Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence*

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Abstract

We examine the hypothesis that the slowdown in productivity following the Great Recession was in significant part an endogenous response to the contraction in demand that induced the downturn. We first present some descriptive evidence in support of our approach. We then augment a workhorse New Keynesian DSGE model with an endogenous TFP mechanism that allows for both costly development and adoption of new technologies. We then estimate the model and use it to assess the sources of the productivity slowdown. We find that the post-Great Recession fall in productivity was a largely endogenous phenomenon. The endogenous productivity mechanism also helps account for the slowdown in productivity prior to the Great Recession. Overall, the results are consistent with the view that demand factors have played a role in the slowdown of capacity growth. More generally, they provide insight into why recoveries from financial crises may be so slow.

JEL: JEL Classification: E3, O3

Keywords: Endogenous Technology, Business Cycles.

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1 Introduction

One of the great challenges for macroeconomists is explaining the slow recovery from major financial crises (see, e.g. Reinhart and Rogoff (2009)). This phenomenon is only partly accounted for by existing theories. The deleveraging process was likely an important cause of persistent reduced spending by borrowers as they saved to lower debt. Constraints on macroeconomic policy likely also contributed to sluggish demand: the zero lower bound on the nominal interest rate limited the ability of monetary policy to stimulate the economy and the political fight over the national debt ceiling effectively removed fiscal policy as source of stimulus.

While these demand side factors have undoubtedly played a central role, it is unlikely that they alone can account for the extraordinarily sluggish movement of the economy back to the pre-crisis trend. This had led a number of authors to explore the contribution of supply-side factors. Both Hall (2014) and Reifschneider et al. (2015) have argued that the huge contraction in economic activity induced by the financial crisis in turn led to an endogenous decline in capacity growth. Hall (2014) emphasizes how the collapse in business investment during the recession brought about a non-trivial drop in the capital stock. Reifschneider et al. (2015) emphasize not only this factor but also the sustained drop in productivity. They make the case that the drop in productivity may be result of a decline in productivity-enhancing investments, and thus an endogenous response to the recession.

Indeed, sustained drops in productivity appear to be a feature of major financial crises. This has been the case for the U.S. in the wake of the Great Recession. A similar phenomenon has occurred recently in Europe. The same phenomenon holds broadly for financial crises in emerging markets: in a sample of East Asian countries that experienced a financial crisis during the 1990s, Queralto (2015) finds a sustained drop in labor productivity in each case to go along with the sustained decline in output.

What accounts for the reduced productivity growth following financial crises? There are two candidate hypotheses: bad luck versus an endogenous response. Fernald (2014) makes a compelling case for the bad luck hypothesis. As he emphasizes, the productivity slowdown began prior to the Great Recession, raising questions on whether it could be a causal factor. Figure 1 illustrates the argument. The figure plots both detrended total factor productivity, specifically Fernald’s utilization corrected measure, along with labor productivity. Both measures show a sustained decline relative to trend in the years after the Great Recession. But the decline appears to begin around 2004-05, prior to the downturn.

There are several different theories of how the productivity slowdown could reflect an
All series are log-linearly detrended. Labor productivity is GDP divided by hours worked (see Appendix A.1 for data sources). TFP is Utilization-Adjusted Total Factor Productivity (available at http://www.frbsf.org/economic-research/total-factor-productivity-tpf/; see Fernald (2012) for details).
endogenous response to the crisis. The one on which we focus, because we think it has
the most empirical promise, involves endogenous growth considerations. Specifically, to the
extent that the crisis induced a large drop in expenditures on research and development as
well as technology adoption, the subsequent decline in productivity could be an endogenous
outcome. Indeed as we document in section 2, during the Great Recession, there was a
sharp decline in private R&D activity as well as in one specific type of new technology
adoption: corporate expenditures in software adoption. There was also a sharp decline in
these measures of endogenous technology upgrading activities during and after the 2001-
2002 recession which raises the possibility that the productivity slowdown prior to the Great
Recession was also in part a response to cyclical factors. Overall, this evidence plus some
other descriptive evidence on technology diffusion that we present is potentially consistent
with endogenous growth factors contributing to the productivity slowdown.

Given this background motivation, we develop and estimate a monetary DSGE model
modified to allow for endogenous technology via R&D and adoption. We then use the
model to address the following three issues: (i) how much of the recent productivity decline
reflects an endogenous response to the Great Recession; (ii) whether the mechanism can
also account for the productivity slowdown prior to the Great Recession; and (iii) more
generally the extent to which endogenous productivity can help account for business cycle
persistence.

The endogenous productivity mechanism we develop is based on Comin and Gertler
(2006), which uses the approach to connect business cycles to growth. The Comin/Gertler
framework, in turn, is a variant of Romer (1990)'s expanding variety model of technolog-
cal change, modified to include an endogenous pace of technology adoption. We include
adoption to allow for a realistic period of diffusion of new technologies, but we allow for
endogenous adoption intensity to capture cyclical movements in productivity that may be
the product of cyclical adoption rates. Evidence from Comin (2009) based on 20 manufac-
turing processes in the U.K. suggests that the speed of technology diffusion strongly varies
over the cycle.

In addition to the literature cited above, there are several other papers related to our
have applied variants of the Comin/Gertler approach to explain the persistence of finan-
cial crises. We differ by estimating as opposed to calibrating a model, by introducing
endogenous adoption (which we find to be the critical channel to explain productivity), by
allowing for monetary policy (which also turns out the be important), and by investigating

\footnote{Guerron-Quintana and Jinnai (2014) also estimate their model.}
quantitatively the ability of the model to explain not only the post- but also the pre-crisis slowdown in U.S. productivity.

The paper most closely related to ours is Bianchi and Kung (2014), who first estimated a monetary DSGE model with endogenous growth using R&D data. In addition to focusing on somewhat different issues, we differ in the following ways: first, we develop an explicit model of R&D and adoption with realistic lags which aids in both the empirical identification and the interpretation of the mechanism; second, we use data on business R&D as opposed to the NIPA measure. The former measure corresponds more closely to the model counterpart of R&D since unlike the latter it excludes public expenditures on R&D and includes expenditures on software development. As a consequence, it exhibits cyclical properties more in keeping with the predictions of the theory. Third, we impose the zero lower bound on monetary policy, which turns out to be an important factor propagating the endogenous decline in productivity in the wake of the Great Recession.

The rest of the paper is organized as follows. Section 2 presents some evidence on the cyclical behavior R&D and technology adoption. In addition, we present some new panel data evidence on the speed of technology diffusion that suggests strong cyclical effects, consistent with the approach we take in our model. Section 3 presents the model. Section 4 describes the econometric implementation and present the estimates. Section 5 analyzes the extent to which the endogenous growth mechanism can account for the evolution of productivity both before and after the Great Recession.

2 Evidence on R&D and the speed of technology diffusion

In this section, we present some evidence on the cyclical behavior of R&D and some various measures of technology adoption. We are interested in exploring not only the general cyclical patterns, but also behavior during the last fifteen years, which is the main focus of our study. While reasonably broad measures are available for R&D, the same is not true for adoption. Accordingly, for the latter we consider several different specific measures.

We begin with figure 2 which plots expenses on R&D conducted by US corporations and one specific measure of adoption: business expenses in own-account software.\(^2\) Own-account software consists of in-house expenditures for new or significantly-enhanced software created by business enterprises for their own use.\(^3\) According to the BEA, the expenditures are made for analysis, design, programming, and testing of software and may be made by

\(^2\)All series are linearly detrended. Data on expenditures in R&D and own account software are deflated by the GDP deflator and divided by the civilian population older than 16.

\(^3\)See http://www.bea.gov/papers/pdf/software.pdf
Any industry. Because these expenditures are directed at new or enhanced software used by a company, they provide a reasonable proxy for some of the costs a company needs to incur to upgrade its technologies, which is what we have in mind by adoption expenditures.

During the Great Recession, there is a sharp decline in R&D and own account software, potentially consistent with endogenous growth factors contributing to the productivity slowdown.⁴ Note also that there was a sharp decline in R&D and our measure for software adoption expenditures following the 2001-2002 recession. Even though this recession overall was mild, the IT producing industries were particularly hard hit. The significance of these R&D and adoption contractions raise the possibility that the productivity slowdown prior to the Great Recession was also in part a response to cyclical factors.

⁴There is a long literature documenting the cyclicality of R&D expenditures (see Barlevy (2007), for a summary). Barlevy (2007) presents evidence based on firm-level data on the importance of both sectoral demand as well as firms’ financial conditions for the pro-cyclicality of R&D expenditures.
The evidence on R&D is consistent with the general procyclical behavior found in other studies (see e.g. Comin and Gertler (2006)). Less however is known about the behavior of adoption. Accordingly, to explore further the cyclicality of the speed of diffusion of new technologies, we resort to datasets that contain traditional measures of diffusion from the productivity literature.\(^5\) The data we have available is a sample of 26 production technologies that diffused at various times over the period 1947-2003 in the US (5) and the UK (21).\(^6\) We use the data to estimate the effect of the business cycle on the speed of diffusion, after controlling for the normal diffusion process.

