

## Chapter 16

# Lakes and Reservoirs

WARNING: This chapter is still under construction.

### 16.1 Definition

Lakes are natural bodies of water, where flow from one or several rivers is impounded by a natural obstacle. A lake differs from the incoming rivers simply by the fact that its flow is far weaker and no longer driven by gravity down a slope. Instead, currents in lakes are driven by surface winds and buoyancy forces.

A reservoir is an artificial lake created by a dam blocking a river. There are several reasons why people build reservoirs: Generation of hydro-electric power, flood control, freshwater supply (for households or irrigation), recreation, and control of water quality. But, there are also disadvantages: Elimination of dry land and obstacle to fish swimming upstream for spawning, among others.

Figure 16.1 shows the location and size of the Wellington Reservoir in Western Australia, which we are taking as a study example. This reservoir covers an approximate area of  $10 \text{ km}^2$  and has an average depth of 30 m. It is fed by several rivers totalling to an inflow rate of  $5 \text{ m}^3/\text{s}$  (annually averaged over significant seasonal variations).

### 16.2 Physical Processes

Lakes and reservoirs differ from rivers by their greater depths and their weaker velocities. They therefore impound water for quite some time, and an important characteristic of a lake is its *residency time*, sometimes also called *retention time*. It is defined as the average time spent by a water parcel from time of

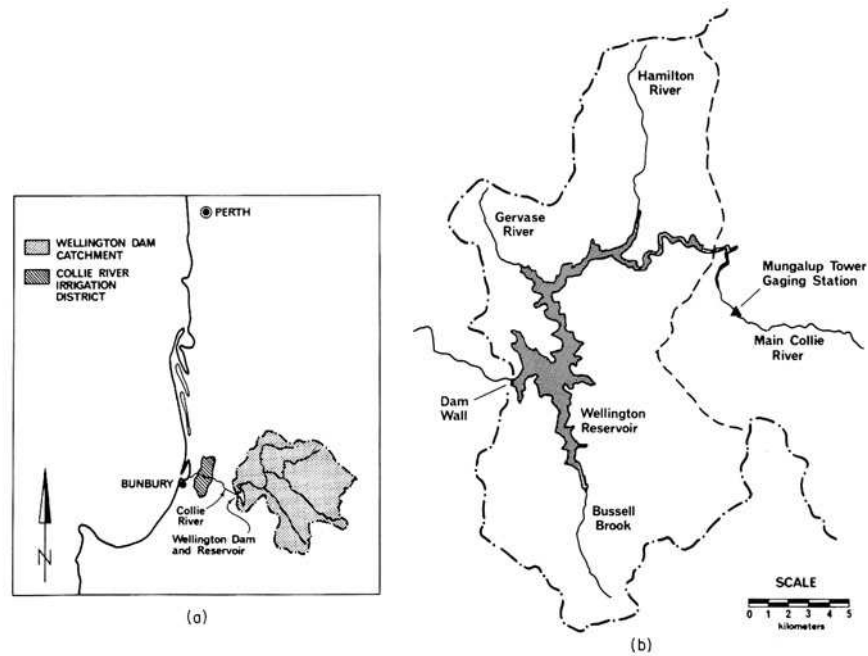


Figure 16.1: Location and geography of Wellington Reservoir in Western Australia: (a) Location on Australia's west coast, (b) Shape of reservoir and boundary of catchment basin. (From Fischer, List, Koh, Imberger & Brooks, 1979)

inflow to that of outflow. The longer a fluid parcel remains in the lake or reservoir, the more likely it is to be subjected to local processes such as heating or cooling, sedimentation, biological or chemical transformations, etc.

If the volume of the reservoir is  $V$  and the input flow rate (defined as the average volumetric inflow rate) is  $Q$ , the residency time is given by

$$T = \frac{V}{Q}. \quad (16.1)$$

For Wellington Reservoir, we have  $V = \text{area} \times \text{depth} = 30 \text{ m} \times 10^7 \text{ m}^2 = 3 \cdot 10^8 \text{ m}^3$ , and  $Q = 5 \text{ m}^3/\text{s}$ , giving  $T = (3 \cdot 10^8 \text{ m}^3)/(5 \text{ m}^3/\text{s}) = 6 \cdot 10^7 \text{ s} = 694$  days, or about 2 years.

When the residency time surpasses one year, as in the preceding example, we expect that seasonal variations are important and that the thermal variations under summer heating and winter cooling may control the lake's dynamics, especially its level of turbulence and hence capacity to mix and disperse pollutants. In addition to the seasonal cycle, lakes and reservoirs are also often subject to diurnal variations.

*Annual variability*

Consider again Wellington Reservoir. Its major seasonal variations are depicted in Figure 16.2 for the period June 1975 to July 1976, which comprises a little more than one full year. There are many things to note here. So, let us take one thing at a time.

In the top panel of Figure 16.2, we note the seasonal variation of the solar short-wave radiation,  $H_{SW}$ , with its summer peak occurring in December–January (Southern Hemisphere!). Accompanying this, but with a delay of a couple of months, is the late summer peak of the temperature of the river inflow. The summer is also the period of very low inflow (with flowrate indistinguishable from zero on the graph). In contrast, the river inflow is largest in winter and early spring (months of July to September), during which period of time it arrives in several big bursts, due to snow melt and episodes of heavy rain. The salt concentration (NaCl) of the inflow exhibits a few simultaneous peaks, showing that large river discharges are accompanied by channel erosion and increased salt.

The middle panel of Figure 16.2 shows two things, the water level (top line) and temperature of the water column. The water level obviously rises in the wake of bursts in the river inflow. The more interesting information here is the way the temperature is behaving over the seasonal cycle. In winter, the temperature is homogeneous in the vertical, due to convection by surface cooling. The temperature also drops from 15°C to 13°C as the winter progresses. The homogenization of the water column is due not only to surface heat loss, making colder and denser water that sinks to the bottom, but also to cold river inflow. That convection occurs and overturns the water is evident from the pattern of dissolved oxygen (lower panel). It reaches full saturation all the way down to the bottom during winter.

The 13°C temperature is about as cold as the water becomes. Through spring and summer, the solar heating warms the surface, generating buoyant waters that float on top and create a vertical thermal stratification. By November (see the temperature section in the top panel of Figure 16.3), the surface temperature has reached 23–24°C, and the stratification below the surface is very pronounced. By February, the middle of the summer, the surface temperature has reached 26°C, while the bottom waters have remained at about 13°C, but there has been some degree of vertical mixing (see the lower panel of Figure 16.3).

When vertical stratification exists, it acts as a shield depriving the lower waters from aeration by contact with the air at the surface. This is manifested by a gradual decrease in dissolved oxygen by biological consumption over the course of spring, summer and early fall, from 100% (saturation) to less than 20% by the next June.

