Escaping the Deadly Waters:
Role of Capital Markets in Accumulating Public and Private Capital Stock

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Abstract. The papers examines how frictions in the credit markets affect a poor country's ability to accumulate public and private capital stock. Governments accumulate infrastructure or public capital stock and entrepreneurs accumulate private capital stock. The capital is differentiated by the end-users. The output of the economy is a convex function of both types of capital and the two types of capital are complementary. Both the private and the public sector leverage their respective income streams to borrow from the capital markets. The public sector's income stream is the revenue stream from the taxing the private sector. The marginal productivity of private capital depends on the level of public capital in the economy and similarly, the marginal productivity of public capital depends on the level of private capital in the economy. Consequently, the private sector's ability to leverage their income stream interacts with the public sector's ability to leverage their incomes stream (tax revenues) to determine the growth path of the economy. The paper pins down the conditions under which an economy may find itself in a poverty trap.

1. Introduction

This paper takes its primary motivation from Lucas (1990), which has a self-explanatory title, "Why Doesn't Capital Flow from Rich to Poor Countries?" The rich countries have higher capital stock and presumably a lower marginal productivity than poor countries, which should lead to capital flows from the rich to the poor countries. Actually, the global capital flows are dominated either by flows between developed countries or between the developed and the emerging countries. (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005) In comparison, capital flows to the poor developing countries are nothing more than a trickle.

Lucas (1990) proposed two main candidates, differences in the complementary inputs, like human capital and total factor productivity, and frictions in the global capital markets. The paper did not satisfactorily answer the question it set for itself. If it is indeed the differences in the complementary inputs that lead to differences in per-capita income, why can't free flow of capital eliminate the differences in the complementary inputs across the world? For instance, if a entrepreneurs starts a production process in a poor country, the human capital of the local labour would get upgraded over time through the process of learning-by-doing. In another influential paper, Lucas (1990) concludes that a very large part of the growth in fast growing economies like the East Asian economies can to be attributed to accumulation of human capital through the process of learning by doing, which, in turn has been stimulated by trade.

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If indeed there are large differences in the complementary inputs across the world, arguing in a similar vein, it is not clear why free flow of capital along with international trade and factor mobility cannot eliminate the differences in factor productivity across countries in the world. If it is the capital market frictions that stem the flow of capital, then the determinants of these frictions have to be pinned down.

This paper attempts to pin down how the lack of fiscal capacity and provision of public goods in conjunction with frictions in the credit market can potentially trap a country in poverty. Our narrow focus is on the potential impact credit market frictions have on economic activity and tax revenues. We assume that the government is benign and conscientiously invests all its tax revenues into public good.[1] Using this assumption, we will try to pin down the full impact of the credit market frictions in terms of how the *government's limited ability to borrow interacts with the entrepreneur's limited ability to borrow*. What emerges out of this interaction is a poverty trap. It may explain why there are negligible capital flows to extremely poor countries. The model predict that once a country crosses a particular output threshold, there are large volume of capital flows to the country as both the government and the private sector starts borrowing simultaneously as their respective leverage factors interact and in turn increase their respective borrowing capacities. Indeed, we saw dramatic increase in capital flows to the BRIC countries at the end of the last century.

Lack of fiscal capacity of state is both the symptom and cause of lack of development. Richer countries tend to raise more taxes, provide better public goods and have more economic activity than poor countries. (Baunsgaard and Keen, 2010; Besley and Persson, 2009) There is an endogenous relationship between provision of public goods or fiscal capacity of the state and the extent of private economic activity in the economy.[2]

Unravelling this endogenous relationship between public goods provision and private economic activity is critical to getting to grips with the problems faced by the poorest countries in the world. Lack of public goods provisions is blamed on inefficiency and corruption of the governments in the poor countries (Acemoglu, 2006, 2008). In this paper, we want to abstract from the political economy questions and focus on the resource constraints that conscientious governments in the poor country face when they aspire to accumulate public capital stock. The objective of the paper is to find conditions under which the poor countries are not able to accumulate public and private capital in spite of access to global capital markets.

Quah (1997) was the first one to emphasise the stratification in world income distribution and a possible shift towards a bimodal distribution. Even if there is some conditional convergence across the world, it is painfully slow. In contrast there is a lot of evidence of convergence within the OECD countries (Acemoglu, 2009). This suggests the economic environment faced by the poor countries are distinct from the rest of the world and there are some specific factors that sustain this

[1] Acemoglu (2005) models the impact of corrupt rulers that divert tax revenues to benefit themselves and finds that neither very weak or very strong states are good for the economy.

[2] Besley and Persson (2008) suggests that external conflicts serve as an exogenous impulse that increases the tax revenue, allowing a country to increase its investment in its fiscal capacity. They find that civil wars lead to smaller investments in fiscal capacity, whereas prospects of external war generally lead to larger investments.

divergence. The poorest of the poor countries thus stagnate while the rest of the world continues to grow. Our wider focus is on the set of conditions that trap a country or a region in a vicious cycle to poverty, independent of the inefficiencies and corruption of the government. The paper thus aims to explore why even conscientious governments in the poor countries may get caught in a poverty trap, in spite of some access to global capital markets.

The piracy emanating out of Somalia is an interesting case to look at when examining the relationship between the fiscal capacity of state and the private economic activity. Somalia is one of the most wretched countries in the world and in the recent past, has become synonymous with piracy. Even through the popular perception is that Somalia is a country that is uniformly chaotic without any significant institutions, the reality is that there are actually three distinct parts of Somalia with very different types of governments and only one of them has become a base for piracy (Bahadur, 2011).