Specifically, we denote by \(m_{it}\) the fraction of potential adopters that have adopted a specific technology \(i\) in \(t\). The ratio of adopters to non-adopters \(r_{it}\) is

\[
 r_{it} = \frac{m_{it}}{1 - m_{it}}. \tag{1}
\]

The speed of diffusion is then the percentage change in \(r_{it}\):

\[
 Speed_{it} = \Delta \ln(r_{it}) \tag{2}
\]

If the diffusion process follows a logistic curve, the speed of diffusion (2) is equal to a constant \(\alpha_i\). In reality, however, the speed of diffusion is not constant, it tends to be faster in the early stages. Therefore, \(r_{it}\) declines with the age of the technology. Additionally, we want to explore whether the speed of technology diffusion varies over the cycle. To this end, we consider the following specification

\[
 Speed_{it} = \alpha_i + G(lag_{it}) + \beta * y_{2200t} + \epsilon_{it}, \tag{3}
\]

where \(G(.)\) is a polynomial in the years since the technology was first introduced, and \(y_{2200t}\) is a measure of the business cycle (in year \(t\)) that captures both high and medium term fluctuations in GDP per person over 16 years old.

Table 1 presents the estimates of equation (3). The main finding is that the estimates of the elasticity of the speed of diffusion with respect to the cycle, \(\beta\), are robustly positive.

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\(^5\)See for example, Griliches (1957) and Mansfield (1961).

\(^6\)The data on the UK technologies comes from Davies (1979) and covers special presses, foils, wet suction boxes, gibberellic acid, automatic size boxes, accelerated drying hoods, basic oxygen process, vacuum degassing, vacuum melting, continuous casting, tunnel kilns, process control by computer, tufted carpets, computer typesetting, photo-electrically controlled cutting, shuttleless looms, numerical control printing presses, numerical control turning machines and numerical control turbines. The data for the five technologies in the US comes from Trajtenberg (1990), and Bartel et al. (2009) and covers the diffusion of CT scanners, Computersized numerical controlled machines, automated inspection sensors, 3-D CAD, and flexible manufacturing systems.
Table 1: Cyclicality of the Speed of Technology Diffusion

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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<tbody>
<tr>
<td>$y_{2200t}$</td>
<td>3.73</td>
<td>3.7</td>
<td>3.64</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>(3.59)</td>
<td>(2.81)</td>
<td>(3.94)</td>
<td>(3.17)</td>
</tr>
<tr>
<td>$y_{2200t} \times \text{US}$</td>
<td>0.07</td>
<td>-0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{lag}_{it}$</td>
<td>-0.057</td>
<td>-0.057</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.22)</td>
<td>(4.76)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{lag}_{it}^2$</td>
<td>0.001</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.52)</td>
<td>(2.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(\text{lag}_{it})$</td>
<td>-0.29</td>
<td>-0.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.68)</td>
<td>(6.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$ (within)</td>
<td>0.11</td>
<td>0.11</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>$N$ technologies</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>$N$ observations</td>
<td>327</td>
<td>327</td>
<td>327</td>
<td>327</td>
</tr>
</tbody>
</table>

Notes: (1) dependent variable is the Speed of diffusion of 26 technologies, (2) all regressions include technology specific fixed effects, (3) t-statistics in parenthesis, (4) $y_{2200}$ denotes the cycle of GDP per capita in the country and represents the high and medium term components of output fluctuations, (5) $y_{2200} \times \text{US}$ denotes the medium term cycle of GDP per capita times a US dummy, (6) lag denotes the years since the technology first started to diffuse.

and significant. In particular, the point estimate is between 3.6 and 4.1 depending on the specification. The effect of the years since the technology started diffusing is negative and convex (i.e. it vanishes over time). The results are robust to specifying the function $G$ as a second order polynomial or in logarithms. Finally, we do not observe any significant differential effect of the cycle in the US technologies vs. the UK technologies.

To further illustrate the cyclicality of the speed of technology diffusion, Figure 3 plots the the speed of diffusion for the balanced panel of four US technologies for which we have data from 1981 to 2003. Specifically, for each of the technologies we remove the acyclical component of the diffusion rate ($\alpha_i + G(\text{lag}_{it})$). When then average the residual ($\beta * y_{2200t} + \epsilon_{it}$) over the four technologies. The dashed line is a plot of this average, while the solid line is a three year centered moving average. The Figure clearly shows a positive correlation between the speed of diffusion and the cycle. The speed of diffusion was lowest in the deep 1981-82 recession. Then it recovered during the 80s and declined again after the 1990 recession. Increased notably during the expansion in the second half of the 90s and declined again with the 2001 recession.

Finally, Andrews et al. (2015) have recently provided evidence that technology diffusion
may have slowed down during the Great Recession in the OECD. In their study, they show that the gap in productivity between the most productive firms in a sector (leaders) and the rest (followers) has increased significantly during the Great Recession. Andrews et al. (2015) show that the most productive firms have much greater stocks of patents which suggests that they engage in more R&D activity. They interpret the increase in the productivity gap as evidence that followers have slowed down the rate at which they incorporate frontier technology, developed by the leaders.

These co-movement patterns between the business cycle and measures of investments in technology as well as their outcomes provides, in our view, sufficiently suggestive evidence to motivate the quantitative exploration we conduct with the help of our model.

3 Model

Our starting point is a New Keynesian DSGE model similar to Christiano et al. (2005) and Smets and Wouters (2007). We include the standard features useful for capturing the data, including: habit formation in consumption, flow investment adjustment costs, variable

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7 In manufacturing the productivity gap increased by 12% from 2007 and 2009, and in services by approximately 20%.
capital utilization and "Calvo" price and wage rigidities. In addition, monetary policy obeys a Taylor rule with a binding zero lower bound constraint.

The key non-standard feature is that total factor productivity depends on two endogenous variables: the creation of new technologies via R&D and the speed of adoption of these new technologies. Skilled labor is used as an input for the R&D and adoption processes.

We do not model financial frictions explicitly; however, we allow for a shock that transmits through the economy like a financial shock, as we discuss below.

We begin with the non-standard features of the model before briefly describing the standard ones.

3.1 Production Sector and Endogenous TFP: Preliminaries

In this section we describe the production sector and sketch how endogenous productivity enters the model. In a subsequent section we present the firm optimization problems.

There are two types of firms: (i) final goods producers and (ii) intermediate goods producers. There is a continuum, measure unity, of monopolistically competitive final goods producers. Each final goods firm $i$ produces a differentiated output $Y^i_t$. A final good composite is then the following CES aggregate of the differentiated final goods:

$$ Y_t = \left( \int_0^1 (Y^i_t)^{1/\mu_t} di \right)^{\mu_t} $$

where $\mu_t > 1$ is given exogenously.

Each final good firm $i$ uses $Y^i_{mt}$ units of intermediate goods composite as input to produce output, according to the following simple linear technology

$$ Y^i_t = Y^i_{mt} $$

Each final good firm $i$ sets nominal price $P^i_t$ on a staggered basis, as we describe later.

There exists a continuum of measure $A_t$ of monopolistically competitive intermediate goods firms that each make a differentiated product. The endogenous predetermined variable $A_t$ is the stock of types of intermediate goods adopted in production, i.e., the stock of adopted technologies. Intermediate goods firm $j$ produces output $Y^j_{mt}$. The intermediate goods composite is the following CES aggregate of individual intermediate goods:

$$ Y_{mt} = \left( \int_0^{A_t} (Y^j_{mt})^{1/\varphi} dj \right)^{\varphi} $$
with ϑ > 1.

Let \( K_j^t \) be the stock of capital firm \( j \) employs, \( U_j^t \) be how intensely this capital is used, and \( L_j^t \) the stock of labor employed. Then firm \( j \) uses capital services \( U_j^t K_j^t \) and unskilled labor \( L_j^t \) as inputs to produce output \( Y_{mt}^j \) according to the following Cobb-Douglas technology:

\[
Y_{mt}^j = \theta_t \left( U_j^t K_j^t \right)^\alpha \left( L_j^t \right)^{1-\alpha}
\]

where \( \theta_t \) is an exogenous random disturbance. As we will make clear shortly, \( \theta_t \) is the exogenous component of total productivity. Finally, we suppose that intermediate goods firms set prices each period. That is, intermediate goods prices are perfectly flexible, in contrast to final good prices.

Let \( \bar{Y}_t^j \) be average output across final goods producers. Then the production function (4) implies the following expression for the final good composite \( Y_t \)

\[
Y_t = \Omega_t \cdot \bar{Y}_t^j
\]

where \( \Omega_t \) is the following measure of output dispersion

\[
\Omega_t = \left( \int_0^1 \frac{1}{\bar{Y}_t^j} \frac{1}{\mu_t} d\mu_t \right)^\mu_t
\]

In a first order approximation, \( \Omega_t \) equals unity, implying that we can express \( Y_t \) simply as \( \bar{Y}_t^j \).

Next, given the total number of final goods firms is unity, given the production function for each final goods producer (5), and given that \( Y_t \) equals \( \bar{Y}_t^j \), it follows that to a first order

\[
Y_t = Y_{mt}
\]

Finally, given a symmetric equilibrium for intermediate goods (recall prices are flexible in this sector) it follows from equation (6) that we can express the aggregate production function for the finally good composite \( Y_t \)

\[
Y_t = \left[ A_t^{\vartheta - 1} \theta_t \right] \cdot (U_t K_t)^\alpha (L_t)^{1-\alpha}
\]

where the term in brackets is total factor productivity, which is the product of a term that reflects endogenous variation, \( A_t^{\vartheta - 1} \), and one that reflects exogenous variation \( \theta_t \). Note that equation (11) holds to a first order since we impose \( \Omega_t \) equals unity.
In sum, endogenous productivity effects enter through through the expansion in the variety of adopted intermediate goods, measured by $A_t$. We next describe the mechanisms through which new intermediate goods are created and adopted.

