TURBULENT BOUNDARY LAYER SEPARATION

OUTLINE

- What we know about Boundary Layers from the physics
- What the physics tell us about separation
- Characteristics of turbulent separation
- Characteristics of turbulent reattachment
TURBULENT BOUNDARY LAYER SEPARATION

TURBULENT vs. LAMINAR BOUNDARY LAYERS

- Greater momentum transport creates a greater $du/dy$ near the wall and therefore greater wall stress for turbulent B.L.s
- Turbulent B.L.s less sensitive to adverse pressure gradients because more momentum is near the wall
- Blunt bodies have lower pressure drag with separated turbulent B.L. vs. separated laminar
TURBULENT Vs. LAMINAR BOUNDARY LAYERS

Kundu fig 9.16

Kundu fig 9.21

laminar separation bubbles: natural ‘trip wire’
BOUNDARY LAYER ASSUMPTIONS

- Simplify the Navier Stokes equations

\[
\frac{\partial \bar{u}}{\partial t} + \nabla \cdot (\bar{u} \bar{V}) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \nu \nabla \cdot \nabla \bar{u} - \left( \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{u}' \bar{v}'}{\partial y} + \frac{\partial \bar{u}' \bar{w}'}{\partial z} \right)
\]

- 2D, steady, fully developed, Re -> infinity, plus assumptions

\[
\frac{v}{u} \ll 1 \\
\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y} \\
\frac{\partial p}{\partial y} \approx 0
\]

- Parabolic - only depend on upstream history
TURBULENT BOUNDARY LAYER SEPARATION

BOUNDARY LAYER ASSUMPTIONS

- the advective and viscous momentum terms have the same order of magnitude in the BL

\[
u \frac{\partial u}{\partial x} \sim \nu \frac{\partial^2 u}{\partial y^2} \rightarrow U_\infty^2/L \sim \nu U_\infty/\delta^2 \rightarrow \delta \sim \sqrt{\nu L/U_\infty} \rightarrow \delta/L \sim 1/\sqrt{Re}
\]

- variations across the BL occur over a much shorter length scale than variations in the streamwise direction

\[
\frac{\partial}{\partial x} \sim 1/L, \quad \frac{\partial}{\partial y} \sim 1/\delta
\]

which, when combined with the continuity equation, yields:

\[
\frac{U_\infty}{L} \sim \frac{v}{\delta} \rightarrow v \sim \delta U_\infty/L = \frac{U_\infty}{\sqrt{Re}}
\]

- The pressure distribution along the BL is the same as at the edge of the boundary layer with the freestream (roughly true in experimental data)

\[
p - p_\infty \sim \rho U_\infty^2
\]

By non-dimensionalizing the Navier-Stokes equations with the above assumptions and taking the limit as \( Re \rightarrow \infty \) we arrive at the Prandtl boundary layer equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \tag{1.2}
\]

\[
0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \rightarrow \frac{\partial p}{\partial x} = \frac{dp}{dx} \tag{1.3}
\]
PHYSICAL PRINCIPLES OF FLOW SEPARATION

- What we can infer from the physics

\[
\frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial y} \left( \nu \frac{\partial \bar{u}}{\partial y} - \bar{u}' \bar{v}' \right)
\]

\[
\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} = \nu \left( \frac{\partial^2 \bar{u}}{\partial y^2} \right)_{\text{wall}}
\]
TURBULENT BOUNDARY LAYER SEPARATION

BOUNDARY LAYER ASSUMPTIONS

- Eliminate the pressure gradient

\[ \frac{\bar{u}}{u} \frac{\partial \bar{u}}{\partial x} + \frac{\bar{v}}{v} \frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \frac{\nu}{\partial y} \left( \frac{\partial \bar{u}}{\partial y} - \bar{u}' \bar{v}' \right) \right) \]

\[ \frac{\bar{u}}{u} \frac{\partial \bar{u}}{\partial x} + \frac{\bar{v}}{v} \frac{\partial \bar{u}}{\partial y} = \bar{U}_e \frac{\partial \bar{U}_e}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} (\tau_v + \tau_t) \]

\[ \tau_v = \rho \nu \frac{\partial \bar{u}}{\partial y}, \quad \tau_t = -\rho \bar{u}' \bar{v}' \]
**Turbulent Boundary Layer Separation**

**Where the Boundary Layer Equations Get Us**

\[
\frac{u}{U_e} \frac{\partial \bar{u}}{\partial x} + \frac{v}{U_e} \frac{\partial \bar{u}}{\partial y} = U_e \frac{\partial \bar{U}_e}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} (\tau_v + \tau_t)
\]

