**Lakes** are natural bodies of water, where flow from one or several rivers is impounded by a natural obstacle.

A lake differs from the incoming rivers simply by the fact that its flow is far weaker and no longer driven by gravity down a slope. Instead, currents in lakes are driven by surface winds and buoyancy forces.

A **reservoir** is an artificial lake created by a dam blocking a river.

There are several reasons why people build reservoirs:
- generation of hydro-electric power,
- flood control,
- freshwater supply (for households or irrigation),
- recreation, and
- control of water quality.

But, there are also disadvantages:
- elimination of dry land (sometimes ancestral lands) and
- obstacle to fish swimming upstream for spawning (sometimes remedied by fish ladders).
Unlike rivers, in which the flow is driven by gravity (downslope), lakes have motions that are generated by various mechanisms. The energy sources are:

**Wind** → wind stress on surface \( (\tau_{\text{wind}} \text{ in } \text{N/m}^2) \)

which creates

- mean currents
- varying currents & waves
- turbulence, leading to mixing in the vertical

\( \text{in 3D geometry} \)

**Heat flux** \( (\vec{\bar{h}}, \text{ in } \text{W/m}^2) \) essentially of 2 types:

- **Heating** (daytime, summer) → vertical stratification
  - inhibits mixing
  - shields intermediate and bottom waters from wind-stirred mixing and surface re-aeration

- **Cooling** (nighttime, winter) → convective overturning
  - creates vigorous mixing
  - ventilates the water (re-aeration of bottom waters by the end of winter)

Lakes and reservoirs differ from rivers by their greater depths and their weaker velocities. They therefore impound water for quite some time, and an important characteristic of a lake is its *residency time*, sometimes also called *retention time*.

It is defined as the average time spent by a water parcel from time of inflow to that of outflow. The longer a fluid parcel remains in the lake or reservoir, the more likely it is to be subjected to local processes such as heating or cooling, sedimentation, biological or chemical transformations, etc.

If the volume of the reservoir is \( V \) and the input flow rate (defined as the average volumetric inflow rate) is \( Q \), the residency time is given by

\[
\theta = \frac{V}{Q}
\]

**Example of Wellington Reservoir in Western Australia:**

\[
\begin{align*}
V &= \text{area} \times \text{average depth} = 30 \text{ m} \times 10^7 \text{ m}^2 = 3 \times 10^8 \text{ m}^3 \\
Q &= 5 \text{ m}^3/\text{s}.
\end{align*}
\]

\[
\theta = \frac{V}{Q} = 6 \times 10^7 \text{ s} = 694 \text{ days},
\]

or about 2 years.

When the residency time surpasses one year, as in the preceding example, we expect that seasonal variations are important and that the thermal variations under summer heating and winter cooling may control the lake’s dynamics, especially its level of turbulence and hence capacity to mix and disperse pollutants. In addition to the seasonal cycle, lakes and reservoirs are also often subject to diurnal variations.
Case study
Wellington reservoir in Western Australia

(Fischer et al., 1979, Chapter 6)

Wellington Reservoir:
Seasonal cycle
Wellington Reservoir:
Examples of thermal stratification

Quantifying vertical stratification

Thermal expansion: $\rho = \rho_0 [1 - \alpha(T - T_0)]$ with $\alpha = 2.57 \times 10^{-4} / ^\circ C$
and $\rho_0 = 999 \text{ kg/m}^3$ for $T_0 = 15 ^\circ C$

Decomposition of density field:
$\rho = \rho_0 + \rho_s(z) + \rho'(x,y,z,t)$

Stratification frequency $N$ defined from

$$N^2 = -\frac{g}{\rho_0} \frac{d\rho_s}{dz} + \alpha \frac{dT}{dz} > 0$$
Penetrative convection

Sinking velocity:

\[ w = -\sqrt{\frac{g h T'}{\rho_0}} \]

\[ = -\sqrt{\frac{\alpha g h T'}{\rho_0 \rho}} \]

essentially a balance between potential energy being converted to kinetic energy.

