EFFECTIVE SMOKESTACK HEIGHT

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This hand-out recapitulates how to estimate the effective height of smokestacks.

1. Quantities:

Smokestack parameters are:

H = Effective smokestack height (in m)

h = Actual smokestack height (in m)

Δh_b = Vertical plume rise due to buoyancy and/or momentum (in m)

Δh_d = Vertical distance of downwash (in m)

g = Gravitational acceleration (= 9.81 m/s^2)

r = Inner radius of stack at its exit (in m)

u = Wind speed at stack height h (in m/s)

w_s = Vertical velocity at which fumes exit the stack (in m/s)

T_s = Temperature at which fumes exit the stack (in absolute degrees Kelvin)

T_a = Ambient temperature at stack height h (in absolute degrees Kelvin)

F = Buoyancy flux parameter (in m^4/s^3)

x_f = Downwind distance over which a bent-over plume rises (in m)

N = Stratification frequency of the atmosphere (in 1/s)

Γ = Adiabatic lapse rate ($= g/C_p = 9.76\ 10^{-3}$ K/m)

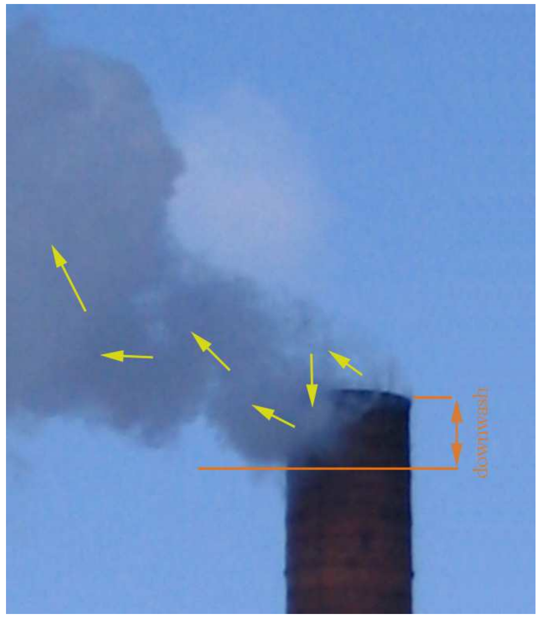

Figure 1. Example of downwash.

2. Downwash:

Downwash results from the formation of a vortex in the wind wake behind the stack. If the wind speed is large and the gas exit velocity is small, the low pressure inside this vortex is capable of pulling the smoke downward, below the rim of the stack.

The rule is:

$$\begin{aligned} &\text{If } w_s \geq 1.5u \quad \text{then} \quad \Delta h_d = 0, \quad \text{there is no downwash} \\ &\text{If } w_s < 1.5u \quad \text{then} \quad \Delta h_d = 4r\left(1.5 - \frac{w_s}{u}\right). \end{aligned}$$

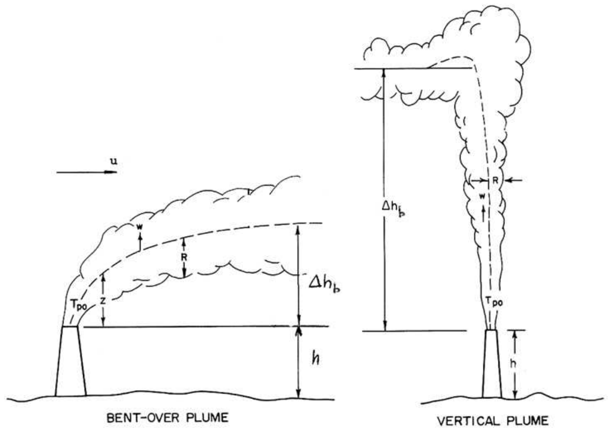

Figure 2. Bent-over plume and vertical plume.

3. Buoyancy Flux:

To determine the buoyancy rise, there is a series of steps. First, one determines the buoyancy flux parameter F, which is defined as:

$$F = gr^2 w_s \left(1 - \frac{T_a}{T_s}\right),$$

4. Plume types:

The second step is to determine the type of plume. Smokestack plumes broadly fall into two categories: bent-over plumes and vertical plumes. One assumes that a bent-over plume occurs in stability classes **A** to **D** whenever there is a significant wind, whereas a vertical plume occurs in stability classes **E** and **F**, or in stability class **D** when the wind is weak (say, less than 2 m/s).

5. Bent-over plumes:

For a bent-over plume, one first calculates the distance x_f over which the plume rises, using one of the following empirical formulas:

$$\begin{aligned} \text{If } F &< 55 \text{ m}^4/\text{s}^3 \quad \text{then} \quad x_f = 49\ F^{5/8} \\ \text{If } F &\geq 55 \text{ m}^4/\text{s}^3 \quad \text{then} \quad x_f = 119\ F^{2/5} \end{aligned}$$

with x_f obtained in meters when F is expressed in m^4/s^3. Once this distance is determined, one can finally calculate the buoyancy rise:

$$\Delta h_b = 1.6 \frac{F^{1/3} x_f^{2/3}}{u} .$$

6. Vertical plumes:

For a vertical plume, it is necessary to determine first the stratification parameter N^2:

$$N^2 = \frac{g}{T_a} \left(\frac{dT_a}{dz} + \Gamma \right),$$

before calculating the buoyancy rise as follows:

$$\begin{aligned} \text{If } u &< 0.275(FN)^{1/4} \quad \text{then} \quad \Delta h_b = 4.0 \left(\frac{F}{N^3} \right)^{\frac{1}{4}} \\ \text{If } u &\geq 0.275(FN)^{1/4} \quad \text{then} \quad \Delta h_b = 2.6 \left(\frac{F}{N^2 u} \right)^{\frac{1}{3}} \end{aligned}$$

7. Putting it all together:

The effective smokestack height is determined by adding the pieces:

$$H = h + \Delta h_b - \Delta h_d.$$