3.2 R&D and Adoption

The processes for creating and adopting new technologies are based on Comin and Gertler (2006). Let $Z_t$ denote the stock of technologies, while as before $A_t$ is the stock of adopted technologies (intermediate goods). In turn, the difference $Z_t - A_t$ is the stock of unadopted technologies. R&D expenditures increase $Z_t$ while adoption expenditure increase $A_t$. We distinguish between creation and adoption because we wish to allow for realistic lags in the adoption of new technologies. We first characterize the R&D process and then turn to adoption.

3.2.1 R&D: Creation of $Z_t$

There are a continuum (measure unity) of innovators that use skilled labor to create new intermediate goods. Let $L_{st}^p$ be skilled labor employed in R&D by innovator $p$ and let $\varphi_t$ be the number of new technologies at time $t+1$ that each unit of skilled labor at $t$ can create. We assume $\varphi_t$ is given by

$$\varphi_t = \chi_t Z_t L_{st}^{\rho_z-1}$$

where $\chi_t$ is an exogenous disturbance to the R&D technology and $L_{st}$ is the aggregate amount of skilled labor working on R&D, which an individual innovator takes as given. Following Romer (1990), the presence of $Z_t$, which the innovator also takes as given, reflects public learning-by-doing in the R&D process. We assume $\rho_z < 1$ which implies that increased R&D in the aggregate reduces the efficiency of R&D at the individual level. We introduce this congestion externality so that we can have constant returns to scale in the creation of new technologies at the individual innovator level, which simplifies aggregation, but diminishing returns at the aggregate level. The advantage of diminishing returns in the aggregate is that the elasticity of the creation of new technologies with respect to R&D becomes a parameter we can estimate, as we make clear shortly.

Let $J_t$ be the value of an unadopted technology, $\Lambda_{t,t+1}$ the representative household’s stochastic discount factor and $w_{st}$ the real wage for a unit of skilled labor. We can then express innovator $p$’s decision problem as choosing $L_{st}^p$ to solve
3.2.1 Optimal R&D Investment

The optimality condition for R&D is then given by

$$\max_{L_{sr}^p} E_t \{ \Lambda_{t,t+1} J_{t+1} \varphi_t L_{sr}^p \} - wstL_{sr}^p = 0$$

(13)

The optimality condition for R&D is then given by

$$E_t \{ \Lambda_{t,t+1} J_{t+1} \varphi_t \} - wst = 0$$

which implies

$$E_t \{ \Lambda_{t,t+1} J_{t+1} \chi_t Z_t \rho_z^{-1} \} = wst$$

(14)

The left side of equation (14) is the discounted marginal benefit from an additional unit of skilled labor, while the right side is the marginal cost.

Given that profits from intermediate goods are pro-cyclical, the value of an unadopted technology, which depends on expected future profits, will be also be pro-cyclical. This consideration, in conjunction with some stickiness in the wages of skilled labor, which we introduce later, will give rise to pro-cyclical movements in $L_{sr}^p$.\(^8\)

Finally, we allow for obsolescence of technologies. Let $\phi$ be the survival rate for any given technology. Then, we can express the evolution of technologies as:

$$Z_{t+1} = \varphi_t L_{sr} + \phi Z_t$$

(15)

where the term $\varphi_t L_{sr}$ reflects the creation of new technologies. Combining equations (15) and (12) yields the following expression for the growth of new technologies:

$$\frac{Z_{t+1}}{Z_t} = \chi t L_{sr}^{\rho_z} + \phi$$

(16)

where $\rho_z$ is the elasticity of the growth rate of technologies with respect to R&D, a parameter that we estimate.

3.2.2 Adoption: From $Z_t$ to $A_t$

We next describe how newly created intermediate goods are adopted, i.e. the process of converting $Z_t$ to $A_t$. Here we capture the fact that technology adoption takes time on average, but the adoption rate can vary pro-cyclically, consistent with evidence in Comin (2009). In addition, we would like to characterize the diffusion process in a way that

\(^8\)To the best of our knowledge, this model is the first that combines a labor intensive R&D technology with wage rigidities to ensure the pro-cyclicality of R&D investments. Other approaches include introducing financial frictions (Aghion, Angeletos, Benarjee, Manova, 2010) or to short term biases of innovators (Barlevy, 2008).
minimizes the complications from aggregation. In particular, we would like to avoid having to keep track, for every available technology, the fraction of firms that have and have not adopted it.

Accordingly, we proceed as follows. We suppose there are a competitive group of "adopters" who convert unadopted technologies into ones that can be used in production. They buy the rights to the technology from the innovator, at the competitive price $J_t$, which is the value of an adopted technology. They then convert the technology into use by using skilled labor as input. This process takes time on average, and the conversion rate may vary endogenously.

In particular, the pace of adoption depends positively on the level of adoption expenditures in the following simple way: an adopter succeeds in making a product usable in any given period with probability $\lambda_t$, which is an increasing and concave function of the amount of skilled labor employed, $L_{sat}$:

$$\lambda_t = \lambda(Z_t L_{sat})$$

with $\lambda' > 0$, $\lambda'' < 0$. We augment $L_{sat}$ by a spillover effect from the total stock of technologies $Z_t$ - think of the adoption process as becoming more efficient as the technological state of the economy improves. The practical need for this spillover is that it ensures a balanced growth path: as technologies grow, the number of new goods requiring adoption increases, but the supply of labor remains unchanged. Hence, the adoption process must become more efficient as the number of technologies expands.

Our adoption process implies that technology diffusion takes time on average, consistent with the evidence. If $\lambda$ is the steady state value of $\lambda_t$, then the average time it takes for a new technology be adopted is $1/\lambda$. Away from the steady state, the pace of adoption will vary with skilled input $L_{sat}$. We turn next to how $L_{sat}$ is determined.

Once in usable form, the adopter sells the rights to the technology to a monopolistically competitive intermediate goods producer that makes the new product using the production function described by equation (11). Let $\Pi_{mt}$ be the profits that the intermediate goods firm makes from producing the good, which arise from monopolistically competitive pricing. The adopter sells the new technology at the competitive price $V_t$, which is the present discounted value of profits from producing the good, given by

$$V_t = \Pi_{mt} + \phi E_t \{ \Lambda_{t,t+1} V_{t+1} \}$$

In the estimation, we assume that

$$\lambda(\bullet) = \bar{\lambda} \ast (\bullet)^{\rho \lambda}.$$
Then we may express the adopter’s maximization problem as choosing $L_{sat}$ to maximize the value $J_t$ of an unadopted technology, given by

$$J_t = \max_{L_{sat}} E_t \{-w_{st}L_{sat} + \phi \Lambda_{t,t+1} [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}] \}$$

subject to equation (17). The first term in the Bellman equation reflects total adoption expenditures, while the second is the discounted benefit: the probability weighted sum of the values of adopted and unadopted technologies.

The first order condition for $L_{sat}$ is

$$Z_t \lambda_t' \cdot \phi E_t \{ \Lambda_{t,t+1} [V_{t+1} - J_{t+1}] \} = w_{st}$$

The term on the left is the marginal gain from adoption expenditures: the increase in the adoption probability $\lambda_t$ times the discounted difference between an adopted versus unadopted technology. The right side is the marginal cost.

The term $V_{t} - J_{t}$ is pro-cyclical, given the greater influence of near term profits on the value of adopted technologies relative to unadopted ones. Given this consideration and the stickiness in $w_{st}$ which we alluded to earlier, $L_{sat}$ varies pro-cyclically. The net implication is that the pace of adoption, given by $\lambda_t$, will also vary pro-cyclically.

Given that $\lambda_t$ does not depend on adopter-specific characteristics, we can sum across adopters to obtain the following relation for the evolution of adopted technologies

$$A_{t+1} = \lambda_t \phi [Z_t - A_t] + \phi A_t$$

where $Z_t - A_t$ is the stock of unadopted technologies.

### 3.3 Households

The representative household consumes and saves in the form of capital and riskless bonds which are in zero net supply. It rents capital to intermediate goods firms. As in the standard DSGE model, there is habit formation in consumption. Also as is standard in DSGE models with wage rigidity, the household is a monopolistically competitive supplier of differentiated types of labor.

The household’s problem differs from the standard setup in two ways. First it supplies two types of labor: unskilled labor $L_t^h$ which is used in the production of intermediate goods and skilled labor which is used either for R&D or adoption, $L_t^{st}$.

