Chapter 7

Mixing in Estuaries

7.1 INTRODUCTION AND CLASSIFICATION

An estuary is where a river meets the sea. Pritchard (1967) has given a more circumscribed definition that "an estuary is a semienclosed coastal body of water which has a free connection with the open sea and within which sea water is measurably diluted with fresh water derived from land drainage." Our purpose in this chapter, however, is to describe mixing in a class of water bodies that do not necessarily fit Pritchard's definition, or any other strict definition of an estuary. We are concerned, for example, with mixing in a tidal bay, like perhaps San Diego Bay, which has so little fresh water inflow that there is no measurable dilution. We are also concerned with mixing in the tidal portions of rivers upstream of the maximum extent of sea water intrusion. The studies we describe are even applicable, to some extent, to mixing on some continental shelves well outside what might be called a semienclosed body of water, but where the effect of river water is still evident. Thus our definition of the bodies of water we are concerned with must be an operational one. In Chapter 5 we discussed mixing in a flow driven by the slope of the water surface; in Chapter 6 we discussed mixing in flows driven primarily by wind stresses and by internal density variations. The flows discussed in this chapter are driven by all three; the primary flow is usually driven by the slope of the tidal wave, but wind stresses and internal density variations are often important. In addition we add a new complication, the flow oscillates. The result is the complex, unsteady and
spatially varying flow which we customarily see in what are broadly referred to as “estuaries.” Our purpose is to give an account of how the various aspects of this flow lead to different types of mixing, to synthesize the results as best we can, and then to describe some practical methods for studying the distribution of pollutants.

Part of the difficulty in describing estuaries is that the term covers such a diversity of sizes and shapes. A method for classifying estuaries into categories would be helpful, but no single scheme has been found sufficient. Bowden (1967a) and Pritchard (1967) distinguished three major hydrodynamic categories; sharply stratified estuaries such as fjords and salt-wedge estuaries; partially stratified estuaries, in which there is a significant vertical density gradient; and well mixed estuaries (see Fig. 7.1). A second method of classification is geomorphological: coastal-plain estuaries; fjord type estuaries; bar-built estuaries; and the rest. Fjord-type estuaries are generally formed by glacial action; they are usually very deep, narrow, and highly stratified. Coastal plain estuaries are generally formed by the gradual drowning of a river system, and are usually long and narrow with many branches. Bar-built estuaries are formed by the closing off of an embayment by a sand bar, and are generally found along coasts with large littoral drift. “The rest” includes such large closed bays as San

![Diagram](image)

Figure 7.1 Salinity distributions along the axes of (a) a “salt wedge” estuary, (b) a “partially mixed” estuary, and (c) a well mixed estuary.
Francisco Bay, and estuaries formed by channels running across aluvial plains, such as the Columbia. A third possibility is to devise analytical methods of classification. Hansen and Rattray (1966) proposed a method based on the vertical variation of salinity and the strength of the internal density-driven circulation; some of their results are discussed in Section 7.2. A simple geometrical classification can be based on the ratios of length $L$, width $W$, and mean depth $d$. None of these schemes are able to express the unique characteristics of any estuary, however. Estuaries have individual personalities, made up of the distribution of sand bars, points of land, man made jetties and harbors, islands, deep channels, shallow bays, the characters of the tributary rivers, and the seasonal variation of the weather. Before doing a mixing study in an estuary it is best to get to know it.

This chapter begins with an enumeration of various causes of mixing in estuaries, and analytical treatments of them. We then discuss cross-sectional mixing and longitudinal dispersion in the same sequence as in the chapter on rivers, although with much less ability to give dependable results. We close with a discussion of the one-dimensional analysis as a practical tool. The discussion provides essential background to the descriptions of the use of numerical and physical estuary models in the next chapter.