Strong winds, as indicated by open dots on top of Figure 16.2, occur during late summer (February–March) and early winter (May–June), creating some stirring and partial homogenization of the water column. This explains the relatively uniform temperature of about 24°C in February (bottom panel of

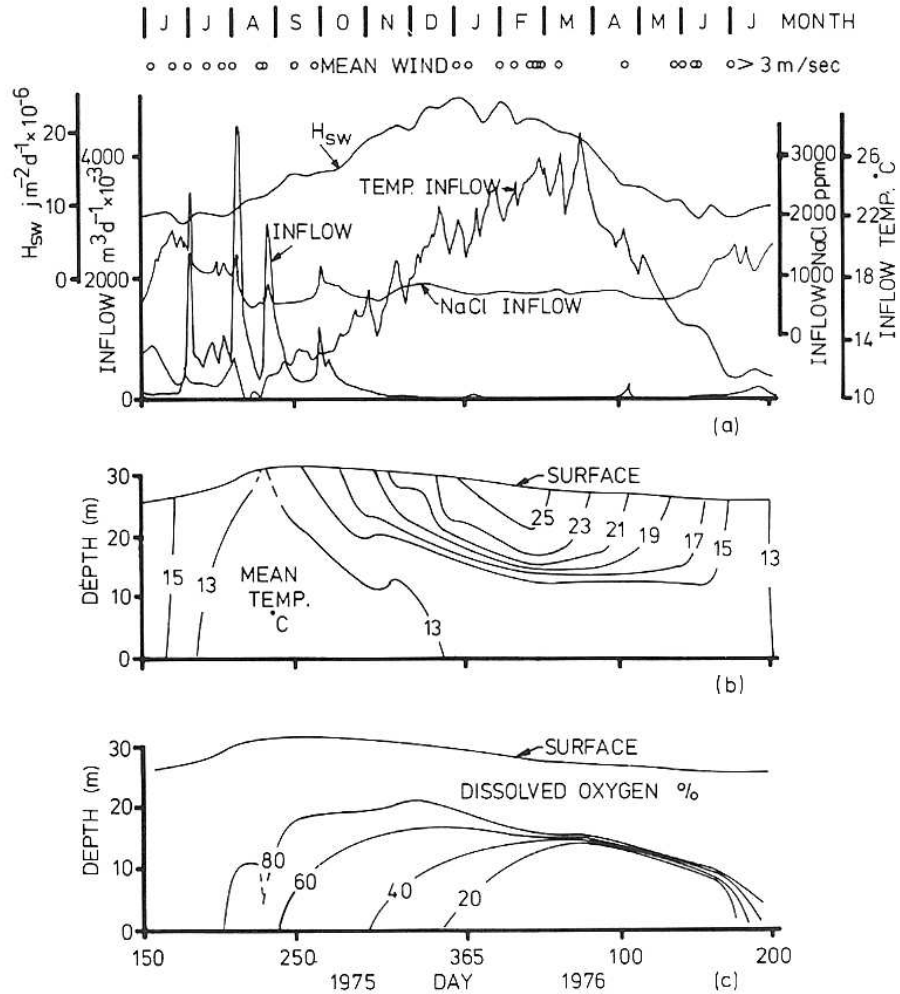


Figure 16.2: Seasonal variations at one point inside Wellington Reservoir: (a) Inflow characteristics and solar radiation, (b) Water level and temperature (in °C), and (c) Dissolved oxygen as a percentage of saturation. (From Fischer, List, Koh, Imberger & Brooks, 1979)

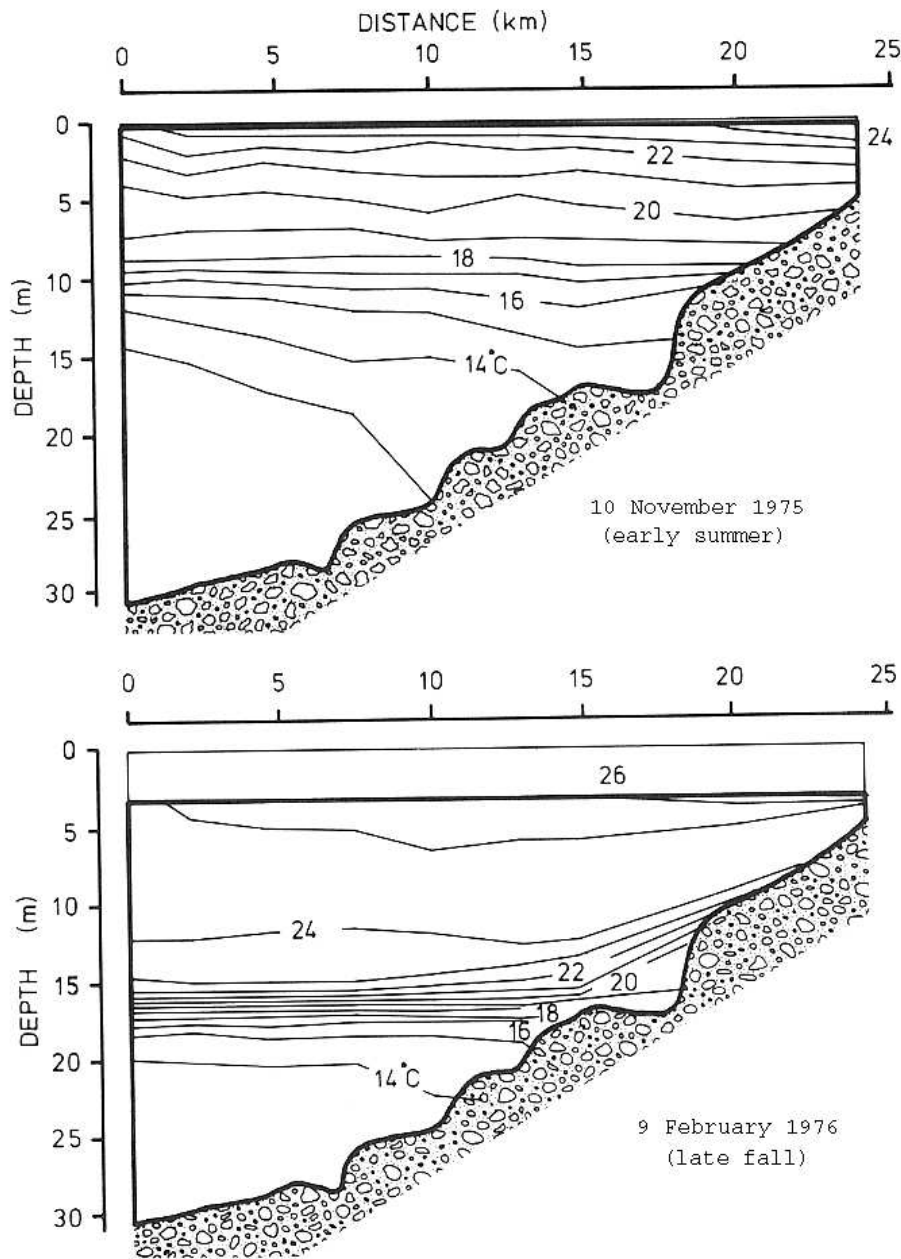


Figure 16.3: Seasonal variation of vertical stratification inside Wellington Reservoir: (a) Early summer, with thermal stratification quite pronounced especially near the surface, and (b) late summer, with erosion of the stratification by wind mixing. Contours mark depths of isothermals (in °C). (From Fischer, List, Koh, Imberger & Brooks, 1979)

Figure 16.3) over the top 15 meters.

The seasonal cycle concludes by late fall and winter cooling, generating convection that erases the stratification over a gradually greater depth. This is manifested by the sharp angles in the isotherms of Figure 16.2 (middle panel): Each isotherm remains nearly horizontal as part of stratification until it is abruptly turning vertical at the time when convection reaches its level. This gradual progression of convection induced by surface cooling is called *penetrative convection*. Penetrative convection lasts until either the bottom is reached or the winter turns into spring, whichever occurs first. In the case of Wellington Reservoir, penetrative convection ends in July when the bottom is reached and the temperature has been homogenized over the entire depth. Note, however, that this is not necessarily the end of convection, only the end of its penetrative phase. Further cooling of the entire water column can still occur, as it did from June to July 1975, when the water cooled from 15°C to 13°C.

### *Stratification*

The preceding considerations point to the existence of thermal stratification over the seasonal cycle. When it occurs, stratification is characterized by less dense, warmer water floating on top of denser, colder water. This creates buoyancy forces, which are an obstacle to turbulence and dispersion of contaminants. What the water contains by way of contaminants tends to remain at the level where it is instead of being spread across the vertical. Because of this, it is important to have a measure of stratification.