Second, we suppose that the household has a preference for the safe asset, which we
motivate loosely as a preference for liquidity and capture by incorporating bonds in the utility function, following Krishnamurthy and Vissing-Jorgensen (2012). Further, following Fisher (2015), we introduce a shock to liquidity demand $\varrho_t > 0$. As we show, the liquidity demand shock transmits through the economy like a financial shock. It is mainly for this reason that we make use of this shock, as opposed to a shock to the discount factor.\footnote{Another consideration is that the liquidity demand shock induces positive co-movement between consumption and investment, while that is not always the case for a discount factor shock.}

Let $C_t$ be consumption, $B_t$ holdings of the riskless bond, $\Pi_t$ profits from ownership of monopolistically competitive firms, $K_t$ capital, $Q_t$ the price of capital, $R_{kt}$ the rate of return, and $D_t$ the rental rate of capital. Then the households’ decision problem is given by

$$\max_{C_t,B_t,L^h_t,L^h_{st},K_t} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ \log(C_{t+\tau} - bC_{t+\tau-1}) + \varrho_t B_t - \nu \left[ \frac{(L^h_t)^{1+\varphi} + (L^h_{st})^{1+\varphi}}{1 + \varphi} \right] \right\}$$

subject to

$$C_t = w^h_t L^h_t + w^h_{st} L^h_{st} + \Pi_t + R_{kt} Q_{t-1} K_t - Q_t K_{t+1} + R_t B_t - B_{t+1}$$

with

$$R_{kt} = \frac{D_t + Q_t}{Q_{t-1}}$$

$\Lambda_{t,t+1}$, the household’s stochastic discount factor, is given by

$$\Lambda_{t,t+1} \equiv \beta u'(C_{t+1})/u'(C_t)$$

where $u'(C_t) = 1/(C_t - bC_{t-1}) - b/(C_{t+1} - bC_t)$. In addition, let $\zeta_t$ be the liquidity preference shock in units of the consumption good:

$$\zeta_t = \varrho_t / u'(C_t)$$

Then we can express the first order necessary conditions for capital and the riskless bond as, respectively:

$$1 = E_t \{ \Lambda_{t,t+1} R_{kt+1} \}$$

$$1 = E_t \{ \Lambda_{t,t+1} R_{t+1} \} + \zeta_t$$

As equation (28) indicates, the liquidity demand shock distorts the first order condition for the riskless bond. A rise in $\zeta_t$ acts like an increase in risk: given the riskless rate $R_{t+1}$ the increase in $\zeta_t$ induces a precautionary saving effect, as households reduce current
consumption in order to satisfy the first order condition (which requires a drop in $\Lambda_{t,t+1}$). It also leads to a drop in investment demand, as the decline in $\Lambda_{t,t+1}$ raises the required return on capital, as equation (27) implies. The decline in the discount factor also induces a drop in R&D and investment.

Overall, the shock to $\zeta_t$ generates positive co-movement between consumption and investment similar to that arising from a monetary shock. To see, combine equations (27) and (28) to obtain

$$E_t\{\Lambda_{t,t+1}(R_{kt+1} - R_t + 1)\} = \zeta_t$$

(29)

To a first order an increase in $\zeta_t$ has an effect on both $R_{kt+1}$ and $\Lambda_{t,t+1}$ that is qualitatively similar to that arising from an increase in $R_{t+1}$. In addition, note that an increase in $\zeta_t$ raises the credit spread $R_{kt+1} - R_{t+1}$. In this respect it transmits through the economy like a financial shock. Indeed, we show later that our identified liquidity demand shock is highly correlated with credit spreads.

Since it is fairly conventional, we defer until later a description of the household’s wage-setting and labor supply behavior.

### 3.4 Firms

#### 3.4.1 Intermediate goods firms: factor demands

Given the CES function for the intermediate good composite (6), in the symmetric equilibrium each of the monopolistically competitive intermediate goods firms charges the markup $\vartheta$. Let $p_{mt}$ be the relative price of the intermediate goods composite. Then from (6) and the production function (7), cost minimization by each intermediate goods producer yields the following standard first order conditions for capital, capital utilization, and unskilled labor:

$$\alpha \frac{p_{mt}Y_{mt}}{K_t} = \vartheta [D_t + \delta(U_t)Q_t]$$

(30)

$$\alpha \frac{p_{mt}Y_{mt}}{U_t} = \vartheta \delta'(U)Q_tK_t$$

(31)

$$\frac{(1 - \alpha) p_{mt}Y_{mt}}{L_t} = \vartheta w_t$$

(32)
3.4.2 Final goods producers: price setting

Let $P_i^t$ be the nominal price of final good $i$ and $P_t$ the nominal price level. Given the CES relation for the final good composite, equation (4), the demand curve facing each final good producer is:

$$Y^i_t = \left( \frac{P^i_t}{P_t} \right)^{-\mu/(\mu-1)} Y_t$$

(33)

where the price index is given by:

$$P_t = \left( \int_0^1 (P^i_t)^{-1/(\mu-1)} \, di \right)^{-(\mu-1)}$$

(34)

Following Smets and Wouters (2007), we assume Calvo pricing with flexible indexing. Let $1 - \xi_p$ be the i.i.d probability that a firm is able to re-optimize its price and let $\pi_t = P_t/P_{t-1}$ be the inflation rate. Firms that are unable to re-optimize during the period adjust their price according to the following indexing rule:

$$P^i_t = P_{t-1}^{\pi_{t-1}^{\iota_p}} P_t^{1-\iota_p}$$

(35)

where $\pi$ is the steady state inflation rate and $\iota_p$ reflects the degree of indexing to lagged inflation.

For firms able to re-optimize, the optimization problem is to choose a new reset price $P^*_t$ to maximize expected discounted profits until the next re-optimization, given by

$$E_t \sum_{\tau=0}^{\infty} \xi^{\tau}_p \Lambda_{t,t+\tau} \left( \frac{P^*_t \Gamma_{t, t+\tau}}{P_{t+\tau}} - p_{mt+\tau} \right) Y_{t+\tau}^i$$

(36)

subject to the demand function (33) and where

$$\Gamma_{t, t+\tau} \equiv \prod_{k=1}^{\tau} \pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p}$$

(37)

The first order condition for $P^*_t$ and the price index that relates $P_t$ to $P^*_t$, $P_{t-1}$ and $\pi_{t-1}$ are then respectively:

$$0 = E_t \sum_{\tau=0}^{\infty} \xi^{\tau}_p \Lambda_{t,t+\tau} \left[ \frac{P^*_t \Gamma_{t, t+\tau}}{P_{t+\tau}} - \mu_{t+\tau} p_{mt+\tau} \right] Y_{t+\tau}^i$$

(38)
\[ P_t = \left[ (1 - \xi_p) (P^*_t)^{-1/(\mu_t-1)} + \xi_p \left( \pi^t_{t-1} \pi^{-1} P_{t-1} \right)^{-1/(\mu_t-1)} \right]^{-(\mu_t-1)} \]  

Equations (38) and (39) jointly determine inflation. In the loglinear equilibrium, current inflation is a function of current real marginal cost \( p_{mt} \), expected future inflation, and lagged inflation.

### 3.4.3 Capital producers: investment

Competitive capital producers use final output to make new capital goods, which they sell to households, who in turn rent the capital to firms. Let \( I_t \) be new capital produced and \( p_{kt} \) the relative price of converting a unit of investment expenditures into new capital (the replacement price of capital), and \( \gamma_y \) the steady state growth in \( I_t \). In addition, following Christiano et al. (2005), we assume flow adjustment costs of investment. The capital producers’ decision problem is to choose \( I_t \) to solve

\[
\max E_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left\{ Q_{t+\tau} I_{t+\tau} - p_{kt+\tau} \left[ 1 + f \left( \frac{I_{t+\tau}}{(1 + \gamma_y)I_{t+\tau-1}} \right) \right] I_{t+\tau} \right\}
\]

where the adjustment cost function is increasing and concave, with \( f(1) = f'(1) = 0 \) and \( f''(1) > 0 \). We assume that \( p_{kt} \) follows an exogenous stochastic process.

The first order condition for \( I_t \) relates the ratio of the market value of capital to the replacement price (i.e. "Tobin’s Q") to investment, as follows:

\[
\frac{Q_t}{p_{kt}} = 1 + f \left( \frac{I_t}{(1 + \gamma_y)I_{t-1}} \right) + \frac{I_t}{(1 + \gamma_y)I_{t-1}} f' \left( \frac{I_t}{(1 + \gamma_y)I_{t-1}} \right) - E_t \Lambda_{t,t+1} \left( \frac{I_{t+1}}{(1 + \gamma_y)I_t} \right)^2 f' \left( \frac{I_{t+1}}{(1 + \gamma_y)I_t} \right)
\]

### 3.4.4 Employment agencies and wage adjustment

As we noted earlier, the household is a monopolistically competitive supplier of labor. Think of the household as supplying its labor to form a labor composite. Firms then hire the labor composite. The only difference from the standard DSGE model with wage rigidity, is that households now supply two types of labor, skilled and unskilled.

Let \( X_t = \{ L_t, L_{st} \} \) denote a labor composite. As is standard, we assume that \( X_t \) is the following CES aggregate of the differentiated types of labor that households provide

\[
X_t = \left[ \int_0^1 X_t^{\mu_{wt}} \frac{1}{\mu_{wt}} dh \right]^{\mu_{wt}}
\]

where \( \mu_{wt} > 1 \) obeys an exogenous stochastic process.
Let $W_{xt}$ denote the wage of the labor composite and let $W^h_{xt}$ be the nominal wage for labor supplied by household $h$. Then profit maximization by competitive employment agencies yields the following demand for type $x$ labor:

$$X^h_t = \left( \frac{W^h_{xt}}{W_{xt}} \right)^{-\mu_{wt}/(\mu_{wt} - 1)} X_t,$$  \hspace{1em} (43)$$

with

$$W_{xt} = \left[ \int_0^1 W^h_{xt} \frac{1}{\mu_{wt} - 1} dh \right]^{-1/(\mu_{wt} - 1)}. \hspace{1em} (44)$$

As with price setting by final goods firms, we assume that households engage in Calvo wage setting with indexation. Each period a fraction $1 - \xi_w$ of households re-optimize their wage. Households who are not able to re-optimize adjust according to the following indexing rule:

$$W_{xt} = W_{xt-1} \pi_{t-1}^{\gamma} \pi^{1-\gamma}.$$  \hspace{1em} (45)$$

where $\gamma$ is the steady state growth rate of labor productivity.