- Boundary conditions necessary to solve:
  - Inlet velocity profile \( u_0(y) \)
  - \( u=v=0 @ y = 0 \)
  - \( U_e(x) \) or \( P_e(x) \)
  - Turbulent addition: relationship for turbulent stress
- Note: BL equ.s break down at leading edge where \( \delta = 0 \)
TURBULENT BOUNDARY LAYER SEPARATION

CANONICAL SOLUTIONS TO THE B.L. EQUATIONS

- **Similarity Solutions**: built on \[ \frac{u}{U_e} = f'(\eta), \quad \eta = \frac{y}{\delta} \]

- **Blasius**: assumes \( \frac{dp}{dx} = 0 \), i.e. \( U_e(x) = \) constant, laminar

- **Falkner-Skan**: extension of Blasius to \( U_e(x) = ax^n \)
  - \( n = -0.0904 \) \( \rightarrow \) \( \tau_w = 0 \) \( \rightarrow \) separation imminent
  - given \( U_e(x) \) can find \( x_{sep} \)
  - \( n < -0.0904 \) not realistic because equations assume \( u \sim U_e \) and after separation \( u = 0 \) somewhere in the flow
  - also laminar
CANONICAL SOLUTIONS TO THE B.L. EQUATIONS

- von Karman Integral Equations & Thwaites’ method
  - mean values across the flow
  - valid for laminar and time-averaged turbulent flows
  - similarly, only tells us when to abandon the boundary layer assumptions, i.e. when separation is likely
SIMILARITY (EQUILIBRIUM) SOLUTIONS - VELOCITY PROFILES

- Turbulent similarity (equilibrium) solutions
  - Law of the wall - Prandtl 1925
    - $u \sim y$ linear profile for $y/\delta < 0.1$ (viscous sublayer, laminar)
    - valid for all turbulent flows with finite wall shear stress
  - Log law - von Karman 1930
    - $u \sim \ln(y)$ logarithmic profile
  - Wake law - Coles 1956

$$\frac{u}{u_r} = f\left(\frac{yu_r}{v}\right) + \frac{\Pi}{\kappa} w\left(\frac{y}{\delta}\right)$$

Fig. 7.29. The velocity-defect law. Symbols, experimental data of Klebanoff (1954); dashed line, log law; solid line, sum of log law and wake contribution $\Pi w(y/\delta)/\kappa$. 
Use a velocity defect law: in place of \( u/U_e = f'(\eta) \) we have that \( (U_e - u) / u_T = f'(\eta) \), where \( u_T = (\tau_w/\rho)^{(1/2)} \)

- yields logarithmic profile as well

Replace \( u_T \) with a different characteristic velocity \( u_0 \)

- \( (U_e - u) / u_0 = f'(\eta) \)

- yields logarithmic and power law profiles (by applying matching condition with linear viscous profile)
TURBULENT BOUNDARY LAYER SEPARATION

SEPARATION CRITERIA

- **MRS Criterion** - Moore ‘58, Rott ‘55, Sears ‘56
  - occurs where \( u \) and \( \frac{du}{dy} = 0 \) (not necessarily at wall)
  - unsteady *laminar*

- **Stratford Criterion** - gives \( x_{\text{sep}} \)
  - laminar and *turbulent* cases
  - applies to flows where a \( U_{\text{max}} \) occurs before separation
  - must know \( U(x) \), \( x_{\text{max}} \), \( \theta_{\text{max}} \), and \( U_{\text{max}} \)

Schlichting and Erster, 2000
TURBULENT BOUNDARY LAYER SEPARATION

WHERE THE BOUNDARY LAYER EQUATIONS GET US

- To the point of separation
- Downstream of BL separation is a new story
- How do we solve for the flow parameters?
- Must drop all prior assumptions
**Figure 1** (a) Traditional view of turbulent boundary-layer separation with the mean backflow coming from far downstream. The dashed line indicates $U = 0$ locations. (b) A flow model with the turbulent structures supplying the small mean backflow. ID, *incipient detachment*; ITD, *intermittent transitory detachment*; D, *detachment*. The dashed line denotes $U = 0$ locations.
EXPERIMENTAL OBSERVATIONS

- What is happening with Reynolds stresses?
- Coherent structures?
- What do the velocity profiles look like?
- How does the flow behave in a separation bubble?
- Reattachment?
**Cuvier et al., 2014**  *Journal of Turbulence*

Figure 1. Schematic view of the ramp.

**Na and Moin, 1998**

*DNS of a separated turbulent boundary layer*

Figure 16. Mean streamlines.

Figure 17. Mean streamwise PIV velocity field $U_{vel}$ normalised with $U_{\infty}$ on the flap in the wind tunnel reference frame.

Figure 1. Computational domain of separated turbulent boundary layer.
FIGURE 17. Skin-friction and mean wall-pressure coefficients based on $U_0$. 