Stratification arises because of thermal expansion: In getting warmer, water expands, holding therefore less mass in the same volume. In other words, density falls as temperature rises. Density differences in turn create buoyancy forces. Since temperature and density variations remain relatively modest in nature, a linear relationship is adequate. We write:

$$\rho = \rho_o [1 - \alpha(T - T_o)], \quad (16.2)$$

where  $\rho_o$  is the density at the reference temperature  $T_o$ , and the factor  $\alpha$  is called the coefficient of thermal expansion. For freshwater in the range of 10–20°C, its value is  $\alpha = 2.57 \cdot 10^{-4} (\text{°C})^{-1}$ . For the reference values, it is convenient to take  $T_o = 15\text{°C}$  for which  $\rho_o$  is 999 kg/m<sup>3</sup>.

The preceding linear relationship breaks down for water at low and near freezing temperatures, because water density reaches a maximum at 4°C and decreases below that. When ice forms, its density is even lower, explaining why ice floats on the surface.

Temperature and density vary vertically due to stratification but also under motions in the water such as currents, waves, and turbulence. Because not all variations are equally strong, we write:

$$\rho = \rho_o + \rho_e(z) + \rho'(x, y, z, t), \quad (16.3)$$

where  $\rho$  is the local and instantaneous density,  $\rho_o$  is the reference density introduced above,  $\rho_e$  the variation due to stratification only (a function of the

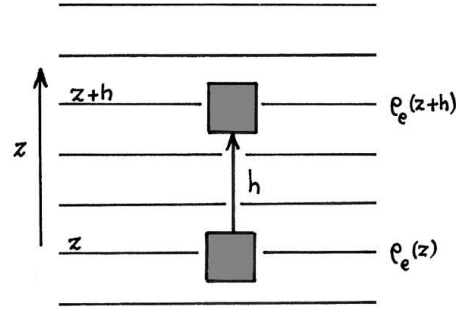


Figure 16.4: Displacement of a fluid parcel in a stratified fluid and the alteration of forces acting upon it.

vertical coordinate  $z$  only), and  $\rho'$  the remaining part caused by motion of the water. Thus,  $\rho_o + \rho_e$  is the density in the absence of motion. Typically,  $\rho'$  is much less than  $\rho_e$ , which itself is much less than  $\rho_o$ . (Water, warmer or colder, is still water and its density does not wander far off, especially in natural settings.)

Consider for a moment a fluid at rest [ $\rho'(x, y, z, t) = 0$ ] and, in it, a small fluid parcel of volume  $V$  at level  $z$  and of density  $\rho_o + \rho_e(z)$ . Imagine next that this parcel is being displaced to a slightly higher level  $z+h$  without modification of its density, as depicted on Figure 16.4.

At the new location, the vertical forces acting on the parcel are its own weight  $mg = [\rho_o + \rho_e(z)]Vg$  and a set of pressure forces, which together create a buoyancy force equal to the weight of the water displaced, namely  $[\rho_o + \rho_e(z+h)]Vg$ . The net upward force is the buoyancy force minus the weight:  $[\rho_e(z+h) - \rho_e(z)]Vg$ . This non-zero force causes a vertical acceleration of the parcel. As the parcel moves in the vertical, its displacement  $h$  becomes a function of time, with  $dh/dt$  its vertical velocity and  $d^2h/dt^2$  its vertical acceleration. Newton's second law (mass times acceleration = sum of forces) requires:

$$[\rho_o + \rho_e(z)]V \frac{d^2h}{dt^2} = [\rho_e(z+h) - \rho_e(z)]Vg.$$

Since  $h$  is small, the difference of densities in the right-hand side can be approximated with the derivative. This and a division by  $V$  yield:

$$[\rho_o + \rho_e(z)] \frac{d^2h}{dt^2} = \frac{d\rho_e}{dz} gh.$$

And, since  $\rho_e$  is much smaller than the reference density  $\rho_o$ , only the first term in the brackets need to be retained on the left. The result is a differential equation prescribing the evolution of vertical displacement  $h(t)$  of the parcel in the vertical:

$$\frac{d^2h}{dt} + N^2 h = 0, \quad (16.4)$$

where the coefficient  $N^2$  has been defined as

$$N^2 = - \frac{g}{\rho_o} \frac{d\rho_e}{dz} . \quad (16.5)$$

If  $N^2 > 0$ , the preceding equation is that of an oscillator:  $h$  varies sinusoidally over time [as a linear combination of  $\sin(Nt)$  and  $\cos(Nt)$ ], meaning that the parcel bounces up and down around its position of equilibrium ( $h = 0$ ). Physically, if  $N^2$  is positive, the density  $\rho_e$  decreases with height and when the parcel is displaced upward, it encounters lighter fluid than itself, in which it sinks. Falling downward, the parcel acquires speed and reaches its position of equilibrium, which it overshoots. Below its position of equilibrium, the parcel is now surrounded by heavier fluid and experiences an upward buoyancy force, which makes it bounce back. The oscillation would continue indefinitely if it were not for some eventual damping by friction. We can call this a *stable* situation. The square root  $N$ , which exists when  $N^2$  is positive and which has the dimension of 1/time, is the frequency of the oscillation and is therefore called the *stratification frequency*.

On the contrary, if  $N^2 < 0$ , Equation (17.4) has an exponentially growing solution [proportional to  $\exp(Nt)$ ]. Physically, the parcel encounters upward of its position of equilibrium heavier fluid than itself, in which a buoyancy force makes it rise even further. Likewise, if the parcel had originally been displaced downward, the parcel would encounter lighter fluid in which it would sink further. Either way, an unlimited displacement ensues. We can call this an *unstable* situation.

The conclusion of this thought exercise is that the quantity  $N^2$  determines the stability of the water column: When the lighter water floats on top,  $N^2$  is positive and the situation is stable, but when the water column is top-heavy,  $N^2$  is negative and the situation is unstable. The limit between these two situations is the case when  $N^2 = 0$ , which corresponds to the absence of stratification ( $d\rho_e/dz = 0$ ) and a neutral water column. By extension, we may consider the value of  $N^2$ , when it is positive, as the strength the stratification: The larger  $N^2$ , the stronger the stratification.

In lakes the stratification is almost always caused by temperature variations. Recalling Equation (17.2) that relates density to temperature, we can express the strength of the stratification in terms of the vertical temperature gradient:

$$N^2 = + \alpha g \frac{dT}{dz} . \quad (16.6)$$

Note that, in a lake, the stratification may depend on the position in the vertical. For example, Wellington Reservoir in November (Figure 16.3 top panel) exhibits a stratification in the top 20 or so meters and a neutral water column below, while in February (Figure 16.3 bottom panel) its water column



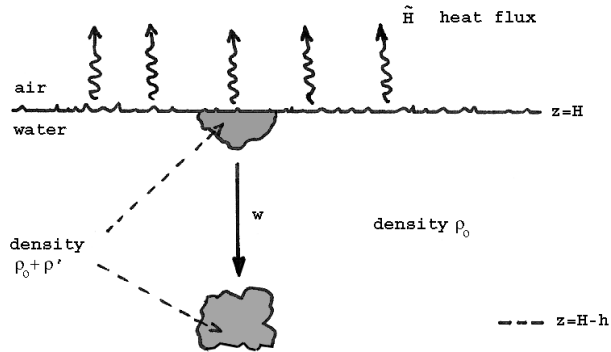


Figure 16.5: Sinking of a water parcel that has been cooled in the proximity to the surface.

consists in three layers, with a stratification from 15 to 20 meters and no stratification on either side.

#### *The convective process*

As summer alternates with winter, the water of a lake or reservoir is alternatively heated and cooled (see Figure 16.2 for data on Wellington Reservoir). To understand the response of the water column, one needs to start somewhere and trace the situation from there. Let us start in late fall when cooling begins to erode the stratification created during the preceding summer.

The scenario is as follows: Cooling at the surface creates water parcels that are slightly colder and hence denser than the water below. An unstable situation exists just below the surface, and the cold fluid sinks (Figure 16.5), to be replaced by a new parcel of slightly warmer fluid, which will be cooled in its turn, and the process goes on. This is convection.

