

Optimal Long Term Contracts in a Costly State Verification Model

Christopher M. Snyder*

George Washington University and M.I.T.

February 5, 2001

Abstract: Webb's (1992) proposal for the optimal long term contract in a multi-period finance model with costly state verification is suboptimal, dominated by a contract that imposes a weaker individual rationality constraint (one holding in expectation across states rather than independently in each state).

JEL codes: D82, G32, L14

Keywords: long term contract, costly state verification, individual rationality

*Current address (visiting M.I.T. August 2000 through August 2001): Department of Economics, Building E52-391c, M.I.T., Cambridge MA 02142, tel. (617) 253-5067, fax. (617) 253-1330, email cmsnyder@mit.edu. Permanent address: Department of Economics, George Washington University, 2201 G Street N.W., Washington DC 20052, tel. (202) 994-6581, fax. (202) 994-6147, email csnyder@gwu.edu. I would like to thank Oliver Hart, David Scharfstein, Richard Schmalensee, Anjan Thakor, Jean Tirole, and seminar participants at M.I.T. for helpful comments. I retain responsibility for errors.

1. Introduction

The costly state verification model involves an entrepreneur, who has a project requiring a fixed investment to produce a random return, and a lender, which finances the project's initial investment. The project's realized return is private information for the entrepreneur; the lender can observe it only if it engages in a costly audit. The costly state verification paradigm has been widely studied and applied in economics and finance¹ in part because the optimal contract in the one-period variant of the model turns out to be quite simple, a standard debt contract (Townsend 1979, Gale and Hellwig 1985).

Webb (1992) extended the model to a dynamic setting with two periods of investment and return. He proposed a long term contract that improved upon a sequence of standard debt, arguing that this established a rationale for long term contracts. This result was important because previous work on the benefits of long term financial contracts (Townsend 1982) was shown by Allen (1985) and Fudenberg, Holmstrom, and Milgrom (1990) to rely on artificial borrowing and lending constraints. Further, Webb claimed that his proposed contract was optimal.

In this paper, I argue that Webb's contract is actually a sequence of short term contracts (different from standard debt, true, but still short term). A rationale for long term contracts can still be resuscitated. As demonstrated with a counterexample, Webb's contract is suboptimal; the optimal contract is a bona fide long term contract. The reason Webb's contract is suboptimal is that its associated contract design problem imposes an excessively strong individual rationality constraint for the lender, requiring it to earn non-negative profit in each state rather than in expectation across states. A bona fide long term contract can perform the sort of cross-subsidization over states required to impose the weaker individual rationality constraint.

¹See, for example, Bond and Crocker (1993), Border and Sobel (1987), Boyd and Smith (1992, 1993), Chang (1990), Krasa and Villamil (1992, 1994), Mookerjee and Png (1989), Townsend (1979, 1982, 1988), Webb (1992), Morgan (1993), Williamson (1994), and Snyder (1999).

2. Model

The entrepreneur can undertake a project in each of two periods $t = 1, 2$ yielding random return X_t . An investment of K_t is required for each project. Suppose $E_{t-1}[X_t] > K_t$, where $E_{t-1}[\cdot]$ is the expectations operator conditional on the information set at time $t - 1$; thus, each project has positive net present value. The entrepreneur has initial wealth $W_0 < K_1$, so external financing is needed to fund the investment. Suppose that the lending sector is competitive, implying that the lender accepts any contract giving it non-negative expected profit. The entrepreneur can observe X_t ; the lender cannot unless it conducts an audits which costs it C . Assume that auditing is deterministic,² not subject to renegotiation,³ and reveals only current assets. Suppose there is no discounting between periods, that the lender has unlimited funds available at a zero interest rate, and that players are risk-neutral. Before the start of the first period, the entrepreneur and lender can sign a long term contract. The entrepreneur is free to sign contracts with competitive outside lenders at any point, but these will have lower priority than the long term contract in case of default.

Applying the revelation principal, appropriate in this context by Townsend (1988), the optimal contract has the entrepreneur make an announcement, \tilde{X}_t , of the project's return, X_t , each period. The optimal contract maximizes the investor's expected profit subject to the incentive compatibility (IC) constraints that the investor's announcements are true ($\tilde{X}_t = X_t$ for $t = 1, 2$) and subject to the individual rationality constraint (IR) that the lender at least break even.

