**Electrostatic Precipitators**
(Nazaroff & Alvarez-Cohen, pages 447-453 + added material)

ESPs are common installations on coal-fired power plants to remove over 99% of ash particles from million ft$^3$ per minute of fumes. They stand tens of meters tall.

A couple more pictures of electrostatic precipitators

Electrostatic precipitators (ESPs) are major pieces of equipment and are expensive.
Electrostatic precipitators work better than the alternative, the fabric filter baghouse…

… especially when the gas to be treated and its particles are hot or wet.

(Note: Mihelcic & Zimmerman, Figure 12.29)

Typical efficiency of an electrostatic precipitator as a function of the corona power ratio, which is power consumed (in Watts) divided by the airflow in cubic feet per minute (cfm).

Note the extremely high efficiencies, nearing 100%.

(Source: Mihelcic & Zimmerman, Figure 12.29)
Comparison:
CYCLONES versus ELECTROSTATIC PRECIPITATORS

Cyclones and electrostatic precipitators are two different types of equipment, each capable of removing particles from an air stream. When the decision arises regarding which type to adopt in a specific situation, one needs to know the advantages and disadvantages of each type of equipment.

**CYCLONES:**

*Advantages:*
- Low capital cost
  (= relatively cheap to buy and install)
- Ability to operate at high temperatures
- Low maintenance requirements
  (absence of moving parts)

*Disadvantages:*
- Relatively low efficiency
  (especially for the smaller particles)
- Limited to dry particles
  (= not operating well on mist)
- High operating cost
  (= expensive to run, because of pressure loss)

**ELECTROSTATIC PRECIPITATORS:**

*Advantages:*
- Low operating cost
  (except at very high efficiencies)
- Very high efficiency, even for smaller particles
- Ability to handle very large gas flow rates
- Ability to remove dry as well as wet particles
  (= mist OK)
- Temperature flexibility in design

*Disadvantages:*
- High capital cost
  (= expensive to purchase and install)
- Taking a lot of space
- Not flexible once installed
- Failure to operate on particles with high electrical resistivity

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*Inside an ESP*

**Principle**: Electrodes at high voltage create a corona effect (ionized atmosphere) surrounding them. This charges the passing particles. Once charged, particles are subject to a transverse electrostatic force that pulls them toward the collecting plates. Plates are periodically “rapped” (vibrated) to make the collected particles fall down into a receiver basket in the bottom of the apparatus.

Various types of charging electrodes and collecting plates

The baffles along the collecting plates are there to catch better the drifting particles.

Note the indentations and sharp corners on some of the electrodes. These are designed to enhance the corona effect.
**Drift speed**

The particle drift speed ($v_d$) results from a balance between the electrostatic force due to the charge ($F_e$) and the resisting drag force ($F_d$) exerted by the air due to the relative motion between air and particle.

For the drag force, we assume that the particles are very small. (The purpose of an ESP is precisely to catch very small particles). So, we use Stokes’ Law with the Cunningham Slip factor correction (refer to slide in lecture on Transport Phenomena).

\[
F_e = \text{electrostatic force} = \text{charge} \times \text{electric field} = qE
\]

\[
F_d = \text{drag force} = \frac{3\pi\mu_f d_p v_d}{C_c}
\]

\[
F_d = F_e \quad \Rightarrow \quad \frac{3\pi\mu_f d_p v_d}{C_c} = qE \quad \Rightarrow \quad v_d = \frac{C_c qE}{3\pi\mu_f d_p}
\]

where

- $q =$ charge acquired by each particle
- $E =$ electrical field = voltage difference divided by electrode-plate distance $d$
- $C_c =$ Cunningham slip factor (to be obtained from graph or formula)
- $\mu_f =$ fluid viscosity = $1.81 \times 10^{-5}$ kg/m.s for air at ambient temperature
- $d_p =$ particle diameter

The charge $q$ acquired by a particle is a certain number times the charge of the electron, which is $1.6 \times 10^{-19}$ C.