The remaining fraction of households choose an optimal reset wage $W^*_t$ by maximizing

$$E_t \left\{ \sum_{\tau=0}^{\infty} \xi_{t+\tau} \beta^\tau \left[ -u^{1+\varphi} X^h_{t+\tau} 1 + \varphi + u'(C_{t+\tau}) \frac{W^*_t \Gamma_{w,t+\tau} X^h_{t+\tau}}{P_{t+\tau}} \right] \right\}$$

subject to the demand for type $h$ labor and where the indexing factor $\Gamma_{w,t+\tau}$ is given by

$$\Gamma_{w,t+\tau} = \prod_{k=1}^\tau \pi^{\xi_w}_{t+k-1} \pi^{1-\gamma}.$$

The first order condition for the re-set wage and the equation for the composite wage index as a function of the reset wage, inflation and the lagged wage are given, respectively, by

$$E_t \left\{ \sum_{\tau=0}^{\infty} \xi_{t+\tau} \beta^\tau \left[ \frac{W^*_t \Gamma_{w,t+\tau}}{P_t} X^h_{t+\tau} \varphi - \mu_{wt} u'(C_{t+\tau}) \right] X^h_{t+\tau} \right\} = 0 \hspace{1em} (48)$$

$$W_{xt} = \left[ (1 - \xi_w) (W^*_t)^{-1/(\mu_{wt} - 1)} + \xi_p \left( \gamma \pi_{t-1} \pi^{1-\gamma} W_{xt-1} \right)^{-1/(\mu_{wt} - 1)} \right]^{-1/(\mu_{wt} - 1)}. \hspace{1em} (49)$$
3.4.5 Fiscal and monetary policy

We assume that government consumption $G_t$ is financed by lump sum taxes $T_t$.

$$G_t = T_t$$  \hspace{1cm} (50)

Further, the (log) deviation of $G_t$ from the deterministic trend of the economy follows an AR(1) process. Formally,

$$\log\left(G_t/(1 + \gamma)^t\right) = (1 - \rho_g) * \bar{g} + \rho_g * \log\left(G_{t-1}/(1 + \gamma)^{t-1}\right) + \epsilon_t^g,$$  \hspace{1cm} (51)

Next, we suppose that monetary policy obeys a Taylor rule. Let $R_{nt+1}$ denote the gross nominal interest rate, $R_n$ the steady state nominal rate, $\pi^0$ the target rate of inflation, and $L$ the steady state employment level. The (nonlinear) Taylor rule for monetary policy that we consider is given by

$$R_{nt+1} = \left[\left(\frac{\pi_t}{\pi^0}\right)^{\phi_{\pi}} \left(\frac{L_t}{L}\right)^{\phi_y} \cdot R_n\right]^{1-\rho} \cdot R_{nt}$$  \hspace{1cm} (52)

where the relation between the nominal and real rate is given by the Fisher relation:

$$R_{nt+1} = R_{t+1} \cdot \pi_{t+1}$$  \hspace{1cm} (53)

and where $\phi_{\pi}$ and $\phi_y$ are the feedback coefficients on the inflation gap and capacity utilization gap respectively. We use the employment gap to measure capacity utilization. In addition, we impose the zero lower bound constraint on the net nominal interest rate, which implies that the gross nominal rate cannot fall below unity.

$$R_{nt+1} \geq 1$$  \hspace{1cm} (54)

We use the employment gap $L_t/L$ to measure capacity utilization as opposed to an output gap in part because estimates using the former predict that the zero lower bound does not bind over the period when it has done so in the data.

3.5 Resource constraints and equilibrium

The resource constraint is given by
\[ Y_t = C_t + \left[ 1 + p_{kt} f \left( \frac{I_{t+\tau}}{(1+\gamma_y)I_{t+\tau-1}} \right) \right] I_t + G_t \quad (55) \]

Capital evolves according to

\[ K_{t+1} = I_t + (1 - \delta(U_t))K_t \quad (56) \]

The market for skilled labor must clear:

\[ L_{st} = L_{sat} + L_{srt} \quad (57) \]

Finally, the market for risk-free bonds must clear, which implies that in equilibrium, risk-free bonds are in zero net supply

\[ B_t = 0 \]

This completes the description of the model.

4 Estimation

We estimate our model using Bayesian methods (see for example An and Schorfheide (2007)). As is common practice in the literature (for example Smets and Wouters (2007) and Justiniano et al. (2010)), we calibrate a subset of the parameters of the model and estimate the remainder.

4.1 Calibrated parameters

We calibrate standard real business cycle model parameters (i.e., the rates of time preference and capital depreciation, and the capital share); the steady state share of government spending in output; the trend growth rate; steady state markups for intermediate and final goods and for wages (\( \vartheta, \mu \) and \( \mu_w \) respectively); and three of the four endogenous technological change parameters.

Of the four endogenous technological change parameters, we calibrate the expenditure elasticity of the adoption probability, \( \rho_\lambda \), the obsolescence rate \( (1 - \phi) \) and the steady state adoption lag \( \bar{\lambda} \). The elasticity of \( \lambda \) with respect to adoption expenditures, \( \rho_\lambda \) is set to 0.9 to induce a ratio of private R&D to GDP consistent with the U.S. post-1970 experience (of approximately 1.9% of GDP). \( \bar{\lambda} \) is set to produce an average adoption lag of 7 years which is consistent with the estimates in Comin and Hobijn (2010) and Cox and Alm (1996).
Finally, the obsolescence rate \( (1 - \phi) \) is set to 8% which falls in the middle of the broad range of estimates for the obsolescence rate in the literature (see Caballero and Jaffe (1993) and Pakes and Schankerman (1984) for the two extremes).

Table 2 presents the calibrated parameters and their values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Capital share</td>
<td>1/3</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation</td>
<td>0.02</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.998</td>
</tr>
<tr>
<td>( \gamma_y )</td>
<td>SS government consumption/output</td>
<td>0.2</td>
</tr>
<tr>
<td>( \mu )</td>
<td>SS output growth</td>
<td>1.8%</td>
</tr>
<tr>
<td>( \mu_w )</td>
<td>SS final goods mark up</td>
<td>1.1</td>
</tr>
<tr>
<td>( \vartheta )</td>
<td>SS wage mark up</td>
<td>1</td>
</tr>
<tr>
<td>( 1 - \phi )</td>
<td>Intermediate goods mark up</td>
<td>1.35</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Obsolescence rate</td>
<td>0.08/4</td>
</tr>
<tr>
<td>( \rho_\lambda )</td>
<td>Adoption elasticity</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 2: Calibrated Parameters

4.2 Data, priors and posteriors

The model is estimated using quarterly data from 1984:I to 2008:III on eight US series: real output, consumption, investment, hours worked, real wages, inflation (as measured by the GDP deflator), nominal risk-free interest rates and expenditures on R&D by US corporations. Unlike the other series, R&D expenditures are annual. We deal with the mixed frequency of the data in estimation using a version of the Kalman filter adapted for this purpose. The data are described in detail in Appendix A.1.

Data beyond 2008:III are not used in the estimation of the structural parameters because the zero lower bound on the nominal interest may have been binding after that period, rendering estimation using a log-linear approximation around the deterministic steady state of our baseline model problematic. We modify the standard log-linear approximation of the model with the technique introduced by Guerrieri and Iacoviello (2015) to deal with occasionally binding constraints, and are able to use data until 2012:IV to identify shocks and other latent variables of our model, as described in Appendix A.2.

Tables 3 and 4 below present the prior and posterior distributions for the parameters that we estimate. We use similar priors to the literature for all parameters. For the elasticity of R&D parameter we use a fairly loose beta prior centered around a mean of 0.6, which is at the lower end of estimates provided in Griliches (1990).
Table 3: Prior and Posterior Distributions of Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Taylor rule smoothing</td>
<td>Beta 0.7 0.15</td>
<td>0.774 0.767 0.034</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Taylor rule inflation</td>
<td>Gamma 1.5 0.25</td>
<td>1.263 1.374 0.122</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Taylor rule labor</td>
<td>Gamma 0.3 0.1</td>
<td>0.336 0.407 0.046</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Inverse Frisch elasticity</td>
<td>Gamma 2 0.75</td>
<td>1.652 2.304 0.604</td>
</tr>
<tr>
<td>$f''$</td>
<td>Investment adjustment cost</td>
<td>Gamma 4 1</td>
<td>1.902 1.948 0.294</td>
</tr>
<tr>
<td>$\delta(U)/\delta$</td>
<td>Capital utilization elasticity</td>
<td>Gamma 4 1</td>
<td>3.900 3.939 0.503</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Calvo prices</td>
<td>Beta 0.5 0.1</td>
<td>0.944 0.944 0.010</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Calvo wages</td>
<td>Beta 0.75 0.1</td>
<td>0.957 0.932 0.024</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>Price indexation</td>
<td>Beta 0.5 0.15</td>
<td>0.155 0.179 0.047</td>
</tr>
<tr>
<td>$\iota_w$</td>
<td>Wage indexation</td>
<td>Beta 0.5 0.15</td>
<td>0.279 0.306 0.065</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit</td>
<td>Beta 0.7 0.1</td>
<td>0.486 0.461 0.037</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>R&amp;D elasticity</td>
<td>Beta 0.6 0.2</td>
<td>0.429 0.403 0.109</td>
</tr>
</tbody>
</table>

Most of our estimates are similar to those in the literature. The price and wage rigidities are higher, while the elasticity of R&D with respect to research labor is lower. This last discrepancy may reflect the fact that (effectively) we use quarterly data while the literature uses annual data, and one would expect greater diminishing returns to R&D at higher frequencies due to the difficulty to adjust this type of labor.