As a benchmark, consider financing the project with a sequence of standard debt contracts. The entrepreneur contributes all of his previous period wealth, W_{t-1} , and borrows the remainder needed for investment, $K_t - W_{t-1}$, from the lender. After X_t has been realized and \tilde{X}_t announced, the investor repays the lender a fixed amount \bar{R}_t if it is solvent (i.e., if $\tilde{X}_t \geq \bar{R}_t$). If the investor

²Deterministic auditing is the standard assumption in the literature and was adopted by Webb (1992), though several authors have analyzed stochastic auditing (Townsend 1979, Border and Sobel 1987, Mookerjee and Png 1989, Boyd and Smith 1994, Snyder 1999).

³See Gale and Hellwig (1989), Dionne and Viala (1994) and Hart and Moore (1998) for models of debt renegotiation.

is insolvent (i.e., if $\tilde{X}_t < \bar{R}_t$) the lender audits and seizes the investor's entire wealth.

The contract proposed by Webb (1992) has the entrepreneur borrow $K_1 - W_0$ in the first period. He makes an announcement \tilde{X}_1 reflecting the first-period return. If $\tilde{X}_1 < \bar{R}_1$, the lender audits and seizes the entrepreneur's wealth. If $\tilde{X}_1 \geq \bar{R}_1$, the lender receives $R_1(\tilde{X}_1)$ and no audit occurs. In the second period, the parties automatically follow the terms of a standard debt contract which yields the lender zero expected profit conditional on lending $K_2 - W_1(\tilde{X}_1) = K_2 - [W_0 + \tilde{X}_1 - R_1(\tilde{X}_1)]$. The advance of Webb's contract over a sequence of standard debt is that the repayment $R_1(\tilde{X}_1)$ need not be fixed but rather can be conditioned on the return announcement. Incentive compatibility can be maintained even though $R_1(\tilde{X}_1)$ varies and no audit is conducted. This is true since the terms of the second period contract are also contingent upon \tilde{X}_1 : if the entrepreneur understates the true project return, then he will have to borrow more than he actually needs, leading to less favorable terms for the second period loan.

3. Discussion

Though Webb's contract is not a sequence of standard debt contracts, it could be structured as a series of short term contracts. The second period contracts could equally well be supplied by outside lenders other than the initial lender. Assuming competing second period lenders receive the same announcement \tilde{X} as the initial lender,⁴ the initial lender would be willing to offer the first period loan specified by Webb even if it were not to make the second period loan. If Webb's contract were optimal as claimed, then his results would show the opposite of what was intended, namely his results would eliminate rather than support a rationale for long term contracts.

Webb's contract is not optimal, however. It implicitly imposes the constraint that the lender earn non-negative continuation profit for each realization of X_1 . Long term contracts allow a

⁴This assumption is implicit in Webb's statement (p. 1117), "With competition between banks in the middle period ... the second stage of the contract problem has to be a standard-debt contract." Absent this assumption, Webb's contract would not be incentive compatible if the entrepreneur can freely resort to outside lenders in the second period.

weaker IR constraint to be imposed, guaranteeing merely that the lender earn non-negative profit in expectation across realizations of X_1 . The gain from imposing a weaker IR constraint is that by transferring utility of the lender across states, its need to engage in wasteful audits can be reduced. The optimal contract can allow for the transfer of utility not just across realizations of X_1 , but across periods as well. Formally, the appropriate IR constraint is $E_0[\Pi_1^L(X_1) + \Pi_2^L(X_1, X_2)] \geq 0$, where Π_t^L denotes the lender's period- t net profit given returns realized up to that point.

4. Counterexample

In this section, I construct an example in which Webb's contract is dominated by a long term contract along the lines of Section 3. The example, depicted in Figure 1, is identical to that in Section IV of Webb (1992) except that the second period variables have been multiplied by 20. In each period, the return for the project has a two point distribution. The first period project requires investment $K_1 = 8$ and returns X_1 equal to 14 or $7\frac{1}{2}$ with equal probability. The second period project requires investment $K_2 = 160$ and returns X_2 equal to 280 or 150 with equal probability. The investor has no initial wealth. The auditing cost is $C = 3\frac{1}{2}$. It can be shown that the entrepreneur's ex ante expected profit in the first best (with costless state verification) is $57\frac{3}{4}$.