So, for example, if a particle acquires 12 electrons, its charge is:

\[
q = 12 \times 1.6 \times 10^{-19} \, \text{C} = 1.92 \times 10^{-18} \, \text{C}.
\]

The number of electrons acquired depends on the intensity of the corona generated around the electrodes, and this is proportional to the electrical field $E$. Thus, $q$ is proportional to $E$, making the electrical force $qE$ proportional to $E^2$.

It follows that the drift speed $v_d$, too, is proportional to the square of the electrical field. This is a useful amplification of the effect.

A rule to determine the charge acquired by a particle is:

\[
q = \pi d_p^2 \varepsilon_0 \frac{3\varepsilon}{2 + \varepsilon} E_{ch}
\]

where

- $\varepsilon_0 =$ 8.85 $\times 10^{-12}$ C/V.m = permittivity of vacuum
- $\varepsilon =$ 3.7 = dielectric constant for the particle relative to vacuum
- $E_{ch} =$ charging field strength (in V/m), different from collecting field but proportional to it.
Nomenclature:

- $U$ = speed of air flow
- $d$ = electrode-plate distance
- $w$ = half of plate separation distance
- $W$ = plate width (height)
- $L$ = plate length

Approach:

Model air flow as a plug flow reactor (PFR).

Budget for an interval $(x, x+dx)$ along the flow, and between electrode and plate (distance $d$) and for the full width $W$ of the plate:

$$\frac{dC}{dx} = Q_{\text{upstream}} C_{\text{upstream}} - Q_{\text{downstream}} C_{\text{downstream}} - Q_{\text{drift}} C_{\text{drift}}$$

steady state

where each volumetric flowrate $Q$ is the product of a velocity with the respective cross-sectional area:

- $Q_{\text{upstream}} = UWd$
- $Q_{\text{downstream}} = Q_{\text{upstream}} = UWd$
- $Q_{\text{drift}} = wWdx$

The budget then becomes:

$$0 = (UWd)C(x) - (UWd)C(x+dx) - (wWdx)C\left(x + \frac{dx}{2}\right)$$
This budget equation can be rewritten as

\[(Ud) \frac{C(x + dx) - C(x)}{dx} = -w C\left(x + \frac{dx}{2}\right).\]

In the limit of \(dx \to 0\), we obtain the differential equation

\[Ud \frac{dC}{dx} = -w C(x) \implies \frac{dC}{dx} = \frac{-w}{Ud} C.\]

The solution of which is

\[C(x) = C(x = 0) e^{-\frac{w}{Ud} x} .\]

With \(C(x = 0) = C_{in}\) and writing the solution for the end point \(x = L\) where \(C(x = L) = C_{out}\), we obtain

\[C(x = L) = C(x = 0) e^{-\frac{w}{Ud} L} \implies C_{out} = C_{in} e^{-\frac{w}{Ud} L}.\]

**Efficiency**

The efficiency \(\eta\) is defined as the percentage of removal. We find it to be:

\[\eta = \frac{\text{amount removed}}{\text{amount entering}} = \frac{C_{in} - C_{out}}{C_{in}} = 1 - \frac{C_{out}}{C_{in}} = 1 - \exp\left(-\frac{w L}{U d}\right).\]

Since the flow speed \(U\) is the volumetric flow \(Q\) of air divided by the cross-sectional area \(Wd\), we can also write the efficiency as:

\[\eta = 1 - \exp\left(-\frac{w_{e} WL}{Q}\right) = 1 - \exp\left(-\frac{w_{e} A}{Q}\right),\]

where \(A = WL\) is the collecting plate area.
Dividing the total collecting area in a set of plates

One passage between two plates:

\[
\eta = 1 - \exp \left( \frac{w_e A_p}{Q/2} \right) = 1 - \exp \left( \frac{w_e (2A_p)}{Q} \right) = 1 - \exp \left( \frac{w_e A_p}{Q} \right)
\]

\[ A = 2 A_p \]

Two passages between three plates:

\[
\eta = 1 - \exp \left( \frac{w_e A_p}{Q/4} \right) = 1 - \exp \left( \frac{w_e (4A_p)}{Q} \right) = 1 - \exp \left( \frac{w_e A_p}{Q} \right)
\]

\[ A = 4 A_p \]

etc. with more plates.
In all cases, \( A \) in the formula stands for the total plate collecting area.

