Ch 6 : The Comparative Statics of Consumption

Comparative Statics: Method.

- **When analyzing equilibria, we ignore intertemporal aspects**
- **Comparative statics means we analyze two equilibria.**
- **Less or no treatment of adjustments**
- **We are particularly interested in exogenous changes of price and income, and how this effects consumer demand.**

6.1 Normal and Inferior Goods

How does consumer's demand for a good change if income changes?

$$\frac{\partial x_i}{\partial m} = \frac{\partial x_i(p_1, p_2, m)}{\partial m}$$

A normal good (superior good): demand increases with income

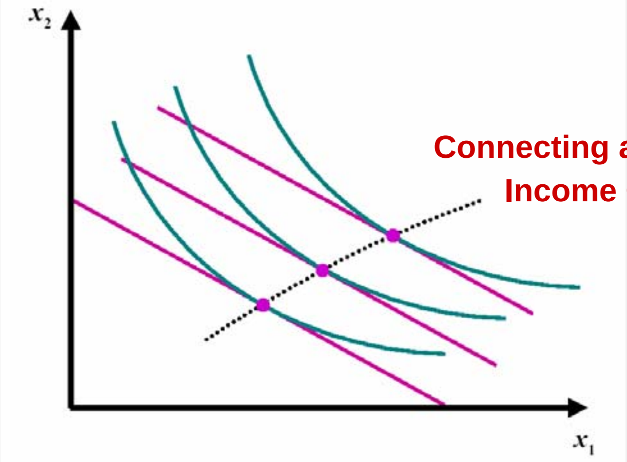
$$\frac{\partial x_i}{\partial m} \geq 0$$

An inferior good: demand decreases with income

$$\frac{\partial x_i}{\partial m} < 0$$

6.2 Income Offer Curves and Engel Curves

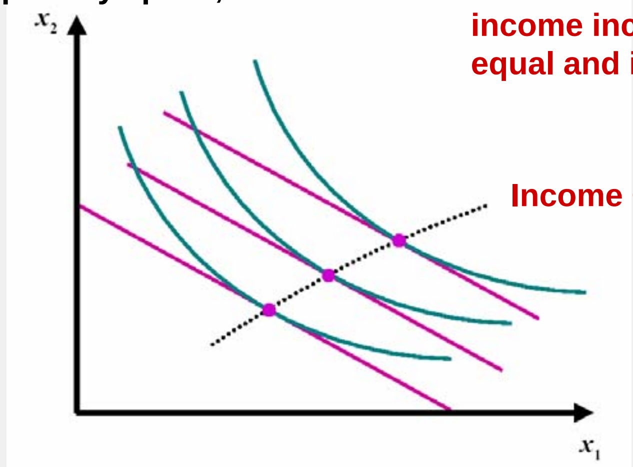
Recall: An increase in income shifts budget line outward, and the new equilibrium is found on the new budget line.



Connecting all equilibria yields the Income Offer Curve

Engel Curves

There is another way to think about this problem. In the traditional quantity space, we see the income increase, and we can read the income increase when prices are equal and income increases.