A sequence of standard debt would have the following form (the calculations follow Webb 1992). In the first period, the repayment required to avoid audit is $\bar{R}_1 = 12$. Following a high realization of first period profit (i.e., continuing from node N_1), the entrepreneur borrows 158 and is required to repay $\bar{R}_2 = 169\frac{1}{2}$ to avoid audit. Following a low realization of first period profit (i.e., continuing from node N_2), the entrepreneur borrows 160 and is required to repay $173\frac{1}{2}$. The entrepreneur's ex ante expected profit is $54\frac{1}{4}$. Auditing occurs at nodes N_2 , N_4 and N_6 .

Webb's proposed contract (again, following the calculations from Webb 1992) involves a first

period repayment of $7\frac{1}{2}$ in the low return state and $8\frac{1}{2}$ in the high return state and no auditing in either state. The second period continuation contract from node N_1 (respectively, N_2) has the form of a standard debt contract with required repayment $162\frac{1}{2}$ (respectively, $173\frac{1}{2}$). The entrepreneur's ex ante expected profit is 56. Auditing occurs at nodes N_4 and N_6 .

The optimal contract involves a first period repayment of $7\frac{1}{2}$ in the low return state and $8\frac{1}{2}$ in the high return state and no audit, exactly as with Webb's contract. In the second period continuation contract from node N_1 , auditing can be avoided entirely by specifying a fixed payment of 150. The lender's continuation payoff is negative with this continuation contract, $-4\frac{1}{2}$ to be precise; so it will have to receive a cross subsidy of $4\frac{1}{2}$ in the continuation contract from node N_2 . This can be done with a standard debt contract requiring the entrepreneur to repay $182\frac{1}{2}$ to avoid audit. The entrepreneur's ex ante expected profit is $56\frac{7}{8}$. Auditing occurs only at node N_6 .

By design, the optimal long term contract is individually rational for the lender. It can be verified that it is incentive compatible. To prove the contract is optimal, note the only possible contract which could dominate it is one with no auditing. But a contract with no auditing cannot satisfy both IR and IC constraints.⁵ The optimal long term contract yields the entrepreneur a higher ex ante expected profit because it involves less auditing.

5. Conclusion

The preceding section provided a single counterexample to the claim that Webb's (1992) contract is optimal in a dynamic, costly state verification model and provided a bona fide long term contract which was optimal. It would be useful in future work to have a full characterization of optimal contracts in the general model. Chang's (1990) solution for optimal contracts in a

⁵To satisfy the IC constraint in the second period, the contract can only require a fixed payment of 150 in all states. To satisfy the IR constraint, then, the contract must raise 18 in the first period. But this is impossible because the highest return in the first period is 14.

simpler dynamic setting may provide an approach to the problem.

Many real world financial contracts appear to exhibit the desirable feature highlighted by this paper, namely they impose a weak version of the lender's IR constraint. Lines of credit stipulate that a variable amount may be borrowed up to a credit ceiling in some future period. They typically set a fixed interest rate on the amount borrowed plus either a fixed fee (a fee independent of the amount borrowed) or a usage fee (a rate charged on the unused portion of the line of credit). Put in terms of the model, a line of credit would tend to subsidize the entrepreneur in low- X_1 states—since then the fixed fee would be spread over a large loan (or the usage fee would be low)—in return for a subsidy from the entrepreneur to the lender in high- X_1 states. Given the importance of lines of credit (by some accounts, they are responsible for over 80 percent of commercial lending in the U.S.), it is not surprising that a substantial literature exists discussing their beneficial properties.⁶

⁶See Bhattacharya and Thakor (1993) and Snyder (1998) for literature reviews. The work closest to the present setting includes Thakor's (1989) model of entrepreneur adverse selection, Houston and Venkataraman's (1994) model of lender moral hazard, and Morgan's (1993) model of costly state verification with convex auditing costs.