7.2 THE CAUSES OF MIXING IN ESTUARIES

Mixing in estuaries results, as it does in rivers, from a combination of small-scale turbulent diffusion and a larger scale variation of the field of advective mean velocities. In rivers the combination is fairly simple, as explained in Chapter 5; the advective velocity field defines a set of approximately steady stream lines. The main role of turbulent diffusion is to transfer mass between stream lines, and longitudinal dispersion comes about mainly because the flow along different stream lines is going at different speeds. In estuaries we can also try to describe mixing in terms of advection by a mean flow along stream lines and turbulent diffusion between stream lines, but matters are nowhere near as simple as in rivers. The first problem is to differentiate diffusion from advection. If a current meter is held at a fixed point in an estuary and a long record is examined, spectral analysis can disclose fluctuations with a wide range of period. Fluctuations with a period of less than a few minutes can be identified as turbulence, and the transport resulting therefrom can be termed diffusive transport, just as we have done in rivers. The term “advection” can then be assigned to the remaining motion. The advective velocity is not constant, however, either in time, space, or direction. The velocity record obtained at a single point will contain semidiurnal and diurnal tidal variations, wind-induced variations of almost any period, an inertial frequency caused by the earth’s rotation, and
fluctuations of longer periods caused by the monthly and longer term variation of the tidal cycle and by seasonal variations of meteorological influences and tributary inflows. The direction of the velocity vector will often not be parallel to the channel axis, even if one can be defined. Often the flow is going in different directions at different depths; often the flow is one way near the shore and the opposite way in the center of the channel. Obviously, the analysis of mixing in terms of the interaction of advection and diffusion is much more complicated in estuaries than in rivers.

The proper way to begin seems to be to make things as simple as possible by considering different mechanisms in turn. Most of what is seen in an estuary can be related to one of three sources, the wind, the tide, and the river. Most of the analyses to be found in the engineering technical literature discuss the effect of only one or at most two sources, for example the current driven by the wind in a tideless bay or the circulation driven by the river inflow in a tideless estuary. Taking the literature as a guide, the next three sections discuss in turn the isolated effects of wind, tide, and river. In each case we will discuss qualitatively why and how the source causes mixing, and will quote what analytical results can be found. Later, in Section 7.4, we attempt an analytical synthesis based on a decomposition of the salinity and velocity profiles.

### 7.2.1 Mixing Caused by the Wind

Wind is usually the dominant source of energy in large lakes, the open ocean, and some coastal areas, but in estuaries it may or may not play a major role.

### 7.2.2 Mixing Caused by the Tide

The tide generates mixing in two ways. Friction of the tidal flow running over the channel bottom generates turbulence and leads to turbulent mixing, and the interaction of the tidal wave with the bathymetry generates larger scale currents. Efforts to quantify the rate of turbulent mixing are discussed in Section 7.3; here we discuss the effects of the larger scale currents. These include shear flow dispersion similar to that found in rivers, and in addition other circulations which we will classify by the terms “pumping” and “trapping” and discuss in detail below.

#### 7.2.2.1 The Shear Effect in Estuaries and Tidal Rivers

The most obvious characteristic of tidal flow in most estuaries is that the flow is like a river, but goes back and forth. In Chapter 5 we showed how to apply shear flow dispersion theory to rivers, and in Section 4.3 we showed analytically
the effect of oscillation on the longitudinal dispersion coefficient. Equations (4.55) and (4.56) combine to give

\[ K = K_0 f(T'), \]

(7.1)

where \( f(T') \) is plotted in Fig. 4.7. \( T' = T/T_c \) is the dimensionless time scale for cross-sectional mixing, \( T \) is the tidal period and \( T_c \) the cross-sectional mixing time. \( K_0 \) is the dispersion coefficient if the tidal period is much longer than \( T_c \); if the cross section is relatively wide and shallow and density effects are absent we can make use of the result given for rivers in Eq. (5.17) that \( K_0 = Iu^{1/2}T_c \), in which \( T_c = W^2/\alpha \) is the time scale for transverse mixing, and \( I \) is a coefficient whose value we found in Chapter 4 was generally approximately equal to 0.1. This result combines with Eq. (7.1) to give a prediction for the longitudinal dispersion coefficient in an estuary due to shear flow as

\[ K = 0.1u^{1/2}T[(1/T')f(T')]. \]

(7.2)

The function \([(1/T')f(T')]\) is plotted in Fig. 7.4. It has a maximum of approximately 0.8 when \( T' \) is approximately one, and shows that the shear flow dispersion coefficient will be small if the estuary is very wide (\( T' \) small) or very narrow (\( T' \) large). Shear flow dispersion will have its maximum effect if the tidal period is similar to the time required for cross sectional mixing; even in that case Eq. (7.2) puts a limit on the magnitude of the coefficient. For example, if the tidal period is 12.5 hr, the tide has a mean velocity of 0.3 m²/sec, and if \( u^{1/2} = 0.2m^2 \), as assumed in Chapter 5, the maximum possible value of the dispersion coefficient is approximately 60 m²/sec.

**Figure 7.4** The quantity \( T^{-1/2}f(T') \) used in Eq. (7.2).
7.2.2.3 Tidal “Trapping”

“Trapping” is a term used by the writers to describe the effects of side embayments and small branching channels. In Section 5.2.2 we discussed the effect of “dead zones” in rivers; similar side embayments exist in estuaries, but their role is enhanced by tidal action in a way apparently first noticed by Schijf and Schonfeld (1953). These writers analyzed what they called a “storing basin” mechanism and concluded that it was responsible for all the diffusive salt flux in some Dutch estuaries. We prefer to call this mechanism “trapping,” because it results from trapping of low velocity water along the sides of an estuary even if physical basins are not present.