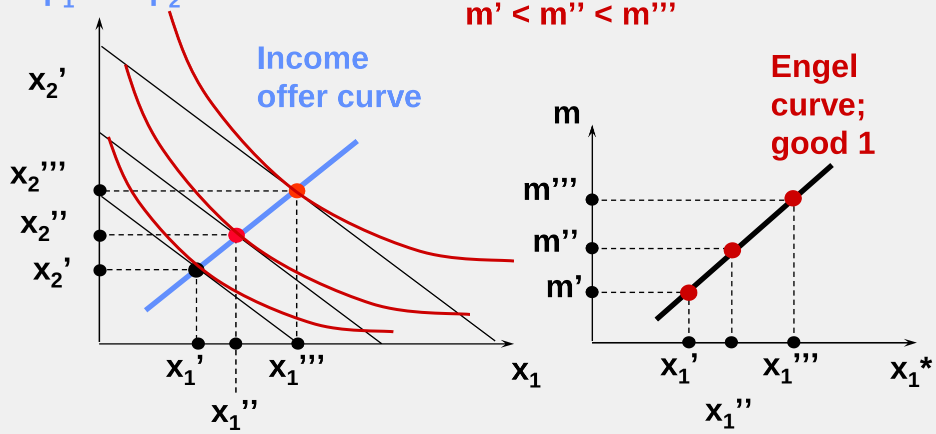


Income Offer Curve

Engel Curves

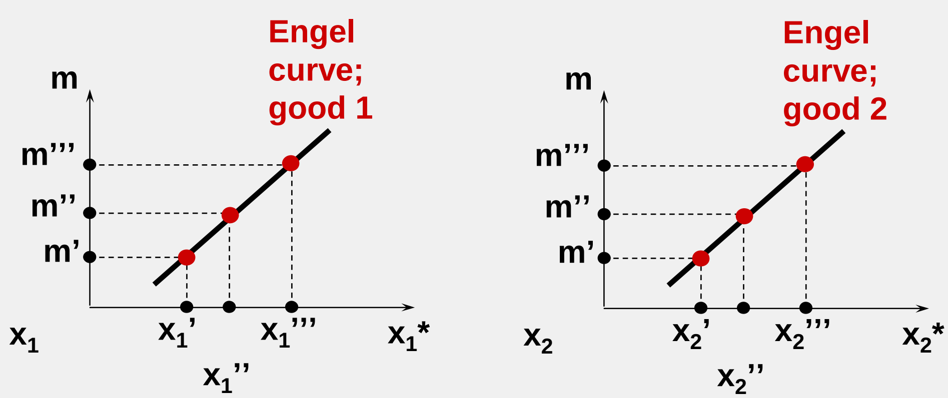
Instead, we could graph directly the demand for one good as a function of income, when prices are (again) equal:

Fixed p_1 and p_2.



Engel Curves

In the same way, we derive the Engel curve for Good 2:



Income Changes & Cobb-Douglas Preferences

Recall the general form of Cobb-Douglas Preferences (PS1 #8):

$$U(x_1, x_2) = x_1^c x_2^d.$$

Cobb-Douglas demands are

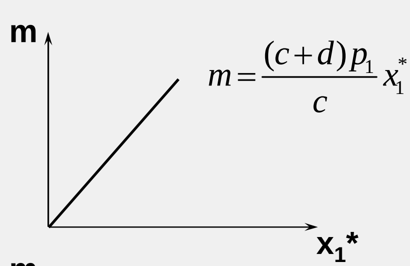
$$x_1^* = \frac{cm}{(c+d)p_1}; \quad x_2^* = \frac{dm}{(c+d)p_2}.$$

Rearranged to isolate *m*, these are:

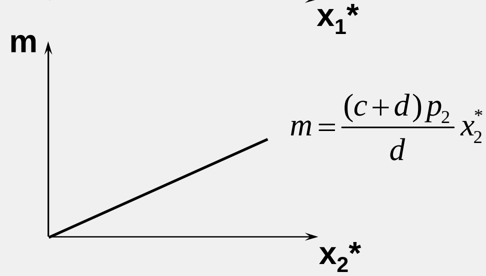
$$m = \frac{(c+d)p_1}{c} x_1^*$$ **Engel curve for good 1**

$$m = \frac{(c+d)p_2}{c} x_2^*$$ **Engel curve for good 2**

Income Changes & Cobb-Douglas Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes & Cobb-Douglas Preferences

You noted that in the Cobb-Douglas case, both the income offer curve and the Engel curves are straight lines.

Whenever the Engel curves are straight lines, the preferences of the consumer under consideration are homothetic. That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin, or:

$$(x_1, x_2) \succ (y_1, y_2) \Leftrightarrow (kx_1, kx_2) \succ (ky_1, ky_2)$$

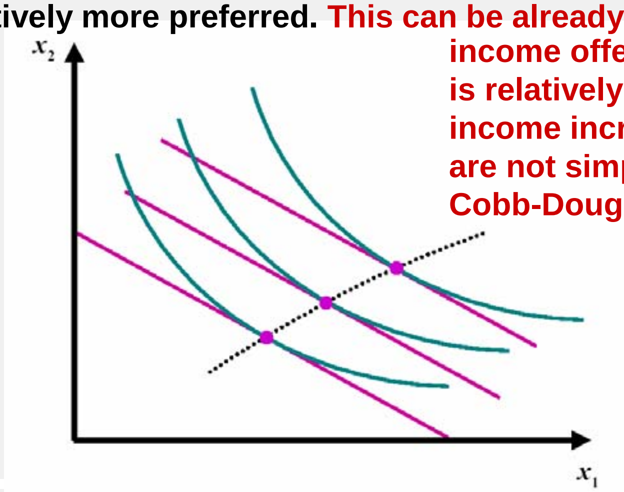
$k>0$ is a constant.