With respect to the shocks, we find lower estimates of the persistence of exogenous TFP than in the literature. This surely reflects the fact that our model produces significant endogenous persistence in TFP. The estimate of the volatility of shocks to the productivity of R&D are also significant suggesting that this is an important element to fit the data. On the demand side, we obtain a high volatility of government shocks and a high estimate for the autocorrelation of liquidity shocks.

4.3 Analysis of variance

Table 5 presents the model (theoretical) standard deviation of the observable variables and compares them with their empirical volatility. Roughly speaking the model is in line with the actual volatilities of the key variables.

To ascertain the relevance of each shock for business cycles, Table 6 presents the variance decomposition. Liquidity shocks are the most significant drivers of fluctuations in output growth, accounting for nearly 40% of its variance. Shocks to the Taylor rule, exogenous TFP and government spending are also significant drivers of output growth, each accounting
Parameter Description Prior Posterior

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\theta}$</td>
<td>TFP</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{pk}$</td>
<td>Investment</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{\varphi}$</td>
<td>Risk premium</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{mp}$</td>
<td>Monetary</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{\mu}$</td>
<td>Mark up</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{g}$</td>
<td>Govt exp</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{\mu w}$</td>
<td>Wage mark up</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\rho_{\chi}$</td>
<td>R&amp;D</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>$\sigma_{g}$</td>
<td>TFP</td>
<td>IGamma</td>
<td>0.1 2</td>
</tr>
<tr>
<td>$\sigma_{pk}$</td>
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<td>Risk premium</td>
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<td>$\sigma_{mp}$</td>
<td>Monetary</td>
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</tr>
<tr>
<td>$\sigma_{\mu}$</td>
<td>Mark up</td>
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<tr>
<td>$\sigma_{g}$</td>
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<tr>
<td>$\sigma_{\mu w}$</td>
<td>Wage mark up</td>
<td>IGamma</td>
<td>0.1 2</td>
</tr>
<tr>
<td>$\sigma_{\chi}$</td>
<td>R&amp;D</td>
<td>IGamma</td>
<td>0.1 2</td>
</tr>
</tbody>
</table>

Table 4: Prior and Posterior Distributions of Shock Processes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.39</td>
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<tr>
<td>Output</td>
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<td>0.67</td>
</tr>
<tr>
<td>Labor</td>
<td>0.74</td>
<td>0.81</td>
</tr>
<tr>
<td>Investment</td>
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<td>1.75</td>
</tr>
<tr>
<td>Nominal R</td>
<td>0.71</td>
<td>0.25</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.25</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 5: Comparison of Standard Deviations

for between 14% and 19%. Interestingly, shocks to the R&D productivity are irrelevant for fluctuations in output growth despite their large volatility.\(^{11}\) This is consistent with Griliches (1990) who surveys estimates of the productivity of R&D and concludes that the consensus is that R&D productivity is a-cyclical.

Liquidity shocks are also important drivers of fluctuations in consumption growth (38% of its variance), investment growth (15%), hours (37%), nominal interest rates (56%), the adoption rate (43%) and endogenous TFP (35%). Other shocks that drive endogenous TFP are the money shock (20%) and the wage markup shock (23%).

\(^{11}\)This is the case because a shock to R&D productivity is similar to a news shock since it does not impact current TFP but will affect future TFP. In our context, an increase in the productivity of R&D has a very small positive contemporaneous effect on output, and a much larger effect in the medium and long run.
Table 6: Variance Decomposition (%)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Liquidity Demand</th>
<th>Money Exp</th>
<th>Govt Price of Capital</th>
<th>TFP</th>
<th>R&amp;D</th>
<th>Mark up</th>
<th>Wage mark up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Growth</td>
<td>39.8</td>
<td>18.6</td>
<td>14.6</td>
<td>6.9</td>
<td>18.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Consumption Growth</td>
<td>38.6</td>
<td>15.6</td>
<td>26.0</td>
<td>3.1</td>
<td>14.1</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Investment Growth</td>
<td>15.1</td>
<td>9.6</td>
<td>1.9</td>
<td>61.5</td>
<td>11.3</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.1</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>1.9</td>
<td>0.0</td>
<td>87.0</td>
</tr>
<tr>
<td>Nominal R</td>
<td>56.1</td>
<td>25.0</td>
<td>1.9</td>
<td>4.9</td>
<td>7.2</td>
<td>0.1</td>
<td>2.3</td>
</tr>
<tr>
<td>Hours</td>
<td>36.9</td>
<td>17.3</td>
<td>13.6</td>
<td>8.6</td>
<td>23.4</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>R&amp;D Growth</td>
<td>18.7</td>
<td>7.5</td>
<td>5.0</td>
<td>2.6</td>
<td>7.1</td>
<td>48.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Endogenous TFP</td>
<td>35.5</td>
<td>20.0</td>
<td>2.3</td>
<td>3.1</td>
<td>5.4</td>
<td>10.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Speed of Diffusion</td>
<td>43.44</td>
<td>23.01</td>
<td>2.44</td>
<td>1.18</td>
<td>4.99</td>
<td>5.93</td>
<td>0.47</td>
</tr>
</tbody>
</table>

The theoretical variance decomposition (HP filter, $\lambda = 1600$). ZLB is not imposed.

5 Analysis

In this section we present and discuss impulse response functions of our estimated model and historical decompositions of the key drivers of endogenous growth in order to elucidate our model's implications for the evolution of productivity over our sample period. We also discuss inflation dynamics during the Great Recession viewed through the lens of our model.

5.1 Impulse response functions

Figure 4 presents the response of some key variables to a one standard deviation shock to liquidity (first column), money (second column) and exogenous TFP (third column). For comparison, we plot the responses in our model and in a version where technology is purely exogenous.

An increase in the demand for liquidity raises the rate of return required to hold “non-liquid” assets in the economy. These include physical capital, the right to adopt intermediate goods, and the right to commercialize adopted intermediate goods. As a result, investment in physical capital and demand for skilled labor services decline lowering aggregate demand. Since prices are rigid, this leads to an output contraction. Note that, because the liquidity shock also affects the return to R&D and adoption activities, the output contraction is larger in the model with endogenous technology.

The reduction in R&D and adoption activities triggers a reduction in the growth rate of endogenous TFP which results in a gradual deterioration in the level of TFP. The decline in TFP is permanent and also leads to permanent contractions in output and consumption.

The pro-cyclical dynamics of productivity growth in the model mute the response of in-
flation to the liquidity shock. As in conventional New Keynesian models, inflation declines when aggregate demand falls. However, once technology becomes endogenous, productivity growth also co-moves with aggregate demand. This second force leads, other things equal, to an increase in the marginal cost of production that partially counters the positive relationship between output growth and inflation that arises from the Philips curve. Hence, there is a more muted response of inflation in the endogenous technology model. This feature of the model can therefore offer at least part of the explanation for the surprising failure of inflation to decline in line with the fall in output experienced during the Great Recession.

Columns 2 and 3 of Figure 4 show that the impulse responses to money and TFP shocks in our model. We note first that the money shock produces responses of the real economy and inflation that are qualitatively similar to the effect of the liquidity demand shock. Both shocks raise the cost of capital. The main difference is that the money shock does so by raising the risk free rate, while the liquidity demand shock does so by increasing the spread between the cost of capital and the risk free rate. Another point to note is that the effects of the money and TFP shocks are qualitatively similar to the exogenous technology model. The main differences are that the endogenous technology mechanisms increase the model’s amplification of the shocks and also produce greater persistence. As discussed in Section 3, this is the case because, in the presence of wage rigidities, expansionary shocks to exogenous TFP, and money have positive effects on the skilled hours devoted to R&D and adoption activities. In this way, we can reconcile the strong pro-cyclicality of these activities in the data with the evidence that these activities are intensive in labor.\(^\text{12}\)

A potentially important feature of the Great Recession is that nominal interest rates have been very close to zero. This observation suggests that the constraint faced by banks and monetary authorities to set nominal interest rates above zero may have been occasionally binding. To understand the impact that a binding zero lower bound (ZLB) may have on the model’s response to shocks, Figure 5 plots the impulse response functions with and without a binding ZLB.\(^\text{13}\) When the ZLB is binding, monetary policy cannot accommodate a recessionary shock. This results in higher interest rates than when the ZLB is not binding. The higher real rates amplify the drops in investment, R&D and adoption intensity. In the short term, this leads to lower aggregate demand and a larger output drop. It also leads to larger declines in the growth rate of the number of adopted technologies and to lower levels of TFP in the medium and long term.