\( T' \) (<0) is the temperature anomaly of the sinking water.

Heat flux at surface:

\[ \tilde{H} = \rho_0 C_v |wT'| \]

Heat flux viewed as heat departing from the surface.

---

Figure 6.9 Pictorial representation of rising and falling plumes crossing the thermocline during penetrative convection. (From Fischl et al., 1979, page 173)
Budgets for penetrative convection

1. Heat budget:
   Overall decrease in temperature is due to heat removed
   \[
   \left[ \int_{H}^{H_h} \rho C_v T(z) \, dz \right]_{\text{before}} = \int_{0}^{\infty} \Delta H \, dt
   \]

2. Mechanical energy budget:
   Only re-arrangement of potential energy
   \[
   \int_{H}^{H_h} \rho g \alpha T(z) \, dz = \text{unchanged}
   \]

3. Solution:
   \[
   h = \sqrt{\frac{6 \rho g H t}{\rho C_v N^2}} \quad \Delta T = \frac{2 N^2 H t}{3 \rho C_v \alpha g} \]

4. Diffusivity:
   \[
   D_z = \frac{1}{10} \frac{d_{\text{max}} u_{\text{max}}}{(d_{\text{max}})} = \frac{1}{10} H \, w = 0.33 \frac{agH}{\rho C_v N^2} (N t)^{2/3}
   \]

When penetrative convection reaches the bottom:
Top-to-bottom convection

This is the ultimate state of penetrative convection. Convection has now reached the bottom but the vertically mixed temperature keeps on decreasing over time as heat is still being removed.

Typical fall velocity:
\[
\nu = \left( \frac{agH}{\rho C_v} \right)^{1/3}
\]

Vertical diffusivity:
\[
D_z = \frac{1}{10} \frac{d_{\text{max}} u_{\text{max}}}{(d_{\text{max}})} = \frac{1}{10} H \, w = 0.33 \left( \frac{agH}{\rho C_v} \right)^{1/3}
\]

Time for complete vertical mixing if substance released at surface:
\[
T = 0.536 \frac{H^2}{D_z} = 5.36 \left( \frac{\rho C_v H^2}{a g H} \right)^{1/3}
\]
Computer simulation of top-to-bottom convection

*Note:* For this simulation, the top and bottom temperatures were held constant over time. Thus, this is not the same as a fixed heat flux at the top boundary, but the structure of the sinking thermals remains qualitatively the same.

**Wind mixing**

Erosion of stratification and agitation of the upper layers of a lake or reservoir is not caused exclusively by convection. Wind stirring can accomplish the same effect.

Let us consider the consequences of irregular winds, namely winds that agitate the water but are not sufficiently coherent to create organized currents across the lake.

Wind momentum in the atmosphere is transferred to the water via the surface stress, denoted $\tau_{\text{wind}}$.

Observational evidence indicates that this stress is nearly proportional to the square of the wind speed $U$:

$$\tau_{\text{wind}} = C_D \rho_{\text{air}} U^2$$

where $C_D$ is a dimensionless drag coefficient and $\rho_{\text{air}}$ the air density at water level. If $U$, which varies with height, is the wind speed at the nominal height of 10 m (a convenient height for ship and buoy measurements, and also sufficiently away from the water waves at the surface to be a more reliable observation), the drag coefficient is about $C_D = 1.0 \times 10^{-3}$ to $1.5 \times 10^{-3}$. 
This wind-induced surface stress $\tau_{\text{wind}}$ creates turbulence in the water.

If we measure this turbulence activity by the eddy velocity scale $u_*$, an argument similar to that use for rivers provides:

$$\tau_{\text{wind}} = \rho_0 u_*^2 \quad \leftrightarrow \quad u_* = \sqrt{\frac{\tau_{\text{wind}}}{\rho_0}}$$

where $\rho_0$ is now the water density ($>> \rho_{\text{air}}$). It follows that $u_* << U$.

Winds on the surface of a water body generate waves, which in turn generate turbulence below the surface. At times, if the wind direction is relatively persistent, the turbulence can organize itself into well defined cells of overturning, called Langmuir cells.