Na and Moin DNS

Y. Na and P. Moin
TURBULENT BOUNDARY LAYER SEPARATION

COMPUTATIONAL OBSERVATIONS

Figure 14. Contours of \( \langle y_u \rangle_{\Delta T} \) (fraction of time that the flow moves downstream for the time period of \( 300\delta_{in}^*/U_0 \)) in an \((x,z)\)-plane very near the wall \((y/\delta_{in}^* = 0.0042)\).
Na and Moin DNS

We're going streaking!!!

Figure 6. Instantaneous skin friction coefficient contours: ————, positive; ————, negative; thick solid lines correspond to $C_f = 0$. (a) $tU_0/\delta^*_in = 4415$; (b) $tU_0/\delta^*_in = 5053$. 
RECALL: TKE BUDGET

p. 769 have described. For steady flows with constant physical properties it reads:

\[
\frac{\partial}{\partial x} \left( \bar{u} \frac{\partial k}{\partial x} + \bar{v} \frac{\partial k}{\partial y} + \bar{w} \frac{\partial k}{\partial z} \right) \quad \text{\{convection\}}
\]

\[
- \frac{\partial}{\partial x} \left[ u' \left( p' + \frac{\rho}{2} q'^2 \right) \right] - \frac{\partial}{\partial y} \left[ v' \left( p' + \frac{\rho}{2} q'^2 \right) \right] - \frac{\partial}{\partial z} \left[ w' \left( p' + \frac{\rho}{2} q'^2 \right) \right] \quad \text{\{turbulent diffusion\}}
\]

\[
+ \mu \left[ \frac{\partial^2}{\partial x^2} (k + \bar{u}^2) + \frac{\partial^2}{\partial y^2} (k + \bar{v}^2) + \frac{\partial^2}{\partial z^2} (k + \bar{w}^2) \right]
+ 2 \left( \frac{\partial^2 u' v'}{\partial x \partial y} + \frac{\partial^2 v' w'}{\partial y \partial z} + \frac{\partial^2 w' u'}{\partial z \partial x} \right) \quad \text{\{viscous diffusion\}}
\]

\[
- \frac{\partial}{\partial x} \left( \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{u} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} \right) + \frac{u' w'}{\partial y} \frac{\partial \bar{u}}{\partial y} + \frac{w' u'}{\partial y} \frac{\partial \bar{w}}{\partial y} + \frac{w' u'}{\partial z} \frac{\partial \bar{w}}{\partial z}
+ \frac{u' w'}{\partial z} \frac{\partial \bar{u}}{\partial z} + \frac{u' w'}{\partial z} \frac{\partial \bar{w}}{\partial z} \quad \text{\{Reynolds stresses\}}
\]

\[-\varrho \tilde{\varepsilon} \quad \text{\{dissipation.\}}\]

The dissipation (cf. Eq. (3.62)) is

\[
\varrho \tilde{\varepsilon} = \mu \left[ 2 \left( \frac{\partial u'}{\partial x} \right)^2 + 2 \left( \frac{\partial u'}{\partial y} \right)^2 + 2 \left( \frac{\partial u'}{\partial z} \right)^2 \right]
+ \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 \right].
\]

\text{Normal viscous stresses}

\text{Shear viscous stresses}
TURBULENT BOUNDARY LAYER SEPARATION

TURBULENT PRODUCTION OF REYNOLDS SHEAR STRESS

Figure 29. Reynolds shear stress field $\overline{uv} = u'v'$ normalised with $U_\infty^2$ on the flap in the local wall reference frame.

Figure 26. Wall-normal turbulence intensity field $v = \sqrt{v'^2}$ normalised with $U_\infty$ on the flap in the local wall reference frame.

Figure 30. Production term $\overline{v'^2 \frac{\partial u'}{\partial y}}$ of $-\overline{u'v'}$ on the flap normalised with $U_\infty^3$ in the local wall reference frame.

Cuvier et al., 2014
TURBULENT BOUNDARY LAYER SEPARATION

REYNOLDS SHEAR STRESS: DNS AND PIV

Figure 22. Contours of turbulence intensities: (a) longitudinal component \((\overline{u^2})^{1/2}/U_0\); (b) wall-normal component \((\overline{v^2})^{1/2}/U_0\); (c) spanwise component \((\overline{w^2})^{1/2}/U_0\); (d) Reynolds shear stress \((-\overline{u'v'}/U_0^2)\).

Figure 29. Reynolds shear stress field \(u'v' = \overline{u'v'}\) normalised with \(U_0^2\) on the flap in the local wall reference frame.
TURBULENCE INTENSITY

TKE production is dominated by the streamwise component

Na and Moin DNS

Cuvier et al., 2014

Figure 22. Turbulence intensity field $u = \sqrt{u'^2}$ normalised with $U_\infty$ on the flap in the local wall reference frame.
COHERENT STRUCTURES AKA LARGE SCALE VORTICES

“We suggest that the outer maximum peaks observed in the Reynolds stress profiles and in the turbulence intensities of APG TBLs are related to the outer streaky structures. “ Lee and Sung 2009

Figure 14. Outer vortical structures in an APG boundary layer ($\beta = 1.68$): (a) perspective view; (b) top view. The vortices are shown as grey isosurface plots of swirling strength (11% of maximum $\lambda_{ci}$). The black and white areas are low- and high-speed regions (10% below and above local free-stream velocity) respectively.