\(^{12}\text{See Barlevy (2007).}\)

\(^{13}\text{In particular, this is achieved with a five-standard deviation positive shock to the liquidity preference.}\)
5.2 Historical decomposition

To further explore the role of liquidity shocks and endogenous technology propagation in business cycle dynamics, we conduct a historical analysis of business cycles through the lens of our estimation. Figure 6 plots\textsuperscript{14} the historical evolution of per capita output growth, together with the contributions of TFP and liquidity shocks. The main take away from the figure is that liquidity shocks play a key role in driving output growth. This is especially the case around recessions. In the three recessions over our sample period, liquidity shocks have caused large drops in output growth which account for the overwhelming majority of the actual contraction. In particular, the contribution of liquidity shocks is much more significant than that of exogenous TFP shocks.

The relevance of liquidity shocks for business cycle fluctuations calls for some external validation of the series of liquidity shocks we have estimated. To this end, Figure 7 compares our estimated liquidity shocks with the measure of the liquidity premium calculated by Gilchrist and Zakrajsek (2012).\textsuperscript{15} The figure shows that the two series are highly correlated (0.69). In particular, both series show increases in liquidity premia around recessions with an absolute peak in the sample around early 2009. The maximum premium we estimate is slightly higher (2.3 vs. 2 in GZ), but in our series there is more persistence in the premium after the Great Recession. This difference may capture the presence of constraints on credit in the post-Great Recession period that are not reflected in the prevailing rates for companies that have access to credit. We consider that the similarity between our estimated liquidity shocks and the GZ series supports our identification strategy.

5.3 Productivity dynamics

We conclude our historical exploration by studying the evolution of productivity. In so doing, we intend to shed light on a number of relevant debates recently opened in the literature. The goal from our analysis is to ascertain why productivity has slowdown since 2005. One hypothesis advanced by Fernald (2014) is that the driver of the slowdown is an exogenous decline in productivity: the bad luck hypothesis. An alternative consistent with

\textsuperscript{14}The decomposition takes into account the ZLB (as described in Appendix A.2), which makes the model nonlinear for the period 2008:I-2012:IV. Because of this nonlinearity, the sum of the contribution of each shock does not equal the value of the smoothed variable being decomposed (output growth in this case) for the mentioned period. This nonlinear residual emerges because the interaction between shocks is relevant in nonlinear models. However, our results indicate that the only shock that moves the economy to the ZLB is the liquidity demand shock. We therefore assign the nonlinear residual to this shock. This comment also applies to Figures 7-11.

\textsuperscript{15}Gilchrist and Zakrajsek (2012) use Compustat to measure the excess interest rate paid on long-term corporate bonds over the 10 year government bonds.
our model is that the slowdown reflects a decline in the agents’ investments in activities that enhance TFP such as adoption and R&D. A related debate posed by Gordon (2012) concerns the potential for future productivity growth in the U.S. economy. Specifically, Gordon argues that this potential has diminished over the last decades.

To start exploring these issues, we use equation (11) to derive the following expression for labor productivity:\footnote{16}{This expression holds to a first order approximation.} \footnote{17}{We focus on labor productivity for two reasons. First, our measure of capital includes both residential investment and consumer durables. Therefore, there is a discrepancy between our measure of TFP and that from standard sources (e.g., BLS). Second, labor productivity also captures the effect of variation in capital per hour. This is another channel by which, fluctuations in demand can affect the potential supply in the economy.}

$$\frac{Y_t}{L_t} = \theta_t \cdot (A_t)^{\theta-1} \cdot (U_t K_t / L_t)^\alpha.$$  

The first component is the exogenous TFP, the second is endogenous TFP and the third is a capital deepening component that includes both capital per hour worked and the utilization rate. Figure 8 plots the evolution of labor productivity together with these three components.

Figure 1 showed that over our sample period, there are three regimes for both (linearly detrended) labor productivity and TFP. Between 1984 and 1995 and between 2005 and 2012, they declined. Between 1995 and 2005, both productivity measures grew faster than trend.

Exogenous TFP and capital deepening are equally important in the decline of labor productivity between 1984 and 1995, with endogenous TFP playing no role (See Figure 8). All three components contributed to the acceleration in productivity from 1995 to 2000. The most important contributor was exogenous TFP which led to an increase by three percentage points, while the other two components each added approximately one percentage point. After 2000, capital deepening continued to grow and was the component that drove the increase in labor productivity between 2000 and 2005. Exogenous TFP first declined and then recovered, but by 2005 it had a level similar to that in 2000. The endogenous component of TFP, instead, started to decline monotonically around 2001.

After 2005, the three components evolved in very different ways. Capital deepening continued to increase until the end of the Great Recession, and only then, declined by two percentage points. Exogenous TFP declined between 2005 and 2009 by four percentage points. But, after mid 2009 it recovered, so that its contribution to the overall decline in labor productivity between 2005 and 2012 is of only one percentage point.
The endogenous component of TFP accounts for most of the decline in labor productivity since 2005. Though the fall in endogenous TFP started in 2001, its rate of decline accelerated during the Great Recession and continued after the 2009:II trough. Overall, the endogenous component of TFP accounts for a decline in labor productivity between 2005 and 2012 of five percentage points versus the overall decline of 7 percentage points. Therefore, the role of bad luck in the form of a drop in exogenous TFP has been quite limited during the 2005-12 period.

This accounting exercise raises two important questions. First, what mechanisms channeled the drop in endogenous TFP since 2001. Second, what shocks account for the evolution of endogenous TFP and labor productivity.

We explore first the mechanisms that drove endogenous TFP. From equation (21), fluctuations in the stock of adopted technologies may come from changes in the stock of unadopted technologies and from changes in the adoption rate. To explore the relevance of these two channels, Figure 9 plots the evolution of the total number of technologies \(Z_t\) and the adoption rate \(\lambda_t\) – measured on the right-hand side axis. The stock of technologies is also a key determinant of the stock of unadopted technologies \(Z_t - A_t\), and is driven by R&D productivity \(\chi_t\) and by R&D hours \(L_{srt}\).

Figure 9 reveals a decline in \(Z_t\) between 1992 and 2001 by four percentage points. Between 2002 and 2007, \(Z_t\) also dropped by almost 5 percentage points and between 2007 and 2012 it remained roughly unchanged.

The evolution of the adoption rate is quite different from \(Z_t\). First, because \(\lambda_t\) is a control variable, it fluctuates significantly at business cycle frequencies. In particular, Figure 9 shows sharp drops in the speed of adoption in 1991, 2001 and 2008. In addition to these pro-cyclical movements, \(\lambda_t\) also fluctuates significantly at medium term frequencies. As a result of these fluctuations, the adoption rate is lower during the 2002-2007 period than over the period 1995-2000, and it is lower after the Great Recession than during the period 2002-2007.

Overall, Figure 9 reveals that the fluctuations in the adoption rate are more important than fluctuations in the stock of developed technologies in accounting for fluctuations in endogenous TFP. During the second half of the 1990s, both adoption and \(A_t\) speed up, while \(Z_t\) declined. During the 2001 recession, the decline in endogenous TFP is entirely driven by a decline in adoption activity. Between 2002 and 2007, both the adoption and R&D margins contributed to the decline in endogenous TFP. However, since the beginning of the Great Recession the drop in endogenous TFP has entirely been the result of the slowdown in adoption activity.
The importance our estimation attributes to fluctuations in the speed of diffusion of new technologies leads us to explore whether the fluctuations induced by our model in $\lambda_t$ are realistic. The estimates from section 2 on the elasticity of the speed of diffusion with respect to output suggests that, indeed, it is reasonable. In particular, we found that the estimates range from 3.6 to 4.1. Over our sample period, the standard deviation of $\lambda_t$ is 3.8 times the standard deviation of output. This statistic suggests that the volatility of the adoption rate produced by our model is in the bracket of estimates available in the data. A similar conclusion can be reached by inspecting Figure 9. After the Great Recession, output and the adoption rate were, respectively, 8 percent and 30 percent below trend. This suggest an elasticity of adoption with respect to output of approximately 3.75, also within the interval of empirical estimates. Hence, we conclude that the volatility of the adoption rates implied by our model is consistent with the evidence.

The final question we address in our historical account of business cycles concerns the sources of fluctuations in productivity measures. By exploring this question, we can quantify the decline in the innovation capacity of the U.S. economy and its consequences for TFP.

We start by studying the contributions of various shocks to the evolution of endogenous TFP (see Figure 10). Liquidity shocks are the main driver of endogenous TFP during and after the Great Recession. Before the Great Recession, both liquidity and money shocks contributed to the slowdown in the number of adopted technologies between 2001 and 2003.

Consistent with Gordon (2012), we find that R&D productivity declined by 11 percentage points between 2001:I until 2005:I. This decline in $\chi_t$ caused a decline in endogenous TFP from 2005 to 2009 of approximately one and a half percentage points. This decline is not irrelevant but it is half of the drop in endogenous TFP caused by liquidity shocks since the beginning of the Great Recession.

Finally, we explore the effect that demand shocks have on the supply side. To this end, Figure 11 plots the contribution of our two main demand side shocks (i.e., the liquidity and money shocks) to the evolution of labor productivity. In particular, the key demand shocks were important drivers of the acceleration in labor productivity between 1995 and 2001 through their effect on capital deepening. They also contributed to the slowdown in productivity between 2001 and 2007 through both endogenous TFP and capital deepening channels. During the Great Recession, the key demand shocks contributed to the decline in productivity only through endogenous TFP. After the Great Recession, the main demand shocks fully account for the decline in labor productivity. Both capital deepening and endogenous technology are significant channels. In sum, we find very strong evidence on the impact that demand shocks has had on the dynamics of productivity over our sample
period and very especially over the last fifteen years.