As parcels of colder water sink through the existing stratification, they encounter increasingly denser water and eventually become buoyant. This arrests their vertical migration. Fluid theory tells that a fluid parcel with a density surplus  $\rho'$  falling over a depth  $h$  acquires during its fall a vertical velocity  $w$  given approximately by:

$$w = -\sqrt{gh \frac{\rho'}{\rho_0}}.$$

In brief, this relation is obtained from invoking a transformation of potential energy into kinetic energy as the parcel falls and accelerates. The negative sign of  $w$  indicates a downward velocity. The density surplus  $\rho'$  that causes the acceleration is related to a temperature deficit  $T'$  of the falling parcels by  $\rho' = -\rho_0 \alpha T'$ , making  $w$  be related to the temperature deficit as:

$$w = -\sqrt{\alpha gh|T'|}. \quad (16.7)$$

The heat content of a water parcel is its mass  $m$  times the heat capacity  $C_v$  of water ( $C_v = 4186 \text{ J/kg.K}$ ) times the temperature  $T$ . Since in first approximation the density of water is  $\rho_o$ , the mass of the parcel is its density  $\rho_o$  times its volume  $V$ , and we can write:

$$\text{Heat content} = \rho_o V C_v T.$$

The *heat content* is measured in Joules (J). An associated but different quantity is the *heat flux*, which is the amount of heat removed from the water per square meter and per second (in  $\text{J/m}^2.\text{s} = \text{W/s}$ ). Let us denote by  $\tilde{H}$  the surface atmospheric cooling heat flux to which the lake water is subjected. In the water, this flux is carried in the form of parcels falling with vertical velocity  $w$  and temperature deficit  $T'$ . The relation between heat content and heat flux is as follows:

$$\begin{aligned} \text{Heat flux} &= \frac{\text{change in heat content}}{\text{horizontal surface area} \times \text{time}} \\ &= \frac{\text{change in heat content} \times \text{vertical extent}}{\text{volume} \times \text{time}} \\ &= \frac{\text{change in heat content}}{\text{volume}} \times \text{vertical velocity} \\ \tilde{H} &= \frac{\rho_o V C_v |T'|}{V} |w| \\ &= \rho_o C_v |w T'|. \end{aligned}$$

Given relation (16.7) between the falling velocity and the temperature deficit, we can solve for each separately in terms of the surface heat flux:

$$w = - \left( \frac{\alpha gh \tilde{H}}{\rho_o C_v} \right)^{\frac{1}{3}} \quad (16.8)$$

$$T' = - \left( \frac{\tilde{H}^2}{\rho_o^2 C_v^2 \alpha gh} \right)^{\frac{1}{3}}. \quad (16.9)$$

Thus, we can estimate the properties of the turbulence created by convection. The vertical diffusivity generated by this turbulence is, according to (3.16), the product of the turbulent velocity, which we take here as  $u_* = |w|$ , and the size of the largest turbulent excursions, which we take here as  $d_{\max} = h$ . A practical numerical coefficient is 0.1. Thus, we have

$$D = 0.1 |w| h = 0.1 \left( \frac{\alpha gh^4 \tilde{H}}{\rho_o C_v} \right)^{\frac{1}{3}}. \quad (16.10)$$

It now remains to determine the vertical extent  $h$  of the convection.

*Penetrative convection*

As stated earlier, heat loss at the surface creates cold water parcels that sink. As they fall, these parcels stir the water column and erase the stratification along the way, from the surface to depth  $h$  of their fall. The region where sinking occurs becomes vertically mixed. Sustained surface cooling during the winter season produces ever colder water parcel, which sink ever further. As time goes on, the mixed zone below the surface becomes deeper and colder. This type of gradually deepening convection is called *penetrative convection*. The process continues until the bottom is reached, or when cooling stops at the surface, whichever occurs first.

To study penetrative convection, let us place ourselves at some time  $t$  after the start of the convection period. At that moment, convection has already reached a depth  $h$ , as depicted in Figure 16.6. The temperature  $T$  in the convecting layer is vertically homogenized, after averaging over the temperature deficit of the falling parcels. But, this temperature typically differs from that of the unperturbed stratification immediately below, and there exists a temperature jump  $\Delta T$  between the convecting zone and the quiet fluid below. We need to determine  $h$ ,  $T$  and  $\Delta T$ .

For simplicity, imagine that the original stratification formed by the end of the summer was uniform over the vertical, with temperature increasing linearly with height:

$$T_{\text{initial}}(z) = T_o + (T_{\text{surface}} - T_o) \frac{z}{H},$$

where  $T_o$  is the initial temperature at the bottom, which is taken as the reference temperature. This stratification is thus characterized by a single value of  $N^2$  from top to bottom:

$$N^2 = \alpha g \frac{dT_{\text{initial}}}{dz} = \alpha g \frac{T_{\text{surface}} - T_o}{H}. \quad (16.11)$$

In this expression,  $H$  is the local water depth. In terms of  $N^2$ , the initial temperature profile can be rewritten as;

$$T_{\text{initial}}(z) = T_o + \frac{N^2}{\alpha g} z. \quad (16.12)$$

When convection has penetrated to depth  $h$ , the unperturbed temperature at the base of the convective zone is  $T_{\text{initial}}(H - h) = T_o + N^2(H - h)/\alpha g$ . Adding the temperature difference  $\Delta T$  gives the temperature  $T$  in the convective layer:

$$T = T_o + \frac{N^2}{\alpha g} (H - h) + \Delta T. \quad (16.13)$$

This forms a first relation between  $h$ ,  $T$  and  $\Delta T$ .

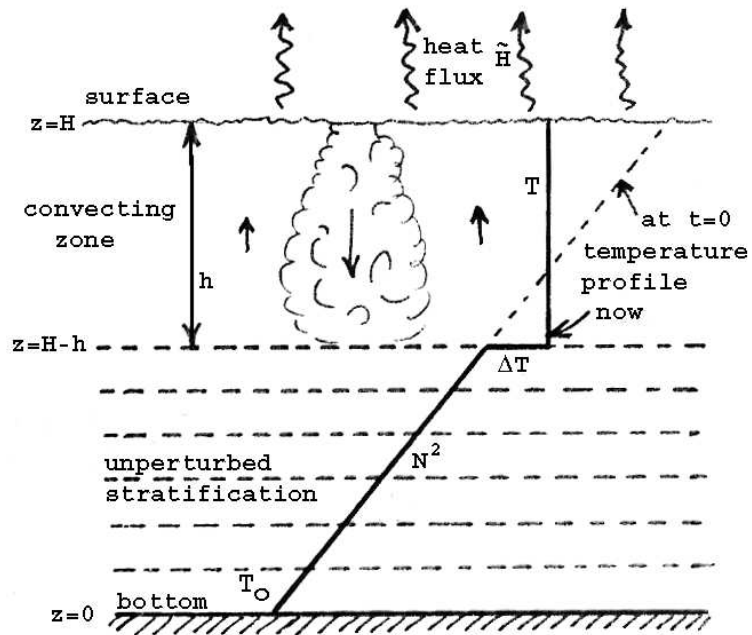


Figure 16.6: Penetrative convection. The initial thermal stratification is gradually eroded over a depth  $h$  that increases over time. The temperature  $T$  inside the convecting zone is uniform, and a temperature jump  $\Delta T$  occurs between the base of the convecting layer and the still unperturbed water below.

