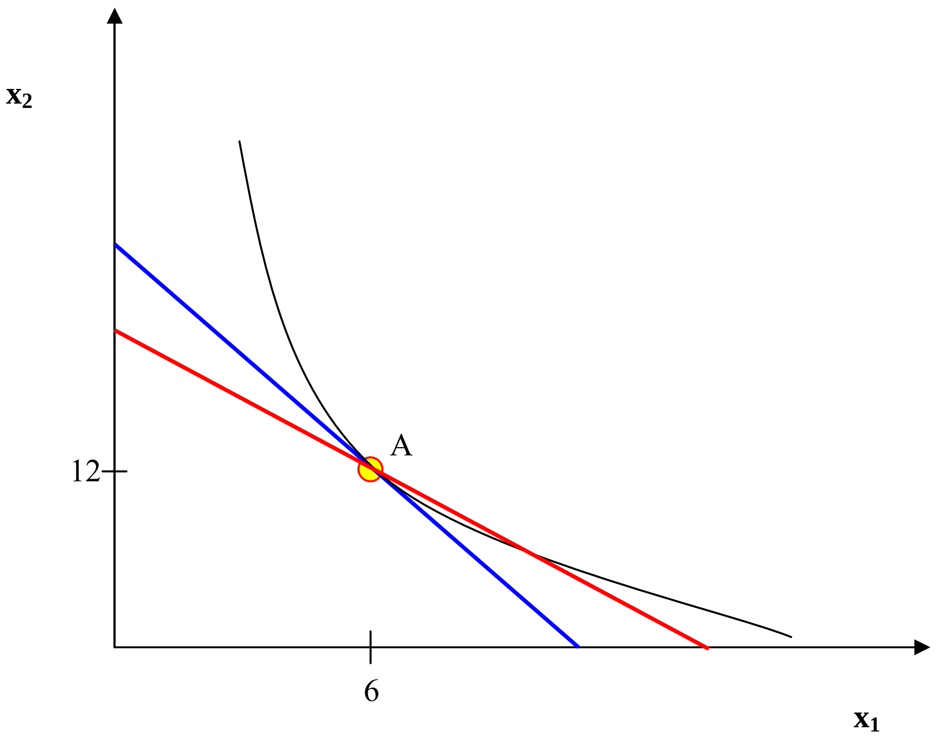
$x_1^A = \frac{m}{2p_1} = \frac{24}{4} = 6$, and $x_2^A = \frac{m}{2p_2} = \frac{24}{2} = 12.$



Note: Point A, the original allocation, has an x_1-demand of $x_1(p_1, m)$, here $p_1 = 2, m = 24$.

Next slide: We assume an exogenous price decrease from $p_1 = 2$ to $p_1' = 1$

First step: We find the money worth in A that the consumer gains through this price decrease would she still stay put in A. This is $\Delta m = \Delta p_1 x_1^A = (1-2)6 = -6.$ **As is easy to see, the consumer only needs $15 to buy 5 apples and 5 bananas at new (lower) prices. We find this value mathematically using** $m' = m + \Delta m = 24 - 6 = 18.$



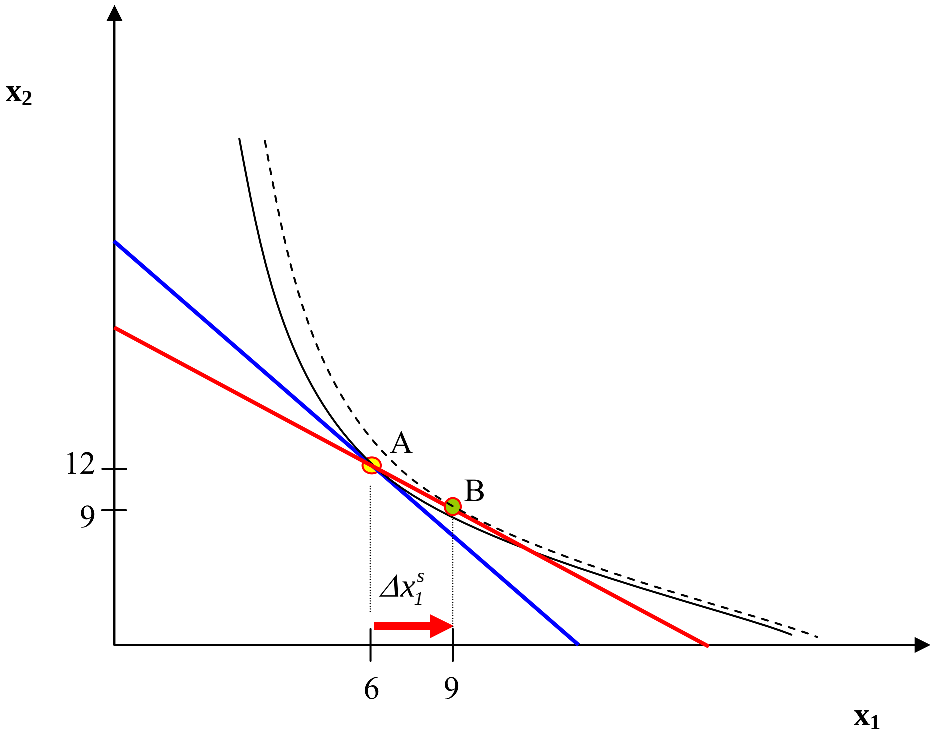
This new (red) "compensated" budget line has $p_1' = 1,\ m' = 18$.