6 Conclusions

We have estimated a monetary DSGE model with endogenous productivity via R&D and adoption. We then used the model to assess the behavior of productivity, with particular emphasis on the slowdown following the Great Recession. Our key result is that this slowdown mainly reflected an endogenous decline in the speed at which new technologies are incorporated in production. The endogenous decline in adoption, further, was a product of the recession. We also find that our endogenous productivity mechanism can help account for the productivity slowdown that preceded the Great Recession. Though here, shocks to the productivity of the R&D process play a role along with demand shocks. Finally, we find a very limited role for an exogenous decline in TFP in the slowdown of productivity. Overall, the results suggest that the post-Great Recession productivity slowdown was not simply bad luck, but rather another unfortunate by-product of the downturn.

Our analysis sheds light on two open debates. First, it provides a time series for the productivity of R&D activities that can be used to explore the hypothesis advanced by Gordon (2012) that the U.S. economy is experiencing a secular deterioration in its innovation capacity. Consistent with Gordon’s hypothesis we find a decline in the productivity of R&D activities between 2001 and 2004 that contributed to the decline in TFP between 2005 and 2009. However, this episode is short-lived and the estimates suggest that the slowdown in productivity reflects medium term cyclical factors rather than secular ones. The second relevant debate concerns the stability of inflation during the Great Recession in spite of the very significant decline in economic activity. Our model and estimates suggest that the endogenous decline in TFP has increased production costs (relative to trend) counteracting the traditional Phillips- curve effect of economic contractions on inflation.

Overall, our results emphasize the importance of the effects that demand shocks have on the supply side over the medium term. This is an important take away that can be used to explain productivity dynamics more generally.
A Appendix

A.1 Data

The data used for estimation are available from the FRED (https://research.stlouisfed.org/fred2/) and NSF (http://www.nsf.gov/statistics/) websites. Descriptions of the data and their correspondence to model observables follow (the standard macro series used are as in Del Negro et al. (2015)). To estimate the model we use data from 1984:I to 2008:III.

Real GDP (GDPC), the GDP deflator (GDPDEF), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) data are produced by the BEA at quarterly frequency. Average weekly hours of production and nonsupervisory employees for total private industries (AWHNONAG), civilian employment 16 and over (CE16OV) and civilian noninstitutional population 16 and over (CNP16OVA) are released at monthly frequency by the Bureau of Labor Statistics (BLS) (we take quarterly averages of monthly data). Nonfarm business sector compensation (COMPNFB) is produced by the BLS every quarter. For the effective federal funds rate (DFF) we take quarterly averages of the annualized daily data (and divide by four to make the rates quarterly).

Letting $\Delta$ denote the temporal difference operator, the correspondence between the standard macro data described above and our model observables is as follows:

- Output growth $= 100 \times \Delta \ln((\text{GDPC})/\ln(\text{INDEX}))$
- Consumption growth $= 100 \times \Delta \ln((\text{PCEC}/\text{GDPDEF})/\ln(\text{INDEX}))$
- Investment growth $= 100 \times \Delta \ln((\text{FPI}/\text{GDPDEF})/\ln(\text{INDEX}))$
- Real Wage growth $= 100 \times \Delta \ln(\text{COMPNFB}/\text{GDPDEF})$
- Hours worked $= 100 \times \ln((\text{AWHNONAG} \times \text{CE16OV}/100)/\ln(\text{INDEX}))$
- Inflation $= 100 \times \Delta \ln(\text{GDPDEF})$
- FFR $= (1/4) \times \text{FEDERAL FUNDS RATE}$

The R&D data used in estimating the model is produced by the NSF and measures R&D expenditure by US corporations. The data is annual, so in estimating the model and extracting model-implied latent variables (see Appendix A.2) we use a version of the Kalman filter adapted for use with mixed frequency data.
A.2 Extracting Model-Implied Latent Variables during ZLB period

The piece-wise linear solution from the OccBin method developed by Guerrieri and Iacoviello (2015) can be represented in state space form as

\[ S_t = C(N_t, \theta) + A(N_t, \theta)S_{t-1} + B(N_t, \theta)\epsilon_t \]

\[ Y_t = H(N_t, \theta)S_t \]

Where \( \theta \) is a vector of structural parameters, \( S_t \) denotes the endogenous variables at time \( t \), \( Y_t \) are observables, and \( \epsilon_t \) are normally and independently distributed shocks. \( N_t \) is a vector that identifies whether the occasionally binding constraint binds at time \( t \) and whether it is expected to do so in the future. In particular, \( N_t \) is a vector of zeros and ones indicating when the constraint is or will be binding. For example, the vector \( N_t = (0, 1, 1, 1, 0, 0, 0,...) \) is a situation in which the constraint does not bind at time \( t \) (denoted by the first zero in the vector), but is expected to bind in \( t+1, t+2 \) and \( t+3 \). Note that the matrices \( A, B \) and \( C \), which in a standard linear approximation depend only on parameters are here also functions of \( N_t \).\(^{18}\)

OccBin provides a way of computing the sequence of endogenous variables \( \{S_t\}_{t=1}^T \) and regimes \( \{N_t\}_{t=1}^T \) for a given initial condition \( S_0 \) and sequence of shocks \( \{\epsilon_t\}_{t=1}^T \). The vector \( N_t \) is computed by a shooting algorithm and its resulting value will depend on the initial state and the shocks at time \( t \). We refer the reader to Guerrieri and Iacoviello (2015) for a detailed description of the method.

We construct the Kalman filter and smoother from the nonlinear state space representation presented above by taking advantage of the fact that a given sequence of regimes, say \( \{\hat{N}_t\}_{t=1}^T \), uniquely defines a sequence of matrices \( \{\hat{C}_t, \hat{A}_t, \hat{B}_t, \hat{H}_t\}_{t=1}^T \). It follows that for that specific set of regimes the state space representation becomes linear:

\[ S_t = \hat{C}_t + \hat{A}_tS_{t-1} + \hat{B}_t\epsilon_t \]

\[ Y_t = \hat{H}_tS_t \]

For this linear state space representation it is straightforward to compute the Kalman filter and smoother. We use this fact in our algorithm by running two blocks: (i) one in

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\(^{18}\)The matrix \( H \) might also be a function of \( N_t \) because some observables might become redundant when the occasionally binding constraint binds. This is the case for the Taylor rule interest rate when the ZLB binds.
which we compute the Kalman filter and smoother for a given set of regimes \( \{N_t\}_{t=1}^T \); and

(ii) another where we use OccBin to compute the regimes given a sequence of shocks \( \{\epsilon_t\}_{t=1}^T \).

The algorithm steps are the following.

1. Guess a sequence of regimes \( \{N_t^{(0)}\}_{t=1}^T \);

2. Use the guess from the previous step and define the sequence of matrices \( \{C_t, A_t, B_t, H_t\}_{t=1}^T \) using OccBin;

3. With the matrices from the previous step, compute the Kalman Filter and Smoother using the observables \( \{Y_t\}_{t=1}^T \), and get the Smoothed shocks \( \{\hat{\epsilon}_t\}_{t=1}^T \) and initial conditions of endogenous variables;

4. Given the smoothed shocks and initial conditions from the previous step, use OccBin to compute a new set of regimes \( \{N_t^{(1)}\}_{t=1}^T \);

5. If \( \{N_t^{(0)}\}_{t=1}^T \) and \( \{N_t^{(1)}\}_{t=1}^T \) are the same, stop. If not, update \( \{N_t^{(0)}\}_{t=1}^T \) and go to step 2.

Once it converges, this algorithm yields a sequence of smoothed variables and shocks, consistent with the observables, and taking into account the occasionally binding constraint.
Figure 4:
Impulse Response Functions (1 std. dev.)

- **Liq Demand**
- **Money**
- **TFP**
  - Output
  - Consumption
  - Invest
  - Endog TFP
  - Inflation
  - Nominal R

Legend:
- **Endogenous TFP**
- **Exogenous TFP**
Figure 5:

Liquidity Demand Shock (5 std. dev.)

- Output
- Consumption
- Investment
- Inflation
- Nominal R
- Hours
- R&D
- Adoption
- Endogenous TFP

Legend:
- ZLB
- Linear model
Data sources are described in Appendix A.1. Smoothed shocks from model estimated using data as described in Section 4.2 and Appendix A.1.
Figure 8:

Labor Productivity by Component

Labor productivity is GDP divided by hours worked (see Appendix A.1 for data sources). Smoothed shocks from model estimated using data as described in Section 4.2 and Appendix A.1.
Figure 9:

Drivers of Endogenous Technology

Legend:
- A
- lamz (right axis)
- Z

Time periods:
- Q1-80 to Q1-85
- Q1-90
- Q1-95
- Q1-00
- Q1-05
- Q1-10
- Q1-15
Figure 10:

Smoothed variables from model estimated using data as described in Section 4.2 and Appendix A.1.
Labor productivity is GDP divided by hours worked (see Appendix A.1 for data sources). Smoothed shocks from model estimated using data as described in Section 4.2 and Appendix A.1.
References


Reinhart, C. M. and K. Rogoff (2009): This time is different: Eight centuries of financial folly, Princeton University Press.