Progress of convection requires entrainment of the fluid below depth  $h$  and some energy must be expended to bring this heavier fluid at temperature  $\Delta T$  lower than that above, upward and into the convecting zone. This causes the center of gravity of the water column to be raised somewhat. During this time, however, cooling causes a net shrinking the water volume, lowering the surface level and therefore also the center of gravity of the system. Calculations not reproduced here show that the two mechanisms balance each other so that the net effect is keeping the level of the center of gravity unchanged. In other words, the potential energy is re-arranged but its total value remains changed. After some mathematical manipulations, the following result is obtained:

$$T = T_o + \frac{N^2}{3\alpha g} (3H - 2h) \quad (16.14)$$

$$\Delta T = \frac{N^2 h}{3\alpha g} . \quad (16.15)$$

These relations give  $T$  and  $\Delta T$  in terms of  $h$ , which is still unknown. To obtain a last relation, we invoke the heat budget, which says that the heat content of the mixed zone at temperature  $T$  is equal to the heat content that was initially present over the same depth  $h$  minus the heat that was removed by the surface heat flux over the elapsed time  $t$ . This yields:

$$\int_{H-h}^H \rho_o C_v T dz = \int_{H-h}^H \rho_o C_v [T_o + \frac{N^2}{\alpha g} z] dz - \tilde{H}t,$$

which, after calculation of the integrals and use of the preceding expression for  $T$ , provides

$$\frac{N^2 h^2}{6\alpha g} = \frac{\tilde{H}t}{\rho_o C_v} . \quad (16.16)$$

From this follows the evolution of the convective depth  $h$  over time  $t$ :

$$h = \sqrt{\frac{6\alpha g \tilde{H}t}{\rho_o C_v N^2}} . \quad (16.17)$$

Now that  $h$  is known, we can obtain the temperature of the convective zone and other physical variables. In particular, the vertical diffusivity is found to be

$$D = 0.1 \frac{\alpha g \tilde{H}}{\rho_o C_v} \left( \frac{6t}{N^2} \right)^{\frac{2}{3}} . \quad (16.18)$$

#### *Surface-to-bottom convection*

Eventually, if the heat loss to the atmosphere persists, penetrative convection reaches the bottom (Figure 16.7). Then,  $h = H$  and the vertical velocity is about

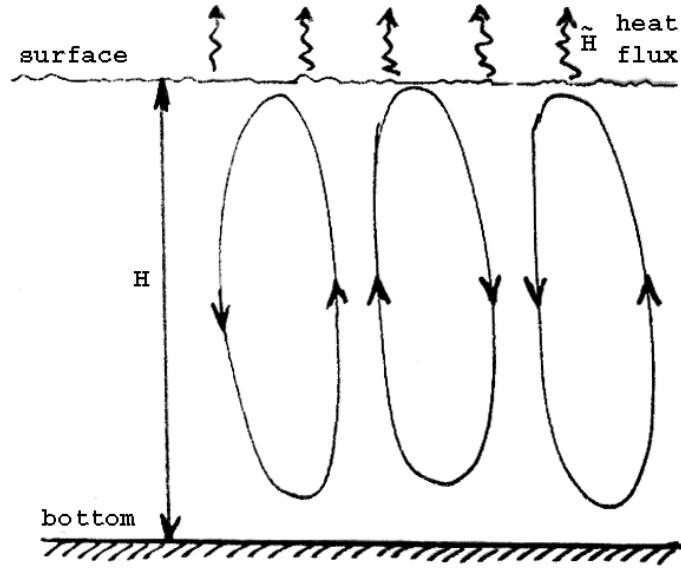


Figure 16.7: When penetrative convection has reached the bottom, there is surface-to-bottom convection. This lasts as long as the surface heat loss persists.

$$w = - \left( \frac{\alpha g H \tilde{H}}{\rho_o C_v} \right)^{\frac{1}{3}} \quad (16.19)$$

and the vertical diffusivity [see (17.10)] reaches the value

$$D = 0.1 \left( \frac{\alpha g H^4 \tilde{H}}{\rho_o C_v} \right)^{\frac{1}{3}}, \quad (16.20)$$

which varies only with the heat flux.

Time for complete vertical mixing of substance released near the surface or bottom is

$$T = 0.134 \frac{H^2}{D} = 1.34 \left( \frac{\rho_o C_v H^2}{\alpha g \tilde{H}} \right)^{\frac{1}{3}}.$$

#### *Wind mixing*

Erosion of stratification and agitation of the upper layers of a lake or reservoir is not caused exclusively by convection. Wind stirring can accomplish the same effect. We now consider the consequences of irregular winds, namely

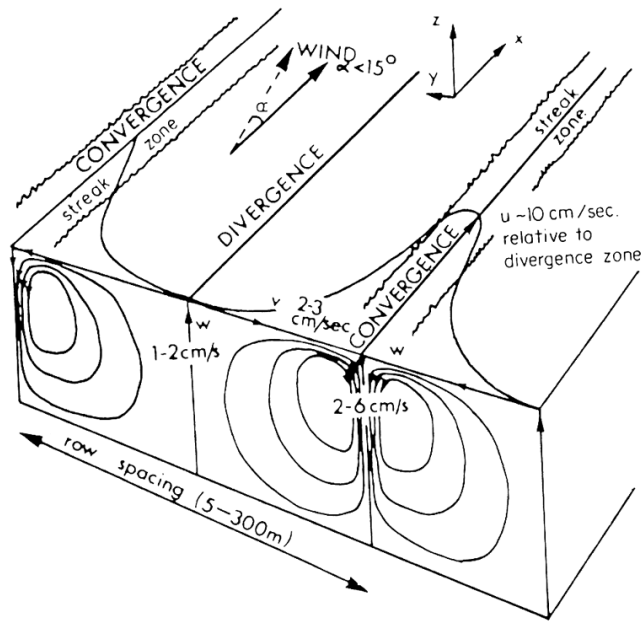


Figure 16.8: Sketch of Langmuir cells, which are helicoidal circulations combining a drift at a slight angle from the wind direction with a cross-wind rotary motion. On the surface appear lines of convergence and divergence (called *windrows*), where surface wind waves behave slightly differently. Also, if there are floating debris, these accumulate along the convergent lines, revealing the Langmuir cells quite clearly.

winds that agitate the water but are not sufficiently coherent to create organized currents across the lake.

Winds on the surface of a water body generate waves, which in turn generate turbulence below the surface. At times, if the wind direction is relatively persistent, the turbulence can organize itself into well defined cells of overturning, called Langmuir cells (Figure 16.8).

Wind momentum in the atmosphere is transferred to the water via the surface stress, denoted  $\tau_{\text{wind}}$ . Observational evidence indicates that this stress is nearly proportional to the square of the wind speed  $U$ :

$$\tau_{\text{wind}} = C_D \rho_{\text{air}} U^2, \quad (16.21)$$

where  $C_D$  is a dimensionless drag coefficient and  $\rho_{\text{air}}$  the air density at water level. If  $U$ , which varies with height, is the wind speed at the nominal height of 10 m (a convenient height for ship and buoy measurements, and also sufficiently away from the water waves at the surface to be a more reliable observation), the drag coefficient is about  $1.0 \times 10^{-3}$  to  $1.5 \times 10^{-3}$ .

This wind-induced surface stress  $\tau_{\text{wind}}$  creates turbulence in the water. If we measure this turbulence activity by the eddy velocity scale  $u_*$ , an argument similar to that use for rivers [see (4.2)] yields:

$$\tau_{\text{wind}} = \rho_o u_*^2,$$

or

$$u_* = \sqrt{\frac{\tau_{\text{wind}}}{\rho_o}}, \quad (16.22)$$

where  $\rho_o$  is now the water density. [It follows that  $u_* \ll U$ .]

The rate of work performed by the wind on the water is the product of the surface stress  $\tau_{\text{wind}}$  with the local, instantaneous velocity in the direction of the stress. The latter is unknown but we can guess that it is proportional to the turbulent velocity  $u_*$ . Therefore, we can express the rate of work done by the wind as:

$$W = m_* \tau_{\text{wind}} u_* = m_* \rho_o u_*^3,$$

where  $m_*$  is a dimensionless coefficient on the order of unity.

The energy budget can be expressed by stating that the work provided by the wind minus a portion lost to dissipation is spent in raising the potential energy by mixing colder waters upward, generating some kinetic energy. The dissipation rate can be taken as a fixed fraction of the work done by the wind (i.e. proportional to  $\rho_o u_*^3$ ), leaving a portion  $m \rho_o u_*^3$  (with  $m < m_*$ ) available for potential and kinetic energy increases. Observational evidence points to  $m = 1.25$ .