Figure 22. Vortical structures of the linearly estimated flow field for the Q2 event vector, $u_2 = (u_m, v_m, 0)$, at $y/\delta \approx 0.45$: (a) isosurface of 60% of maximum $\lambda_{ci}$; (b) isosurface of $\lambda_{ci} \delta/U_{\infty} = 0.25$. 
“Although several types of structures are present in the outer layers of TBLs, it appears that elongated LMRs and hairpin packets are the dominant outer layer structures associated with turbulence production (Ganapathisubramani et al. 2003) “ from Lee and Sung 2009

LMR = low momentum region
TURBULENT BOUNDARY LAYER SEPARATION

COMPUTATIONAL OBSERVATIONS: VORTICITY

Na and Moin DNS approach detachment center of bubble downstream of reattachment

**Figure 10.** Contours of instantaneous streamwise vorticity: ---, positive; ----, negative. (a) $x/\delta_{in} = 120$; (b) $x/\delta_{in} = 160$; (c) $x/\delta_{in} = 220$; (d) $x/\delta_{in} = 320$. Contours: from $-0.5$ to $0.46$ ($U_0/\delta_{in}^*$) with increments of 0.08 ($U_0/\delta_{in}^*$).
COHERENT STRUCTURES aka LARGE SCALE VORTICES

Similarity with cylinder?

(a) Na and Moin DNS

(b) 

(c) 

(d) 

Figure 12. Contours of spanwise-averaged pressure fluctuations in an (x, y)-plane: ————, positive; ————, negative. (a) $tU_0/\delta_{in}^* = 3238$; (b) $tU_0/\delta_{in}^* = 3350$; (c) $tU_0/\delta_{in}^* = 3463$; (d) $tU_0/\delta_{in}^* = 3575$. Contours: $p'/\rho U_0^2$ are from $-0.013$ to $0.012$ with increments of $0.0014$. 
“The flow in the detached shear layer appears to be qualitatively similar to a plane mixing layer”

Na and Moin p.390

**Kinetic energy budget for a mixing layer**

(Pope p.431)

![Kinetic energy budget](image)

**Figure 4.** Longitudinal Reynolds-stress budget normalized by $U_0^3/\delta^*_m$ ($\times10^2$). (a) $x/\delta^*_m = 220$; production; dissipation; viscous diffusion; convection; turbulent transport; velocity pressure gradient; balance of terms.
TURBULENT BOUNDARY LAYER SEPARATION

VELOCITY PROFILES: BEFORE SEPARATION

Na and Moin DNS

Figure 20. Mean streamwise velocity profiles in wall coordinates before detachment.

Cuvier et al., 2014

Figure 10. Mean streamwise velocity profiles at the five hot-wire stations in wall units, $\alpha = -2^\circ$, $\beta = -22^\circ$ and $U_\infty = 10$ m/s.
Figure 5. Instantaneous velocity vectors at $z/\delta^*_\infty = 25$ in the $(x,y)$-plane. (a) $tU_0/\delta^*_\infty = 3350$; (b) $tU_0/\delta^*_\infty = 4415$. 

Na and Moin DNS
TURBULENT BOUNDARY LAYER SEPARATION

VELOCITY PROFILES: AFTER REATTACHMENT

Figure 21. Mean streamwise velocity profiles in wall coordinates after reattachment.

Figure 38. Mean streamwise velocity profiles after the reattachment point in wall units and compared to the log-law and FP case at 5 m/s.
REATTACHMENT - THE COANDA EFFECT

- Separation bubbles
- Entrainment
- Flying saucers

Coanda’s “Lenticular Aerodyne” is a perfect disc, achieving lift by creation of a vacuum around the edge of the wing. Bubbles canopy in the center houses the passenger compartment.

Tritton 1988 Physical Fluid Dynamics, laesieworks.com/ifo/lib/Henri_Coanda.html
CONCLUSIONS: MORE QUESTIONS THAN ANSWERS

- Adverse pressure gradient creates greater normal turbulent stress, leading to increased shear turbulent stress downstream of separation.
- Increased turbulent stresses lead to increased production of large scale vortices - detached BL produces eddies on scale of bubble height - relation to hairpin vortices?
- Separated boundary layers similar to shear layers?
- 3D effects important
- Periodic vortex shedding from bubble
- What can we capture with a vortex lattice method?