Effect of fly-ash resistivity on effective drift velocity in an electrostatic precipitator

Particles of high electrical resistivity lose their charge slowly after hitting the collecting plate. This creates an electrical shield on the plates that lowers the ambient electric field. As a result, particles of high electrical resistivity drift more slowly and are harder to collect.

SOURCE: Adapted from White, "Control of Particulates by Electrostatic Precipitation," Handbook of Air Pollution Technology. Copyright © 1984 by John Wiley & Sons, Inc.
Larger particles are removed more efficiently because they acquire a greater electric charge, whereas smaller particles, too, are removed more efficiently because they are subjected to less drag and thus drift more easily, leaving intermediate particles as those that are less efficiently collected. Nonetheless, efficiency easily exceeds 90% for most particles.

An example

*Given situation*

Airflow $Q = 2,000 \text{ m}^3/\text{min}$  
Particle diameter $d_p = 0.5 \mu\text{m}$  
Average particle charge $q = 10$ electron charges  
Electric field $E = 50,000 \text{ V/m}$  
Each plate has dimensions 6 m by 3 m.

*Design requirement*  
Device must achieve an efficiency of 99%.

*Solution*

One-micron particles are quite small. So, we include the correction due to the Cunningham slip factor: 

$$ C_c = 1 + \frac{\lambda}{d_p} \left[ 2.51 + 0.80 \exp \left( -\frac{0.55 d_p}{\lambda} \right) \right] $$

With $\lambda = 0.066 \mu\text{m}$ and $d_p = 0.5 \mu\text{m}$, we get $C_c = 1.333$. 

Figure 7.C.4 Measured collection efficiency as a function of particle size for an electrostatic precipitator installed on a pulverized coal boiler. (Reprinted with permission of the Air & Waste Management Association from J.D. McCain et al. [1975].)
Next, we calculate the electric charge on each particle. It is
\[ q = 10e = 10 \times (1.6 \times 10^{-19} \text{ C}) = 1.6 \times 10^{-18} \text{ C}. \]

The drift speed can now be estimated:
\[
w_d = \frac{qEC_c}{3\pi \mu d_p} = \frac{(1.6 \times 10^{-18} \text{ C})(5 \times 10^4 \text{ V/m})(1.333)}{(3\pi)(1.81 \times 10^{-2} \text{ kg/m.s})(0.5 \times 10^{-6} \text{ m})} = 1.250 \times 10^{-3} \text{ m/s}
\]
(Note how small the drift speed actually is, about 15 ft per hour...)

For an efficiency of 99% (\( \eta = 0.99 \)), we must have
\[
\exp \left( -\frac{Aw_d}{Q} \right) = 1 - 0.99 = 0.01 \quad \Rightarrow \quad \frac{Aw_d}{Q} = 4.61
\]

With \( Q \) given (= 2000 m³/min = 33.33 m³/s) and \( w_d \) already determined, we can deduce the needed collecting area \( A \):
\[
A = \frac{(4.61)(33.33 \text{ m}^3/\text{s})}{(1.250 \times 10^{-3} \text{ m/s})} = 122,783 \text{ m}^2 \quad \text{(just about 23 football fields)}
\]

Since a single plate offers a collecting area of \( 2 \times 6 \times 3 = 36 \text{ m}^2 \) (counting both sides), the required number of plates is
\[
n = \frac{122,783 \text{ m}^2}{36 \text{ m}^2} + 1 = 3,412
\]
(Note: Need to add 1 because each of the two terminal plates offers only a single collecting side.)