Langmuir streaks on Quake Lake, Montana
Diurnal mixed layer (combination of daytime wind and heating/cooling)

Turbulent mixing generated by wind stress at the surface

Define a turbulent velocity from the wind stress:

\[ u_* = \sqrt{\frac{\tau_{\text{wind}}}{\rho_0}} \]

Energy budget reduces to:

Energy input from wind – fraction thereof consumed by frictional dissipation = energy necessary to increase potential energy (to lift denser fluid from below and push down lighter fluid from above).

Mathematical analysis yields:

\[ h = \left( \frac{12 m u_*^4 t^{1/3}}{N^2} \right) \]

with \( m = 1.25 \)

Vertical diffusivity:

\[ D_z = \frac{1}{10} \frac{d_{\text{max}} u_* (d_{\text{max}})}{N} = \frac{1}{10} \frac{h u_*}{N} = 0.25 \frac{u_*^2}{N} (N t)^{1/3} \]
Wind-driven circulation in Lake Michigan

A dye-release experiment in a small lake

Photos of an experiment in the lake of Muren and scheme of the stream
Lake Eyre in South Australia:

Australia's Lake Eyre is the 5th largest terminal lake in the world, with a drainage basin stretching 1.2 million km² from the Northern Territory to South Australia. The lake is mainly dry except in the wake of a rare, steady rainy season.

In early 2009, intense rains fell over northern Australia. A total of 17m megalitres of water flowed into the lake - which has no outlet - soaking into the soil and sustaining grasses. By June 10, when this satellite image was captured, the flow of water had slowed: Lake Eyre was as full as it was going to get in 2009.

(Photograph: Landsat/NASA)

Computer simulation of currents in Lake Constance, showing the insignificant effect of river inflow and outflow

Wind alone

Westerly wind only

Wind + Rhine River

Westerly wind + Rhine River
Computer simulation of summer surface currents and temperature in Lake Superior

Animation shows the evolution after a hypothetical one-time release of two tracers at the mouths of the Ontonagon and St. Louis Rivers over a period of about two years.

Mixing by wind-driven currents

Wind stress over the lake
→ computer model simulation
→ current distribution as function of time and space

Note that the water current $U$ is not directly depending on the local wind stress value but is a function of the wind stress distribution and history over the entire lake as well as the basin geometry.

Use billowing theory:

$$h = 0.3 \frac{U^2}{\alpha g \Delta T}$$

Vertical diffusivity:

$$D_v = \frac{1}{10} d_{max} u_*(d_{max}) = \frac{1}{10} h u = 0.07 \frac{U^2}{N}$$

Surface seiche in Lake Michigan

1st mode

2nd mode
Fluid mechanics at home: The fun “seiche”

Lake seiching can also occur internally with the wave being supported by thermal stratification

Figure 6.31 Formation of baroclinic motions in a lake exposed to wind stresses at the surface.
(a) Initiation of motion. (b) Position of maximum shear across the thermocline. (c) Steady state baroclinic circulation.
(From Fischer et al., 1979, page 182)
Velocity distribution in the lowest (gravest) internal seiche mode of a confined basin

Conservation of mass requires that what flows up and down on the sides be equal to what flows back and forth in the middle:

\[
\frac{wL}{2} \quad \frac{uH}{2} \Rightarrow \quad u = w \frac{L}{H}
\]

with the vertical velocity \( w \) given in terms of vertical displacement at the edges:

\[
w = \frac{d(\text{vertical displacement})}{dt}
\]

Large vertical displacements of thermal stratification in Cayuga Lake (Ithaca, New York). This is presumed to be due to a resonant internal wave (internal seiche).

(Graph courtesy of Professor Todd Cowen at Cornell University)
The Loch Ness mystery

Elongated lake → susceptible to seiching
Stratification → internal seiching as well

Dense waters happen to be much darker, and as this dark water undulates under the surface it gives the impression that there is a gigantic snake-like creature under the surface. “Nessie” is explained by hydrodynamics!