The trapping mechanism can be explained as follows. The propagation of the tide in an estuary represents a balance between the inertia of the water mass, the pressure force due to the slope of the water surface (the shape of the tidal wave) and the retarding force of bottom friction. As a first example, consider the system shown in Fig. 7.9, a typical coastal plain estuary with one major channel and a number of side branches. In the main channel tidal elevations and velocities are usually not in phase; high water occurs before high slack tide and low water before low slack tide. This is because of the momentum of the flow in the main channel, which causes the current to continue to flow against an opposing pressure gradient. The side channel, in contrast, has less momentum, and the current direction changes when the water level begins to drop. Figure 7.9a shows a cloud of tracer particles being carried upstream by a flooding tide. Some of the particles go into the side channel and some continue upstream in the main channel (Fig. 7.9b). When the water surface begins to drop the particles in the side channel return into the main stream but now they are separated from their previous neighbors by unmarked water originally downstream of the whole cloud (Fig. 7.9c). The separation distance can be as much as the distance of travel in the main channel between high water and slack water, perhaps as much as a mile in a typical coastal plain estuary. Something resembling this effect probably occurs in almost all estuaries, and in many tidal rivers. In San Francisco Bay, for instance, the shoreline consists of irregularly shaped shallow basins, which probably play the same role as side channels. As another example, numerical studies of Jamaica Bay, New York, have shown that although in
overall plan the bay is almost round, trapping in the interior irregularities and inlets is an important dispersion mechanism.

Okubo (1973) has given an analysis which can be applied to the trapping mechanism. Okubo found that for a uniform velocity of flow in the main channel of velocity \( u = u_0 \cos \sigma t \) and a uniform distribution of traps along the sides having a ratio of trap volume to channel volume of \( r \), and a characteristic exchange time between traps and main flow of \( k^{-1} \), the effective longitudinal diffusivity is given by

\[
K = \frac{K'}{1 + r} + \frac{r u_0^2}{2k(1 + r)^2(1 + r + \sigma/k)},
\]

where \( K' \) is the longitudinal diffusivity in the main channel itself. Taking the Mersey as an example, \( u_0 = 1.5 \text{ m/sec} \) and \( \sigma = 1.4 \times 10^{-4} \text{ sec}^{-1} \); if we assume reasonable values of trap-volume ratio \( r = 0.1 \) and characteristic exchange time \( k^{-1} = 10^4 \text{ sec} \), \( K = 0.9K' + 360 \text{ m}^2/\text{sec} \). The second term is equal to the larger of two values of effective diffusivity reported by Bowden and Gilligan (1971). Thus it appears that the trapping mechanism alone can account for longitudinal dispersion in the Mersey.

### 7.2.3 Mixing Caused by the River

The river, or rivers if more than one enter the same estuary, delivers a discharge of fresh water \( Q_r \). Analytical and laboratory studies usually assume that all the fresh water passing a given section comes from a single upstream source, and throughout this discussion we will do the same. The complications that arise when a number of rivers supply fresh water around the periphery do not
change the qualitative description of how fresh water affects mixing, and we prefer to concentrate on what is known of the effect of a single source.

If a river discharges into an estuary connected to a nearly tideless sea, such as the Sea of Japan or the Mediterranean, the fresh water overrides the salt water and flows as a nearly undiluted layer into the sea. Salt water intrudes under-neath the fresh water layer in the form of a wedge, as illustrated in Fig. 7.1. If there is some tide the wedge moves back and forth; the more the wedge motion the more kinetic energy is available to break down the interface and turbulently mix the fresh and saline layers. The river may be thought of as a source of deficit of potential energy, and the tide as a source of kinetic energy to overcome the deficit. More precisely, the river is a source of buoyancy, of amount \( \Delta \rho g Q_t \), where \( \Delta \rho \) is the difference in density between the river and ocean water. The dimensionless ratio

\[
R = \frac{(\Delta \rho / \rho) g Q_t}{W U_t^3},
\]

where \( U_t \) is the rms tidal velocity and \( W \) the channel width, expresses the ratio of the input of buoyancy per unit width of channel to the mixing power available from the tide. \( R \) is a sort of Richardson Number; Fischer (1972a) called it the “Estuarine Richardson Number” in analogy with a “Pipe Richardson Number” defined by Ellison and Turner (1960). It is also equivalent to the ratio we used in Section 5.1.5 to express the likelihood that a buoyant discharge mixes vertically in a river flow. If \( R \) is very large we expect the estuary to be strongly stratified and the flow to be dominated by density currents. If \( R \) is very small we expect the estuary to be well mixed, and we might be able to neglect density effects. Observations of real estuaries suggest that, very approximately, the transition from a well mixed to a strongly stratified estuary occurs in the range \( 0.08 < R < 0.8 \).