References

- Allen, F. (1985) "Repeated Principal-Agent Relationships with Lending and Borrowing," *Economics Letters* 17: 27–31.
- Bhattacharya, Sudipto and Anjan V. Thakor. (1993) "Contemporary Banking Theory," *Journal of Financial Intermediation* 3: 2–50.
- Bond, Eric W. and Keith J. Crocker. (1993) "Bank Capitalization, Deposit Insurance, and Risk Categorization," *Journal of Risk and Insurance* 60: 547–569.
- Border, Kim C. and Joel Sobel. (1987) "Samurai Accountant: A Theory of Auditing and Plunder," *Review of Economic Studies* 54: 535–539.
- Boyd, John H. and Bruce D. Smith. (1992) "Intermediation and the Equilibrium Allocation of Investment Capital: Implications for Economic Development," *Journal of Monetary Economics* 30: 409–423.
- Boyd, John H. and Bruce D. Smith. (1993) "The Equilibrium Allocation of Investment Capital in the Presence of Adverse Selection and Costly State Verification," *Economic Theory* 3: 427–451.
- Chang, Chun. (1990) "The Dynamic Structure of Optimal Debt Contracts," *Journal of Economic Theory* 52: 68–86.
- Dionne, Georges and Pascale Viala. (1994) "Moral Hazard, Renegotiation and Debt," *Economics Letters* 46: 113–119.
- Fudenberg, Drew, Bengt Holmstrom, and Paul Milgrom. (1990) "Short-Term Contracts and Long-Term Agency Relationships," *Journal of Economic Theory* 51: 1–31.
- Gale, Douglas and Martin Hellwig. (1985) "Incentive-Compatible Debt Contracts: The One-Period Problem," *Review of Economic Studies* 52: 647–663.
- Gale, Douglas and Martin Hellwig. (1989) "Repudiation and Renegotiation: The Case of Sovereign Debt," *International Economic Review* 30: 3–31.
- Hart, Oliver D. and John Moore. (1998) "Default and Renegotiation: A Dynamic Model of Debt," *Quarterly Journal of Economics* 113: 1–41.
- Krasa, Stefan and Anne P. Villamil. (1992) "Monitoring the Monitor: An Incentive Structure for a Financial Intermediary," *Journal of Economic Theory* 57: 197–221.
- Krasa, Stefan and Anne P. Villamil. (1994) "Optimal Multilateral Contracts," *Economic Theory* 4: 167–187.
- Mookerjee, Dilip and Ivan Png. (1989) "Optimal Auditing, Insurance, and Redistribution," *Quarterly Journal of Economics* 104: 399–415.

- Morgan, Donald P. (1993) "Financial Contracts When Costs and Returns Are Private," *Journal of Monetary Economics* 31: 129–146.
- Snyder, Christopher M. (1998) "Loan Commitments and the Debt Overhang Problem," *Journal of Financial and Quantitative Analysis* 33: 87–116.
- Snyder, Christopher M. (1999) "Bounding the Benefits of Stochastic Auditing: The Case of Risk Neutral Agents," *Economic Theory* 14: 247–253.
- Thakor, Anjan. V. (1989) "Competitive Equilibrium with Type Convergence in an Asymmetrically Informed Market," *Review of Financial Studies* 2: 49–71.
- Townsend, Robert M. (1979) "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory* 20: 265–293.
- Townsend, Robert M. (1982) "Optimal Multiperiod Contracts and the Gain from Enduring Relationships Under Private Information," *Journal of Political Economy* 90: 1166–1186.
- Townsend, Robert M. (1988) "Information Constrained Insurance: The Revelation Principle Extended," *Journal of Monetary Economics* 21: 411–450.
- Webb, David C. (1992) "Two-Period Financial Contracts with Private Information and Costly State Verification," *Quarterly Journal of Economics* 107: 1113–1123.
- Williamson, Stephen D. (1994) "Do Informational Frictions Justify Federal Credit Programs?" *Journal of Money, Credit, and Banking* 26: 523–544.

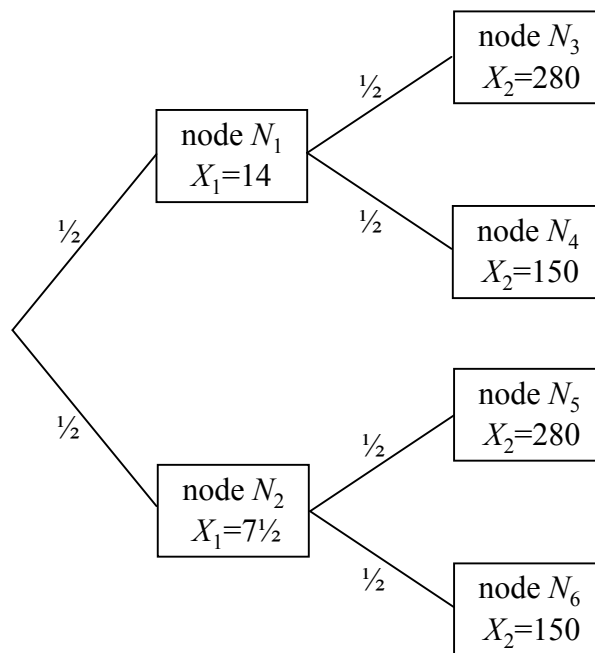


Figure 1: Counterexample