What homotheticity tells is: This consumer's preferences are such that they only depend on the ratio of good 1 and good 2.

Cobb-Douglas Preferences are homothetic.

Realistic?

No. Usually, we would expect that some goods become relatively less preferred if income increases, and other goods become relatively more preferred. This can be already seen following the income offer curve below: Good 1 is relatively more preferred if income increases, and the bundles are not simply "scaled up" as under Cobb-Douglas preferences.



Remark: Good 2 in this graph is sometimes called *relatively inferior* (slope of income-offer curve decreasing)

Quasilinear Preferences are NOT homothetic

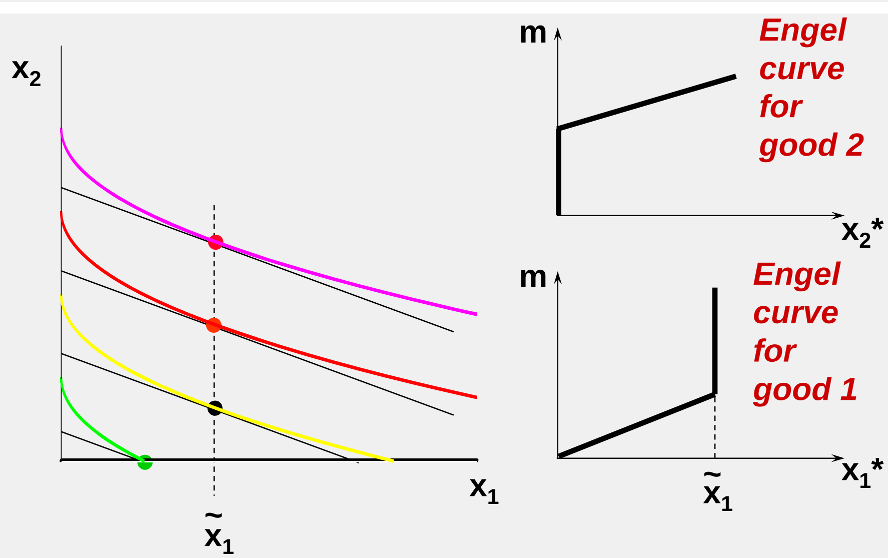
This is easy to show: Consider the general quasilinear function

$$U(x_1, x_2) = f(x_1) + x_2.$$

and as a special case general quasilinear function

$$U(x_1, x_2) = \sqrt{x_1} + x_2.$$

Quasilinear Preferences are NOT homothetic

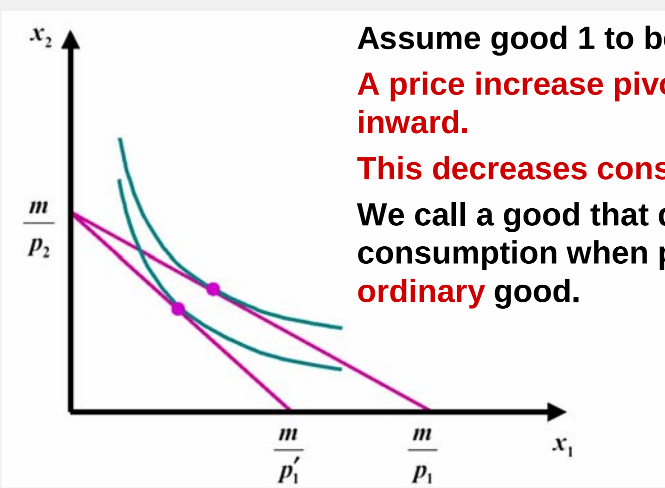


6.4. Price Changes

The important criterion determining for consumer's demand is the relative price of the two goods, not the absolute level of good 1 or good 2

It is thus sufficient to consider a price change of good 1, while price p_2 as well as budget m are kept constant.

