Glacier Flow:
A Silly Putty Analog

1 Introduction

This worksheet will help you record your data and derive equations for the experiment we will do with silly putty as an analog to glacier flow. As we talked about in class, the amount of pressure a material is subject to (i.e., stress) is related to the amount of deformation it undergoes (i.e., strain). In addition, for viscous materials or fluids, stress and deformation type are also related to strain rate through a parameter called viscosity.

2 Focus Question

The focus of this activity is to understand the different aspects of glacier flow, and the material parameters that affect flow velocity.

3 Objectives

Through this activity you will:

1. Research the difference between Newtonian and non-Newtonian fluids.

2. Learn material science vocabulary terms and relate them to calculus terms.
3. Review for your BC exam by integrating the equation for shear rate

4. As an analogue to ice flow, investigate the viscosity of Silly Putty and relate it to flow through a half channel

4 Useful Terms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Units</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>strain</td>
<td>none</td>
<td>$\frac{L_f - L_0}{L_0}$</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>velocity</td>
<td>m/s</td>
<td>$\frac{1}{L_0} \frac{dL_0}{dt} = \frac{\epsilon(t)}{L_0}$</td>
</tr>
<tr>
<td>$\dot{\epsilon}$</td>
<td>strain rate</td>
<td>1/s</td>
<td>$\rho g d \sin \alpha$ for inclined plane</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress</td>
<td>Pa</td>
<td>$\frac{\partial \tau}{\partial y}$ for Newtonian Fluid</td>
</tr>
<tr>
<td>$\dot{\gamma}$</td>
<td>shear rate</td>
<td>1/s</td>
<td>$\left(\frac{\sigma}{\dot{\gamma}}\right)^n$ for non-Newtonian Fluid</td>
</tr>
<tr>
<td>$\dot{B}$</td>
<td>“apparent viscosity” term</td>
<td>Pa-s</td>
<td>empirical</td>
</tr>
<tr>
<td>$n$</td>
<td>stress exponent</td>
<td>none</td>
<td>empirical ($\approx 3$ for glacier flow)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>viscosity</td>
<td>Pa-s</td>
<td>$\frac{\sigma}{\dot{\gamma}}$ for Newtonian Fluid</td>
</tr>
</tbody>
</table>

5 Velocity from Shear Rate: x-y effects

Research the term “non-Newtonian fluid” I introduced briefly in lecture, and describe why Silly Putty fits this description. Draw a graph of stress vs. strain rate for both a Newtonian and non-Newtonian fluid.
Recall from our discussion of laminar flow that viscosity is due to friction between areas of flow with different strain rates (i.e. velocities), which are not constant with $y$. Therefore, when considering differences of flow with respect to $y$, we are more interested in a term called shear rate, which relates how velocity changes with $y$. Figure 1 illustrates these concepts.

The force exerted on a fluid is dependent also on $y$, and is designated by $\sigma$, which can be resolved into force components for an inclined plane. Show that

$$\sigma = \rho gy \sin \alpha$$

where $\rho$ is the density of the material, $g$ is gravity, $y$ is distance from top to bottom of the channel, and $\alpha$ is the angle of incline.
From the definition of shear rate in the above table, we have:

\[ \dot{\gamma} = \left( \frac{\sigma}{B} \right)^n \]

Where again, B is an effective viscosity parameter, and as you showed above, \( \sigma \) is a function of \( y \).

In order to obtain the difference in velocity between the top and bottom layers \( v_0(t) - v_b(t) \), integrate \( \dot{\gamma} \) with respect to \( y \), substituting in the equation you derived for \( \sigma \) of an inclined plane.
6 Velocity from Shear Rate: x-z effects

Similarly to the laminar flow case in the x - y plane, shear rate affects velocity profiles in the x - z plane as well (or, top down view of the open channel). The terms remain the same, only now we consider velocity in the z direction, or across the top of the channel. Now, both boundaries of the channel are fixed (see Figure 2).

Note that σ is the same as you derived for the x - y case.

The difference in velocity is now with respect to the center line. The velocities at the margins are the same, and so for convenience the entire length is changed to 2z. Again integrating shear rate, the difference in velocity between the center line and the margin is:

\[ v_b(t) - v_h(t) = \left( \frac{1}{2} \right)^n \frac{k}{n+1} \sin^n \alpha \cdot z^{n+1} \]

where

\[ k = \left( \frac{\rho g}{B} \right)^n \]
and $z$ is the distance from the center line.

7 Experiments

In the experimental portion of this project, you will compare your calculated values for velocity of a "glacier" with actual measurements made from your silly putty analog. In order to calculate these values, you will draw contour lines on your silly putty "half cylinder," let it flow for a few days, and observe the contours after a set amount of time, for several iterations. Fig. 3 shows one possible configuration of contour lines, drawn with a sharpie on the putty.

You should be able to observe laminar flow in both the $x - y$ and $x - z$ planes by measuring the changes in your contour lines.

1. Find the density $\rho$ of silly putty ________
2. Find the value of $y$ for your half-cylinder ________
3. Find the value of $z$ for your half-cylinder ________
4. Pick an incline angle: $\alpha =$ ________

6
5. What are the velocities $v_\theta(t)$ and $v_\phi(t)$ of your contour lines in the $x - y$ plane?

$v_\theta(t) =$ __________ 
$v_\phi(t) =$ __________ 

6. What are the velocities $v_\theta(t)$ and $v_\psi(t)$ of your contour lines in the $x - z$ plane?

$v_\theta(t) =$ __________ 
$v_\psi(t) =$ __________ 

Based on your integration of the power law of glacier flow in the $x - y$ plane and your above values, calculate the effective viscosity parameter $B$ at $y = y_0$ and $y = y_b$.

Based on your integration of the power law of glacier flow in the $x - z$ plane and your above values, calculate the effective viscosity parameter $B$ at $z = z_0$ and $z = z_b$. 

7
Is viscosity for a non-Newtonian fluid constant with strain rate or stress? How do you know?

Can you think of any other factors that would affect these values?

How would changes in viscosity affect where a glacier flows and where it shatters?

8 Assessment

You will be assessed based on:

1. Your correct integration of the shear rate equation
2. Your correct resolving of forces for an inclined plane
3. That your experimental set-up makes sense for the desired measurements
4. Your thoughtful explanations about viscosity and strain rate

9 Extensions

Extensions for this lab can include:

1. Incorporation of time-lapse photography to capture the flow of the silly putty.
2. Hit the silly putty with differing forces using a load cell and hammer and determine the stress/strain rate at which Silly Putty changes from a viscous to a brittle material
3. Determine the “exact” viscosity of Silly Putty through a pitch-drop experiment