Challenge: For any number of boxes in a row, can you write down a formula for the number of ways that you fill the boxes with stars that each fill one box each and candy bars that each fill two boxes each?

For example, if you have two boxes and are given one candy bar and as many stars as you need, one way to fill the boxes is to have a star in each box. Another way is to have one candy bar that spans the two boxes.

For each person in your group, designate one of the following colors: Red, Blue, and Black. Next to the color, write your name in that color:

Red _______________________________

Blue _______________________________

Black _______________________________

Hold onto these pens; you'll need them for the rest of this activity. Each question will specify which person in your group should write the answer for that question. But for each question, you should work together as a group to work out each answer.
1. Starting with three boxes, labeled Box 1, Box 2, and Box 3, how many ways can you fill the boxes using just stars? The person holding the RED pen should write this answer.

\[
\binom{3}{0} = \frac{3!}{0!(3 - 0)!} = 1
\]

2. Now you are given one candy bar (that fills two boxes) and as many stars as you want.

(a) What is the number of stars that you need to use to fill all three boxes? Why? The person holding the BLUE pen should write this answer.

*There are three boxes. Since each candy bar fills two boxes, then we need only one star to fill all three boxes.*

(b) What is the total number of candy bars and stars that you have? The person holding the BLACK pen should write this answer.

*The total number of candy bars and stars is two since we have one candy bar and one star.*

(c) Write the number of ways to arrange the stars and the candy bar. The person holding the RED pen should write this answer.

*There are two ways to arrange the one star and the one candy bar, either the star followed by the candy bar or the candy bar followed by the star.*

(d) Now returning to your three boxes, how many ways can you fill three boxes with stars and exactly one candy bar? Why? The person holding the BLUE pen should write this answer.

*There are two ways to arrange the one star and the one candy bar in the boxes, either the star in Box 1 with the candy bar in boxes 2 and 3 or the candy bar in boxes 1 and 2 with the star in Box 3.*

(e) Explain how you could write the number of ways you can fill three boxes with stars and exactly one candy bar? The person holding the BLACK pen should write this answer.

\[
\binom{1 + 1}{1} = \binom{2}{1} = \frac{2!}{1!(2 - 1)!} = 2
\]
3. Starting with four boxes, labeled Box 1, Box 2, Box 3, and Box 4, how many ways can you fill the boxes using just stars? The person holding the BLUE pen should write this answer.

\[
\binom{4}{0} = \frac{4!}{0!(4-0)!} = 1
\]

4. Now you are given one candy bar (that fills two boxes) and as many stars as you want.

(a) What is the number of stars that you need to use to fill all four boxes? Why? The person holding the BLACK pen should write this answer.

*There are four boxes. Since each candy bar fills two boxes, then we need two stars to fill all four boxes.*

(b) What is the total number of candy bars and stars that you have? The person holding the RED pen should write this answer.

_The total number of candy bars and stars is three since we have one candy bar and two stars._

(c) Write the number of ways to arrange the stars and the candy bar. The person holding the BLUE pen should write this answer.

_There are three ways to arrange the two stars and the one candy bar: 1) star, star, candy bar, 2) star, candy bar, star, and 3) candy bar, star, star._

(d) Now returning to your four boxes, how many ways can you fill four boxes with stars and exactly one candy bar? Why? The person holding the BLACK pen should write this answer.

_There are three ways to arrange the two stars and the one candy bar in the boxes, 1) star in Box 1, star in Box 2, candy bar in Boxes 3 and 4, 2) star in Box 1, candy bar in Boxes 2 and 3, star in Box 4, and 3) candy bar in Boxes 1 and 2, star in Box 3, star in Box 4._

(e) Explain how you could write the number of ways you can fill four boxes with stars and exactly one candy bar? The person holding the RED pen should write this answer.

\[
\binom{2+1}{1} = \binom{3}{1} = \frac{3!}{1!(3-1)!} = 3
\]
5. Now you are given two candy bars (where each candy bar fills two boxes) and as many stars as you want.

(a) What is the number of stars that you need to use to fill all four boxes? Why? The person holding the **RED** pen should write this answer.

*There are four boxes. Since each candy bar fills two boxes, then we need zero stars to fill all four boxes.*

(b) What is the total number of candy bars and stars that you have? The person holding the **BLUE** pen should write this answer.

*The total number of candy bars and stars is two since we have two candy bars and zero stars.*

(c) Write the number of ways to arrange the stars and the candy bars. The person holding the **BLACK** pen should write this answer.

*There is one way to arrange the two candy bars: candy bar and candy bar*

(d) Now returning to your four boxes, how many ways can you fill four boxes with stars and exactly two candy bar? Why? The person holding the **RED** pen should write this answer.

*There is one way to arrange the two candy bars: candy bar in Boxes 1 and 2 and candy bar in Boxes 3 and 4.*

(e) Explain how you could write the number of ways you can fill four boxes with stars and exactly two candy bars? The person holding the **BLUE** pen should write this answer.

\[
\binom{0 + 2}{0} = \binom{2}{0} = \frac{2!}{0!(2 - 0)!} = 1
\]
6. If you have \( k \) candy bars and \( r \) stars, how many ways can you arrange them in a line? Why? The person holding the **BLACK** pen should write this answer.

\[
\binom{k + r}{r} = \binom{k + r}{k}
\]

7. Now your group has \( n \) boxes and \( k \) candy bars (where each candy bar fills two boxes).

(a) How many stars will you need to fill the boxes? Why? The person holding the **RED** pen should write this answer.

"There are \( n \) boxes. Since each candy bar fills two boxes, then we need \( (n - 2 \cdot k) \) stars to fill \( n \) boxes."

(b) If you have \( n \) boxes and \( k \) candy bars (where each candy bar fills two boxes), how many ways can you fill the boxes? Why? The person holding the **BLUE** pen should write this answer.

\[
\binom{k + (n - 2 \cdot k)}{(n - 2 \cdot k)} = \binom{k + (n - 2 \cdot k)}{k} = \binom{(k + n - 2 \cdot k)}{k} = \binom{(n - k)}{k}
\]