If the initial situation is that of a uniform stratification (constant  $N^2$ ) and the mixing has progressed to depth  $h$  by time  $t$ , as depicted in Figure 16.9, the energy budget becomes:

$$\frac{1}{4} N^2 h^2 \frac{dh}{dt} + \frac{C_*}{2} u_*^2 \frac{dh}{dt} = m u_*^3.$$

Because  $u_*^2$  is usually much less than  $N^2 h^2$  ( $u_*^2 \sim 10^{-4} \text{ m}^2/\text{s}^2$ ,  $N^2 h^2 \sim 0.1 \text{ m}^2/\text{s}^2$ ), the kinetic-energy production (the middle term in preceding equation) is negligible compared to potential-energy increase (the first term), leaving the balance between supply of energy by the surface wind stress (discounted for dissipation) and a consumption of potential energy incurred by the upward mixing of denser fluid from below:

$$\frac{1}{4} N^2 h^2 \frac{dh}{dt} = m u_*^3. \quad (16.23)$$

Under constant wind (fixed  $u_*$ ), the solution is immediate:

$$h = \left( \frac{12 m u_*^3 t}{N^2} \right)^{\frac{1}{3}}. \quad (16.24)$$



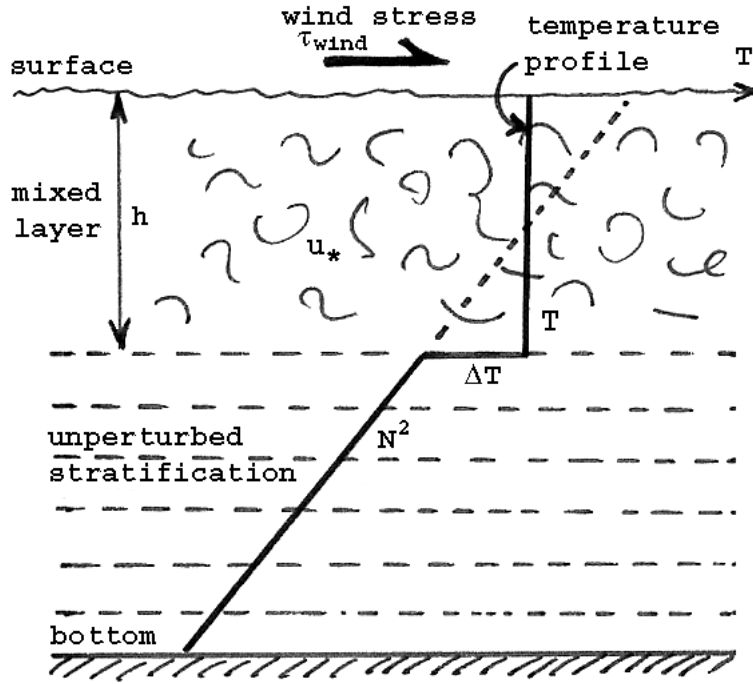


Figure 16.9: Development of a mixed layer under the action of a surface wind stress.

The corresponding vertical diffusivity is given by

$$D = 0.1 u_* h = 0.1 u_*^2 \left( \frac{12mt}{N^2} \right)^{\frac{1}{3}}, \quad (16.25)$$

where, we recall,  $u_* = \sqrt{\tau_{\text{wind}}/\rho_0}$ ,  $\tau_{\text{wind}} = C_D \rho_{\text{air}} U^2$ ,  $U$  = wind speed at 10 m above the water surface,  $C_D = (1.0 \text{ to } 1.5) 10^{-3}$  and  $m = 1.25$ .

Comparing results (17.18) to (17.25) for the convective and wind-stirring cases, respectively, we note that the diffusion coefficient increases faster in the convective case (like  $t^{2/3}$ ) than in the wind-stirring case (like  $t^{1/3}$ ). This implies that, if cooling and wind are simultaneously present, the dispersion induced by convective motions will generally dominate after some time.

Generally, winds over lakes and reservoirs have enough coherence not only to create turbulence by local agitation but also to generate mean currents, creating a circulation across the water body. Unlike the local, wind-stirred turbulence, wind-driven currents are strongly affected by the geometry of the basin, namely its shape and depth. Moreover, in a confined water body, any downwind current must be balanced by some returning current in the opposite direction, either in the vertical or in the horizontal.

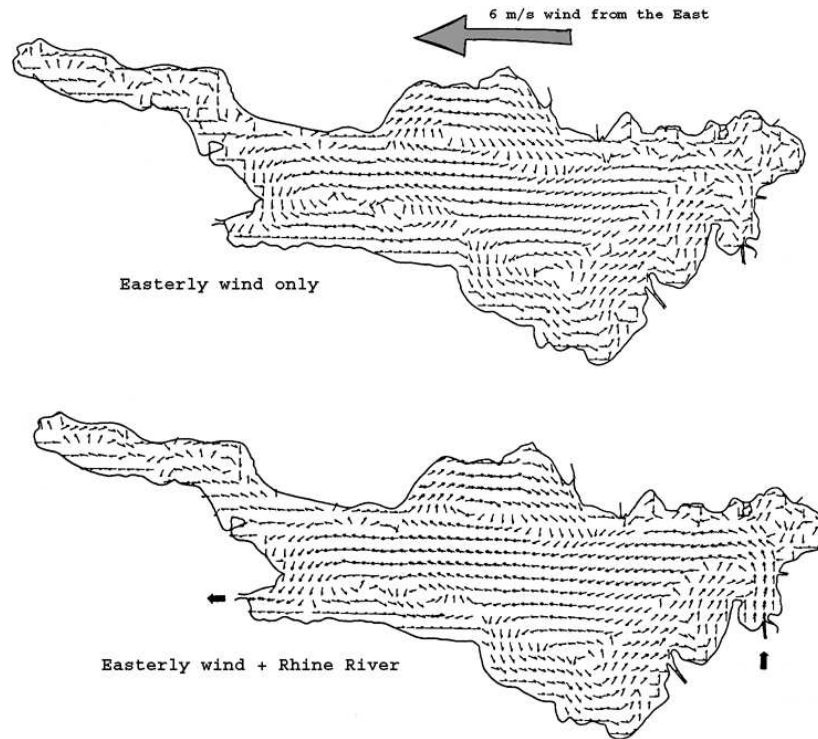


Figure 16.10: Wind-driven circulation in Lake Constance on the Rhine River. The top plot shows the current system under a 6 m/s easterly wind only, while the bottom plot shows the circulation generated by both wind and river throughflow. (From Graf and Mortimer, 1979)

The determination of the current structure under a given wind in a given basin necessitates the use of a computer model, and this is a relatively complicated task. Figure 16.10 gives an example, which shows the circulation pattern in Lake Constance (Germany) under a uniform easterly wind, without and with the currents due to the Rhine River, which passes through it. It is quite clear from this simulation that the circulation pattern is affected by the river only locally, at the river discharge and outlet, and that currents can be directed both with and against the wind. In the case of Lake Constance, currents are directed downwind in the open area and upwind in the embayments.

Since discussion of computer models for the calculation of wind-driven current patterns is beyond the present scope, we shall limit ourselves here to investigating the mixing produced locally by a current, which we take as given.

