A simple example to illustrate why $MR(x)=\frac{\partial R}{\partial x}=p+x\frac{dp}{dx}$

Recall from the slides set on price elasticity of demand that we defined

$$R(p(x))=R(p,x)$$

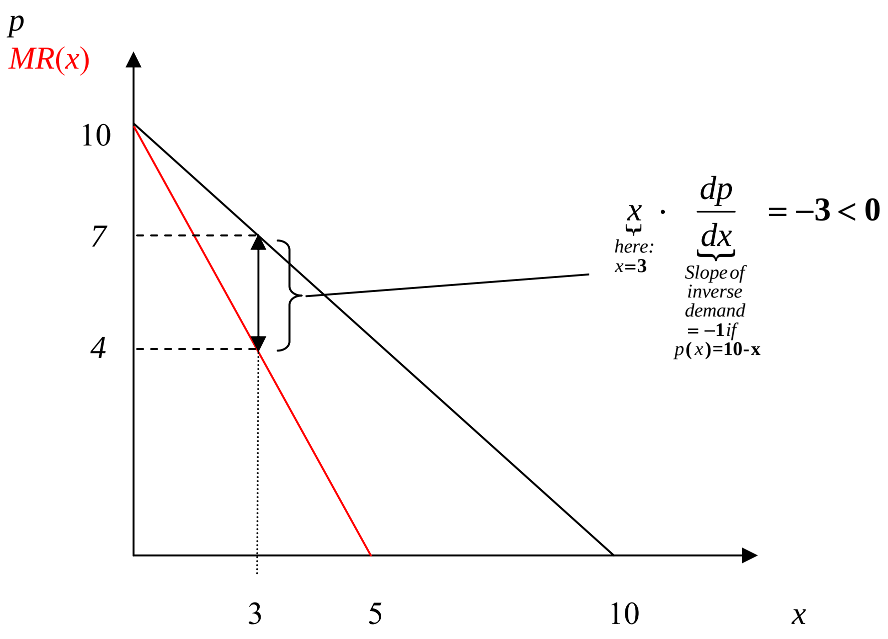
Math part: Check page A9 in the appendix: you need to derive directly, plus use the indirect change that follows the application of the chain rule. This leads to

$$MR(x)=\frac{\partial R(p,x)}{\partial x}=\frac{\partial R(x)}{\partial x}+\underbrace{\frac{\partial R(p(x))}{\partial p}\cdot\underbrace{\frac{dp(x)}{dx}}_{\substack{\text{inner}\\ \text{derivative}}}}_{\substack{\text{chain rule because inverse}\\ \text{demand is downward sloped,}\\ \text{therefore a qty-change involves}\\ \text{a price change.}}} \quad \text{or simply} \quad MR(x)=\frac{\partial R}{\partial x}=p+x\frac{dp}{dx}$$

Trivial: if you increase *x* by one unit, you do not increase *R* just by the product of *px* as under a horizontal demand where *p*=*MR*, but while you increase *x* by one unit, *p* moves down.

Illustration:

Assume a 45-degree inverse demand curve of say $p(x)=10-x$.



For all linear demands, the derivative $\frac{dp}{dx}$ is a constant. That is, the x (horizontal distance) you move from the origin to the right is multiplied by this derivative and gives a mark-down from demand (or a mark-up from *MR*(*x*)).

So, back to the math part: $\frac{\partial R}{\partial x}=p+x\frac{dp}{dx}$ can now be interpreted as follows:

p is the derivative of $R = px$ w.r.t. to x

x is the amount of good x (how many units you are away from the origin)

$\frac{dp}{dx}$ is the multiplier that determines how p influences x (other than under perfect competition where a firm faces a horizontal demand, now we cannot move up and down with quantity and not changing price, but price follows quantity change in a constant rate of $\frac{dp}{dx}$ as long as we have a linear demand). For $p(x) = 10 - x$ $\frac{dp}{dx} = -1$. ■