Second step: obviously, the now flatter red budget line through A shows that A is no longer optimal (tangency condition violated). We find B by again using individual demands, now with $p_1' = 1,\ m' = 18$:

$x_1^B = \frac{m'}{2p_1'} = \frac{18}{2} = 9$, and $x_2^B = \frac{m'}{2p_2} = \frac{18}{2} = 9$.

We mark B in the chart below and find the Slutsky Substitution effect as indicated on the slide (see graph below):

$\Delta x_1^s = x_1(m', p_1') - x_1(m, p_1) = x_1^B - x_1^A = 9 - 6 = 3$.



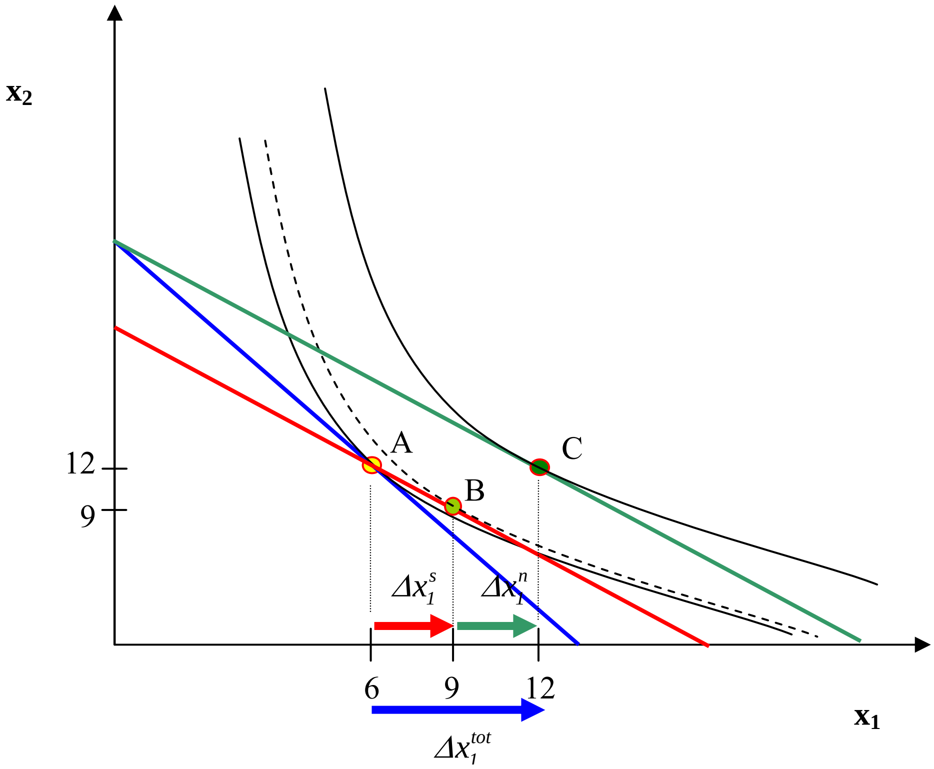
Third step: We find the final allocation and the income effect by now giving back the $6 we took away first to keep the consumer compensated in A. This leads to her final allocation C(12,12) with original income *m* but at new prices.

$$x_1^C = x_1(m, p_1') = \frac{m}{2p_1'} = \frac{24}{2} = 12, \text{ and } x_2^C = x_2^A = x_2(m, p_2) = \frac{m}{2p_2} = \frac{24}{2} = 12 \ .$$

We mark C in the chart below and find the income effect as indicated on the slide (see again graph below):

$$\Delta x_1^n = x_1(m, p_1') - x_1(m', p_1') = x_1^C - x_1^B = 12 - 9 = 3 \ .$$

Note that the final (green) budget line is parallel shifted compared to the second (red) one because it has the same price relation but the old (higher, not compensated) income *m*.



Finally, as in the next slide, we can check if the Slutsky identity holds: total effect (from A to C is the sum of the two single effects Δx_1^s and Δx_1^n :

$$\Delta x_1^{tot} = \Delta x_1^s + \Delta x_1^n$$

$$= x_1(m', p_1') - x_1(m, p_1) + x_1(m, p_1') - x_1(m', p_1')$$

or $= x_1^B - x_1^A + x_1^C - x_1^B$

Since the first and last term cancel out, we have

$$\Delta x_1^{tot} = x_1(m, p_1') - x_1(m, p_1)$$

$$= x_1^C - x_1^A$$ (blue arrow above).

Last remark: we assumed Cobb-Douglas preferences. Thus, we have superior (=normal, not inferior) goods and both the substitution and the income effects point into the same direction. **Note that inferiority has nothing to do with the substitution effect. That effect is ALWAYS NEGATIVE as opposed to the price change (here: price decrease, demand increase). Would we have an inferior good, the green arrow would point to the left, starting in B. As long as C is right of A, we still have a total increase in demand (blue arrow pointing right). If the income effect outweighs the substitution effect, we would have a Giffen good, and the final allocation C would lie on the green budget line but LEFT of A (violating law of demand: total price decreases, demand decreases (!)).** ■