6.4. Price Changes: Ordinary vs. Giffen Goods



Assume good 1 to be a normal good.

A price increase pivots the budget line inward.

This decreases consumption for good 1.

We call a good that decreases in consumption when price increases, an ordinary good.

6.7 Substitutes and Complements

We know what substitutes are and what complements are.

Is there a way to formally define this property?

From our definition of Marshall demand we know that

$$x_1(p_1, p_2, m)$$
$$x_2(p_1, p_2, m)$$

If demand for good 1 increases if price for good 2 increases, we have a substitute,

or: $\frac{dx_1}{dp_2} > 0$

If demand for good 1 decreases if price for good 2 increases, we have a complement,

or: $\frac{dx_1}{dp_2} < 0$

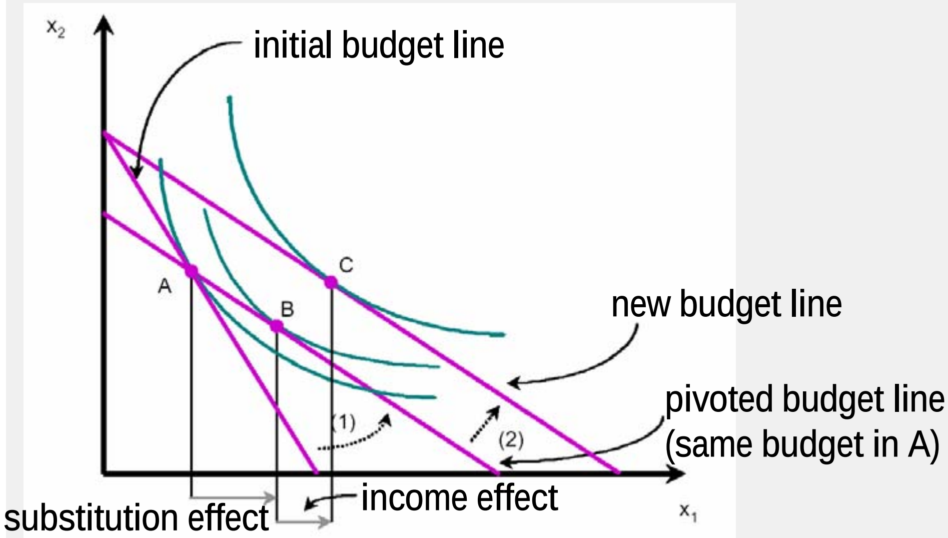
Ch 8 Slutsky equation

Intuition: Imagine price of good 1 increases. What follows?

- Relative price changes: Substitution effect
- Purchasing power of the budget changes: Income effect

We want to disentangle the total effect into the substitution and the income effect.

The Total Effect (for a decrease of p_1)



What happens? Part 1

1. In Point A, the original optimum, budget line pivots to the left, reflecting the new relative price, but at the same purchasing power as before (new line still goes through A)

2. Since price of good 1 has decreased, the pivoted budget line represents a lower budget, namely *m'*.

In A we have:

$$\left.\begin{aligned} p_1 x_1 + p_2 x_2 &= m \\ p_1' x_1 + p_2 x_2 &= m' \end{aligned}\right\} \Rightarrow \quad \Delta m = m' - m = (p_1' - p_1) x_1 = \Delta p_1 x_1$$

Part 2: Substitution effect

3. Although both budget lines go through A, point A is no longer the optimum along the pivoted budget line.

4. The consumer increases utility by moving to Point B.

This is the **substitution effect.** Good 2 is substituted in parts by Good 1.