Figure 7.10 shows a vertical section along a typical partially stratified estuary. The isohalines (lines of constant salinity) slope upward towards the ocean. The natural tendency of isohalines is to become horizontal, because that is the condition of a stratified water body at rest; as illustrated in Fig. 6.4c sloping isohalines imply pressure gradients which, in the absence of other forces, will drive a current tending to bring the isohalines to the horizontal. In the case shown in the figure the necessary currents are a flow landward along the bottom and seaward at the surface. Such a flow is often referred to as the “classical estuarine circulation,” or sometimes as “gravitational circulation.” Internal flows driven by density variations are more properly called “baroclinic circulation,” as distinguished from “barotropic circulation” which occurs in flows of constant density. The tide-driven flows discussed in the previous section are barotropic flows; the river-driven flows discussed here are baroclinic.

It should be noticed that if the estuary is perfectly well mixed vertically but has a horizontal density gradient the isohalines are vertical. Hence an internal baroclinic circulation is driven by a longitudinal density gradient even if the estuary is vertically well mixed.
Several attempts have been made to analyze density-driven circulation, but none are entirely satisfactory. Hansen and Rattray (1965, 1966) analyzed circulation in a vertical two-dimensional plane, assuming no variation across the channel. They visualized the steady flow illustrated in Fig. 7.11a, and assumed that the only tidal effect was to induce vertical and longitudinal turbulent mixing. Graphs were obtained showing the vertical distribution of velocity and salinity as a function of the river inflow, depth, and width of the channel, and vertical and longitudinal mixing coefficients. Going further, they used data from six real estuaries to relate the effect of the interior mixing to bulk channel parameters and eliminate the need to specify the mixing coefficients. The main

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Figure 7.10 Isohalines in a typical partially stratified estuary.

Figure 7.11 The internal circulation driven by the river discharge in a partially stratified estuary. Currents shown are after averaging over the tidal cycle, and are superimposed on the back-and-forth tidal flow. (a) A vertical section along the deepest part of the channel axis. (b) A transverse section showing the transverse distribution of net currents. [After Fischer (1976a).]
7.2.4 Synthesis and Summary

We have discussed three main causes of mixing and several effects resulting from each cause. In real estuaries motions resulting from all three causes are superposed, although one or two causes may dominate. Moreover, the main cause may change from season to season, or even from week to week. Many estuaries change from partially stratified or salt wedge in the wet season to well mixed in the dry season. A flood of a week or so may stratify a previously well-mixed estuary and the stratification may persist or be slowly eroded away over perhaps the following month. On the other hand, a stratified estuary may be suddenly well mixed by a passing hurricane, or a storm at sea may raise the mean tidal level at the coastline and force in a large quantity of ocean water. All of these seasonal or catastrophic events may be categorized as coming from the wind, tide, or river, but most of them are not adequately described by the simple steady-state analyses reviewed in the previous sections. Therefore, the engineer must beware that even if the main causes of mixing can be analyzed for a particular site and set of conditions, most estuaries are hardly ever in steady state and an analysis that is suitable for one season may not be suitable in another season or to describe the sudden effects of a massive change like a storm or flood.

Nevertheless, it is helpful and sometimes essential to at least conceptualize the different mechanisms. There are only a few ways to make practical engineering studies of estuaries: they include physical modeling, numerical modeling, and one-dimensional analytical modeling. Physical and numerical models inevitably neglect certain mechanisms; it is essential to know what mechanisms are important in the estuary at hand, and which are neglected in the model, to make sure that the two don't match. One-dimensional analytical models generally lump all of the mixing mechanisms into a single longitudinal dispersion coefficient; sometimes the magnitude of the coefficient can be determined from natural tracers, but often it must be estimated, or an estimate must be made of what will happen if natural conditions change. We have given several formulas for estimating the value of the longitudinal dispersion coefficient, but each one has been based on an analysis of one mechanism at the neglect of others. In the following chapter we will discuss numerical and physical models. Throughout the discussion it will be helpful to bear in mind that almost all practical methods now in use are based on recent and possibly incomplete research results, and that what really happens in estuaries may be much more complex than our present ability at description.