In the vertical (Figure 16.11), the system is composed of a turbulent upper layer of depth  $h$ , and within this mixed layer there is a uniform current  $U$  and a uniform temperature  $T$ . Below, the water is stably stratified (with uniform

stratification frequency  $N$ ) and at rest (no current). Assuming that, before the wind blew, the waters were stratified continuously to the surface and that there is no surface heat flux, the temperature inside the mixed layer must be the average of the temperatures that initially existed over the depth  $h$ . This implies that the temperature jump  $\Delta T$  at the base of the mixed layer (Figure 16.11) is half of the initial temperature difference between surface and depth  $h$ , i.e.

$$\Delta T = \frac{1}{2} [T_{\text{init}}(z = H) - T_{\text{init}}(z = H - h)] = \frac{N^2 h}{2\alpha g} .$$

The depth  $h$  of the mixed layer can be considered as the mixed zone that results from Kelvin-Helmholtz billows (recall Section 3.4), and therefore,

$$h = 0.3 \frac{\Delta u^2}{\alpha g \Delta T} = 0.3 \frac{U^2}{\alpha g (N^2 h / 2\alpha g)} = 0.6 \frac{U^2}{N^2 h} .$$

From this, we can extract  $h = \sqrt{0.6} U/N$ . But, there is a fundamental problem, which is that we have been confusing here the initial and final states of the billowing process. So, the depth  $h$  is not exactly that found above, but has the same formulation, except for the value of the front coefficient. The better value is:

$$h = 1.4 \frac{U}{N} . \quad (16.26)$$

Note that this value is time independent.

The turbulent diffusivity  $D$  is as always proportional to the turbulent velocity and the diameter of the largest vortices. In the vertical, the largest vortex that can exist is one with diameter  $h$ , while the velocity difference  $U$  between the moving mixed layer and the resting stratified flow below can serve as the vortex orbital speed. After adjustment of the front coefficient, the result is:

$$D = 0.05 U h = 0.07 \frac{U^2}{N} , \quad (16.27)$$

where  $U$  is the local current in the mixed layer and  $N$  the stratification frequency of the original stratification.

## 16.3 Biochemical Processes

### *Water quality in lakes and reservoirs*

All lakes go through an aging process called *eutrophication*, which literally means “well fed”. The cause is a gradual accumulation of silt and organic matter in the lake. A young lake is characterized by a low nutrient content, low plant productivity and clear water. With time, nutrients are brought in by the streams feeding the lake, vegetal productivity increases and the waters

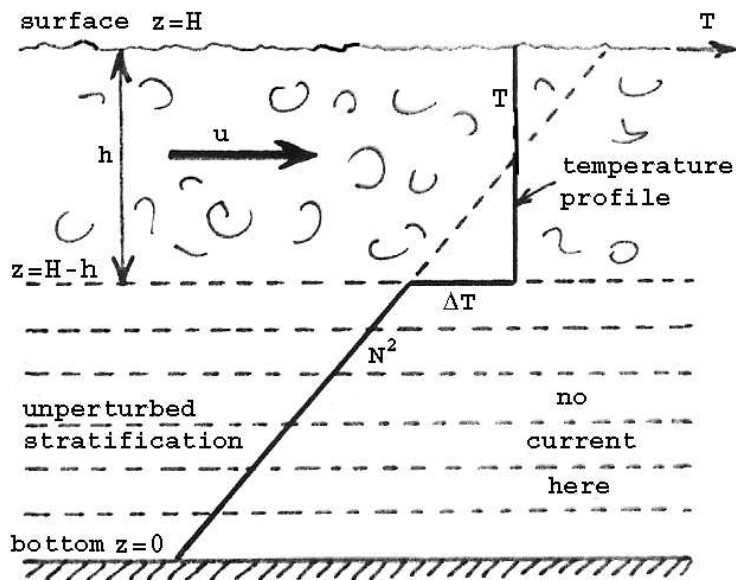


Figure 16.11: Development of a mixed layer under the action of a surface current.

become increasingly murky. The process is entirely natural but taking place over a long, geological time scale, and the ultimate state is the disappearance of the lake and its replacement by a marsh, a bog or a meadow.

What water pollution can do is to increase several times over the nutrient loading of the lake and accelerate the natural eutrophication, leading to premature death of the lake. The culprits are insufficiently treated municipal waste (directly discharged in the lake or indirectly in a tributary stream), industrial wastes, and agricultural runoff of fertilizer. The nutrients (mostly carbonates, nitrates and phosphates) stimulate growth of algae. These algae bloom, die and decay, taking oxygen with them and creating unsightly, malodorous rotting debris (Figure 16.12). Diminution of dissolved oxygen, if sufficient, drives out fish, the most desirable species first.

About the only way to control nutrient loading in a lake or reservoir is to prevent it, that is, pollution must be reduced at the source, including at the discharge locations along the tributary streams. Material budgets (as laid out in Section 1.3) are usually helpful first steps in measuring the amplitude of the problem and progress toward its remediation.

Lakes and reservoirs, basins in which residency time is long, are vulnerable not only to nutrient loading but also to many other forms of pollution, including persistent chemicals and heavy metals. Also, because of their large surface area open to the air, they are exposed to atmospheric pollution, including acid rain.

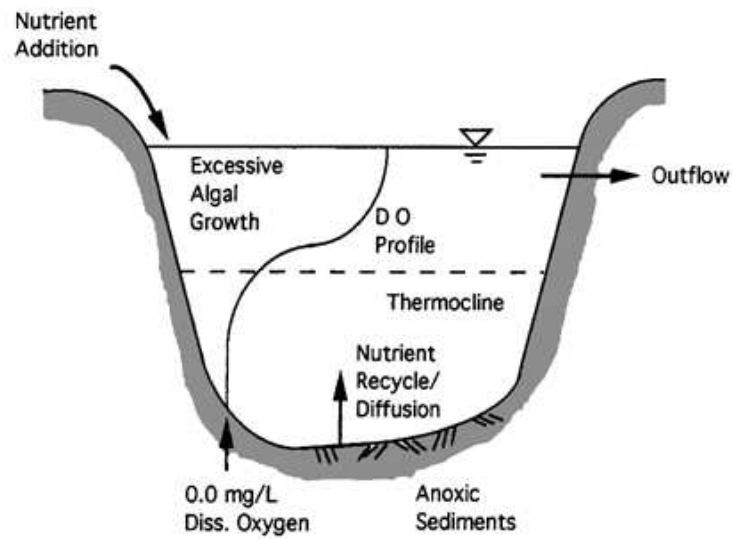


Figure 16.12: Schematic of lake eutrophication and nutrient recycle. Thermal stratification causes a high concentration of oxygen in near-surface waters, but the dissolved oxygen cannot mix vertically, and deep water may become anoxic. (From Schnoor, 1996)

*Acidification*

Atmospheric precipitation is naturally acidic. This is due to the natural presence of carbon dioxide ( $\text{CO}_2$ ) forming carbonic acid ( $\text{H}_2\text{CO}_3$ ) in water droplets, with a pH of approximately 5.6. To obtain the acidity of pristine water and thus natural rain, we apply Henry's Law to carbon dioxide ( $\text{CO}_2$ ). The case, however, is complicated by the fact that  $\text{CO}_2$  reacts in and with water. The first reaction taking place is binding with water to form carbonic acid  $\text{H}_2\text{CO}_3$  with subsequent decomposition into the bicarbonate ion  $\text{HCO}_3^-$  and a proton  $\text{H}^+$ :



The equilibrium constants of these two reactions are:

$$K_1 = \frac{[\text{H}_2\text{CO}_3]}{[\text{CO}_2]} = 1.58 \cdot 10^{-3} \quad (16.28)$$

$$K_2 = \frac{[\text{HCO}_3^-][\text{H}^+]}{[\text{H}_2\text{CO}_3]} = 2.83 \cdot 10^{-4} \text{ M.} \quad (16.29)$$

And, a second reaction is the further decomposition of  $\text{HCO}_3^-$  into the carbonate ion  $\text{CO}_3^{--}$  and another proton:



The equilibrium constant of this last reaction is

$$K_3 = \frac{[\text{CO}_3^{--}][\text{H}^+]}{[\text{HCO}_3^-]} = 4.68 \cdot 10^{-11} \text{ M.} \quad (16.30)$$

The atmosphere currently contains 370 ppm (= parts per million,  $10^{-6}$ ) of  $\text{CO}_2$ , corresponding to a partial pressure of  $370 \cdot 10^{-6} = 3.70 \cdot 10^{-4}$  atm. At  $15^\circ\text{C}$ , the Henry's Law constant is 0.045463 M/atm (see Table 17.2), and the concentration of dissolved undissociated carbon dioxide is  $[\text{CO}_2] = 0.045463 \text{ M/atm} \times 3.70 \cdot 10^{-4} \text{ atm} = 1.682 \cdot 10^{-5} \text{ M}$ .

