Effects of environmental variability on physiological processes: Jensen's Inequality

A generalized function for poikilotherm metabolic rate as a function of temperature is:

\[ MR = a \cdot 10^{b T} \quad b = \frac{\log_{10}(Q_{10})}{10} \]

where \( MR \) = metabolic rate (e.g., J/h) at temperature \( T \), \( a = MR \) at 20°C, and \( b \) = a parameter related to \( Q_{10} \) as in Eq. 2.

1. Imagine a single poikilotherm phenotype, which is a perfect thermal conformer, inhabiting two different thermal environments with the same average temperature, but one being invariant (environment A) and the other being variable (environment B). Predict how the energetic requirements of the poikilotherm would compare between these environments. (Circle one)

\[ A > B \quad A = B \quad A < B \]

2. Design and conduct a simple mathematical experiment to test your prediction. Briefly describe the test.

Briefly state the results.

A generalized function for photosynthetic rate as a function of irradiance is:

\[ Ps = \frac{b \cdot PAR}{1 + a \cdot PAR} - R \]

where \( Ps \) = photosynthetic rate (e.g., \( \mu \text{mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \)), \( b \) is the initial slope of increase in \( Ps \), \( a \) is inversely related to \( Ps \) at light saturation, \( PAR \) = photosynthetically active radiation, and \( R \) = dark respiration.

3. Imagine a single plant phenotype inhabiting two different light environments with the same average \( PAR \), but one being invariant (environment A) and the other being variable (environment B). Predict how the net daily carbon gain of the plant would compare between these environments. (Circle one)

\[ A > B \quad A = B \quad A < B \]

4. Design and conduct a simple mathematical experiment to test your prediction. Briefly describe the test.

Briefly state the results.
A generalized function for assimilable energy as a function of the mass of a food item is:

\[ A = b \cdot M \quad \text{Eq. 4} \]

where \( A \) = assimilable energy (e.g., J), \( b \) is a coefficient of assimilation efficiency, and \( M \) is the mass of the food item.

5. Imagine a single predator phenotype inhabiting two different environments with the same average prey size, but one being invariant with respect to prey size (environment \( A \)) and the other having variably sized prey items (environment \( B \)). If predators capture the same number of prey items in both environments, predict how the predator’s total energy assimilation would compare between these environments. (Circle one)

- \( A > B \)
- \( A = B \)
- \( A < B \)

6. Design and conduct a simple mathematical experiment to test your prediction.

Briefly state the results.

7. Identify another physiological response function besides the three above. Sketch a likely form for the function (with both axes fully labelled, including units). Without doing a simulation experiment, correctly predict the effect of variation in the environmental variable (\( X \) axis) on the average physiological status (\( Y \) axis).

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<thead>
<tr>
<th>Physiological response function</th>
<th>Effect of variance in ( X ) on average of ( Y )</th>
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8. Be able to explain and apply the general mathematical proof which is “Jensen’s Inequality”.

Excel has a convenient and powerful random number generator. click Data Analysis in the Analysis group on the Data tab. In older versions look under “Tools... Data analysis... ”. Open the GUI for “Random Number Generation” and fill in the fields as appropriate (below).

- Number of variables: e.g., 1 (yields 1 column of random numbers)
- Number of random numbers: e.g., 500 (yields 500 random numbers per column)
- Distribution: normal (invokes normal distribution defined by mean and variance)
- Mean: e.g., 14.5
- Standard deviation: e.g., 7.7
- Random seed: make up a number (the same seed will always give the same set of “random” numbers)
- Output range: e.g., h10 (position for 1st random number of the set that will be returned). Click OK.

If the Data Analysis command is not available, you need to load the Analysis ToolPak add-in program.

Click the File tab, click Options, and then click the Add-Ins category.

In the Manage box, select Excel Add-ins and then click Go.

In the Add-Ins available box, select the Analysis ToolPak check box, and then click OK.