Quantity change due to the substitution effect is

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$$

Part 3: Income effect

5. In a fictitious next step we now change the pivoted budget line going through A and B such that it **reaches again the initial budget level *m*.**

6. The consumer thus reaches the **new optimum in C.** This "shift" is called the **income effect**, stemming from the purchasing power increase due to the price decrease of good 1. We measure the income effect as follows:`

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$$

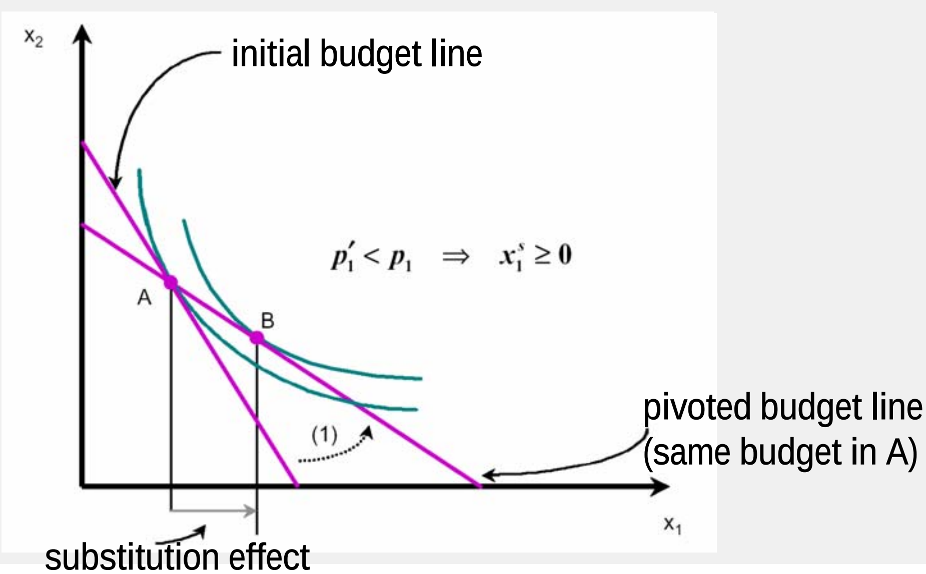
Signs, Total Change, and Rates of Change

We know from our analysis of <u>income changes</u> that the income effect for a normal (inferior) good is positive (negative), such that an increase in income leads to an increase (decrease) in demand.

Thus, the sign of the income effect depends on the properties of the good (normal or inferior).

The sign of the substitution effect is always negative (means here: has the opposite sign of the price change).

Signs, Total Change, and Rates of Change



Signs, Total Change, and Rates of Change

- All points on the pivoted budget line left of A but the consumer preferred A over them. So A was preferred.
- After the budget line pivoted into the flatter one going through A, we can similarly conjecture that this consumer will again prefer A over those points left of A.
- Thus, a price decrease can never lead to a decrease of demand of good 1.

A price increase thus leads either to an increase of good 1 (or to a constant demand of good 1).

The Slutsky Equation

Adding the two effects together yields:

$$\begin{aligned}\Delta x_1 &= x_1(p_1', m) - x_1(p_1, m)\\ &= \Delta x_1^s + \Delta x_1^n\\ &= \underbrace{x_1(p_1', m') - x_1(p_1, m)}_{\Delta x_1^s} + \underbrace{x_1(p_1', m) - x_1(p_1', m')}_{\Delta x_1^n}\end{aligned}$$

We do not focus on the identity, the important point is the division of the total effect into the two effects.

Slutsky Equation II

If price p_1 increases, we thus have

$$\underbrace{\Delta x_1}_{-} = \underbrace{\Delta x_1^s}_{-} + \underbrace{\Delta x_1^n}_{-}$$ **normal good**

$$\underbrace{\Delta x_1}_{?} = \underbrace{\Delta x_1^s}_{-} + \underbrace{\Delta x_1^n}_{+}$$ **inferior good**

$$\underbrace{\Delta x_1}_{+} = \underbrace{\Delta x_1^s}_{-} + \underbrace{\Delta x_1^n}_{+}$$ **Giffen good**

Important for an understanding; Should the initial effect be a price decrease of p_1, all signs would change.

Slutsky Equation III

Inferior goods vs. Giffen goods:

$$\underbrace{\Delta x_1}_{+} = \underbrace{\Delta x_1^s}_{-} + \underbrace{\Delta x_1^n}_{+}$$ **Giffen good**

The income effect in the Giffen case overcompensates the substitution effect such that the total effect is positive (an increase in demand if price increases).

A necessary condition for a Giffen good is inferiority. Not every inferior good is a Giffen good, but every Giffen good is inferior.