The equilibrium constants yield successively

$$[\text{H}_2\text{CO}_3] = K_1[\text{CO}_2] = 2.658 \cdot 10^{-8} \text{ M}$$

$$[\text{HCO}_3^-] = \frac{K_2 [\text{H}_2\text{CO}_3]}{[\text{H}^+]} = \frac{7.521 \cdot 10^{-12} \text{ M}^2}{[\text{H}^+]}$$

$$[\text{CO}_3^{--}] = \frac{K_3 [\text{HCO}_3^-]}{[\text{H}^+]} = \frac{3.520 \cdot 10^{-22} \text{ M}^3}{[\text{H}^+]^2}$$

While these reactions occur, water simultaneously dissociates, as it always does, providing another source or sink of  $\text{H}^+$  ions:

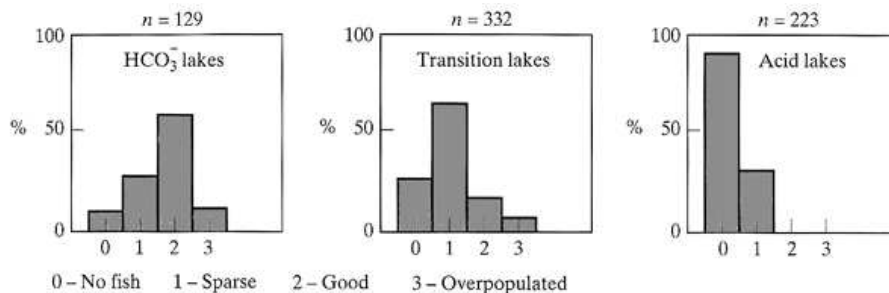


Figure 16.13: Fish populations in 684 Norwegian lakes categorized according to their level of acidity (with number of lakes in each category noted as  $n$  on top of each panel). The greater the acidity, the lower the fish population. (From Wright, 1984)



with constant of dissociation equal to:

$$K_w = [OH^-][H^+] = 10^{-14} \text{ M}^2.$$

To determine the acidity of the water, we need to calculate the concentration of  $H^+$  ions. This is accomplished by enforcing conservation of electrons, otherwise called the equation of *electroneutrality*. This holds true because the negative ions have acquired electrons at the expense of the positive ions. Thus,

$$[H^+] = [OH^-] + [HCO_3^-] + 2[CO_3^{2-}]$$

Replacing the various concentration values in terms of  $[H^+]$ , we obtain

$$[H^+] = \frac{10^{-14} \text{ M}^2}{[H^+]} + \frac{7.521 \cdot 10^{-12} \text{ M}^2}{[H^+]} + 2 \frac{3.520 \cdot 10^{-22} \text{ M}^3}{[H^+]^2}$$

The solution is  $[H^+] = 2.74 \cdot 10^{-6} \text{ M}$ , and the corresponding pH value is

$$\text{pH} = -\log_{10}[H^+] = 5.56. \quad (16.31)$$

Some air emissions are capable of further reducing the pH (increase the acidity) of precipitation. The primary culprits are sulfur dioxide ( $SO_2$ ) and nitrogen oxides ( $NO$  and  $NO_2$ ) create respectively sulfuric acid ( $H_2SO_4$ ) and nitric acid ( $HNO_3$ ). The formation of acid may take place in airborne water droplets, which later falls as acid rain, sleet or snow, or the pollutants may create particles that settle and, once in contact with water at the surface, such as in a lake, turn into acid.

Once in a lake, acids may or may not be neutralized by carbonate (see following subsection). If it is not, the lower than normal pH can create disaster

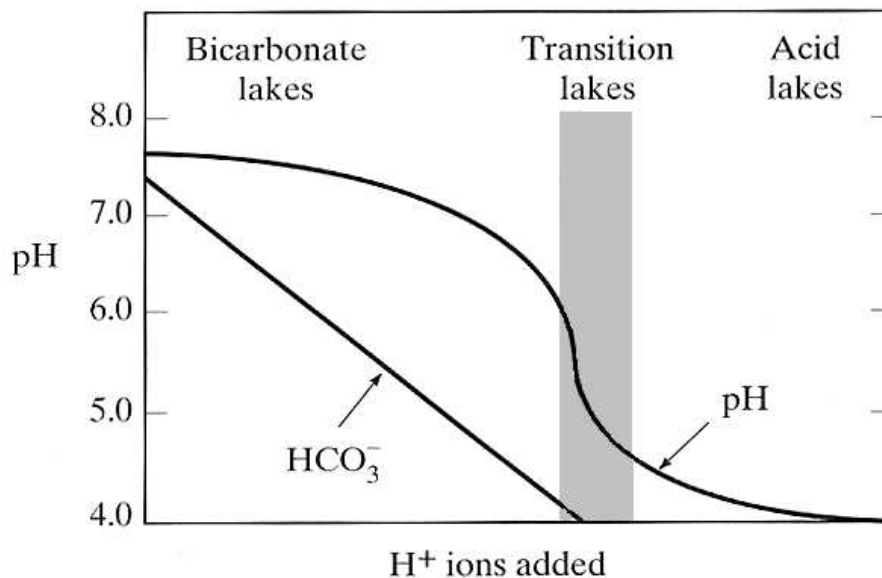


Figure 16.14: The effect bicarbonate ( $\text{HCO}_3^-$ ) in lake water on its acidity. (From Henriksen, 1980)

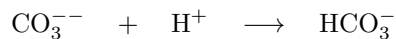
in the lake's ecosystem. The primary victims seem to be fish. Evidence of decreasing fish populations have been reported in the northern United States and Europe. Dramatic evidence was provided in a 1984 study of several hundred lakes in Norway. The data clearly demonstrate that acid lakes are far more likely to have inadequate fish populations than non-acid lakes (Figure 16.13).

#### *Carbonate buffering*

Acid in lake water can be effectively neutralized by carbonate ( $\text{CO}_3^{--}$ ). The most common situation is when the lake sits on a limestone bedrock. Limestone ( $\text{CaCO}_3$ ) easily dissolves in water, creating the carbonate ion ( $\text{CO}_3^{--}$ ) according to

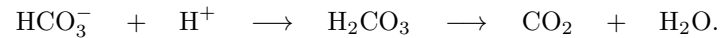


In turn, the carbonate ion binds with a proton  $\text{H}^+$ , to form the bicarbonate ion ( $\text{HCO}_3^-$ )



and the bicarbonate absorbs another proton to form undissociated carbonic acid, which decomposes into carbon dioxide and water, in a process that exactly reverses the formation of naturally acid rain by carbon dioxide:





As we can see, the net transformation is to replace 2  $\text{H}^+$  ions by one  $\text{Ca}^{++}$  ion. The net loss of  $\text{H}^+$  effects a restoration toward better pH values. The key to this restoration is the supply of limestone. Should the amount of limestone present ever be consumed in its entirety, any additional acidity will not be overcome, the drop in pH can be disastrous, as shown in Figure 16.14.

A proposed remedy to restore acid lakes, especially those on granite bedrock (granite is a silicon-based stone, with no calcium), has been the dumping of limestone or hydrated lime ( $\text{Ca}(\text{OH})_2$ ) in its feeding rivers, for slow but continuous release of calcium ion in lake water. However, accumulation of the white powder on the river beds impairs the habitats of bottom organisms and causes an unsightly aspect.

## Problems

17-1. Problem statement

17-2.

17-3.

17-4.

17-5.

17-6.

17-7.

17-8.

17-9.



## Chapter 17

# NEXT CHAPTER

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