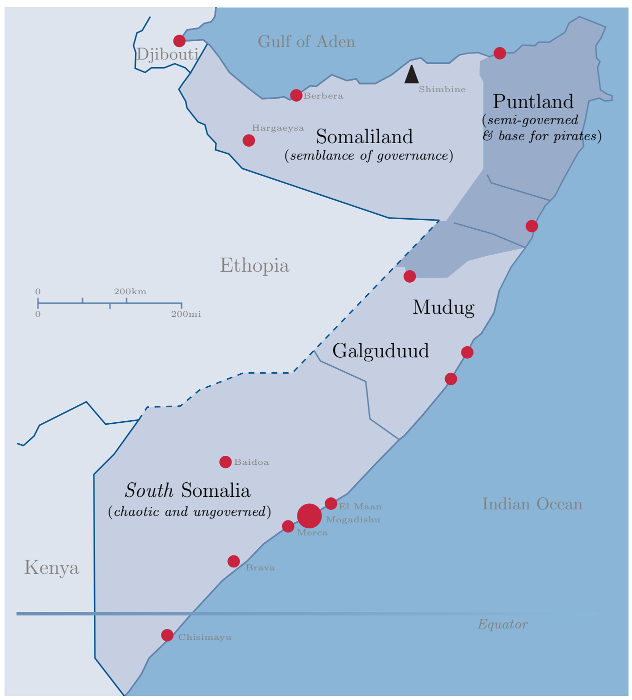

Figure 1. Semi-autonomous regions of Somalia

There is the chaotic *South Somalia* that has almost no semblance of a government. *Somaliland*, on the other hand, has semblance of a government that has some fiscal capacity and some ability to impose its will. *Puntaland* is semi-governed which is stable enough for some economic activity to take place but with a state that is not effective even by the standards of a developing country. The fiscal capacity of the government is less than Somaliland but more than the chaotic South Somalia.

In his book *Deadly Waters*, Bahadur finds that out of the three regions, Puntaland presented the prefect environment for piracy to flourish. According to Bahadur (2011), contrary to popular perception, the piracy is carried out by number of units that have requisite training for the pirates, paramilitary-like structure within the units and equipment, which requires significant amount of upfront investment by entrepreneurs.

> "Puntaland was the perfect area for the pirates to operate because it's just stable enough. You don't have the chronic instability that you have further south ..."[3]
>
> "Somaliland, in contrast, possess a Gulf of Aden coastline comparable to Puntland's, yet the few pirates originating from the region have been swiftly arrested and incarcerated by the local authorities. The difference is due to Somaliland's greater political stability, a product of its robust history of democracy and inter-clan consensus. ... In the south, in short, the pirates had to fear other criminals; in Somaliland, the danger came from a more traditional source: the police." (Bahadur, 2011, page 39)

The flow of capital has been able to exploit economic opportunity in one of the most desperate environments in the world. It suggests that there is some complementarity between the fiscal capacity of the state and the type of economic activity. There is more conventional economic activity in Somaliland but even Puntaland, with its nascent government structure are able to support some economic activity that requires upfront investment. Even though this example presents selective anecdotal evidence, it is interesting because it suggests that all economic activity requires some semblance of either a government structure or provision of public goods before it can flourish.

Given the level of fiscal capacity, we can always rely on human ingenuity to find economic activity that can exist in conjunction with the available public goods. Conversely, the economic activity shapes the incentives for accumulation of public capital. Thus, *poverty traps for economy results from inter-dependent relationship between public capital and private economic activity.*

Lucas (1990) looks at private capital flows but the same question can be rephrased in term of why does capital not flow to the governments of poor countries for accumulation of public goods. Trefler (1993) and Caselli and Feyrer (2007) find very little difference in returns to capital at the margin across across the countries. Yet, at the same time, there is considerable variation in the stock of infrastructure capital across the world. Why can't the conscientious governments of poor countries borrow and invest in public capital stock?

The fact that capital flows are dominated either between the developed countries or from the developed to emerging countries suggests that the returns to capital maybe increasing in capital stock. Only countries that cross a threshold in terms of capital stock can attract the international flow of capital and the flow of capital is dominated by the countries that have the highest capital stock.

In our model, each country has two types of capital stock, public (infrastructure) and private capital stock. The two types of capital stock are complementary with respect to each other. The process that determines the accumulation of one type of capital stock is influenced by the level of the other type of capital stock. The accumulation of private capital stock moves hand in hand with public capital stock.

The poor countries are the ones that are not able to accumulate sufficient infrastructure capital stock. Their low infrastructure capital stock keeps their marginal product of private capital low. Consequently, they fail to attract any capital flows. Conversely, the marginal capital stock is higher

[3] Quote from Roger Middleton from Chatham House in (Bahadur, 2011, page 39)

in the developed and emerging countries. The objective of the paper is to look at why capital flows do not allow the poor countries to accumulate sufficient infrastructure capital stock and kick start the process of development.

Even though we constantly refer to public capital as infrastructure, investment in public capital can easily be interpreted as the cost the government of a country has to bear to create an effective bureaucracy or judicial system. Like infrastructure, once created, the government has to bear the cost of maintaining these institutions. Thus, we can take interpret the stock of public capital as a concept synonymous with the fiscal capacity of the state.

The paper models poverty trap with an aggregate production function that is convex in both public infrastructure and private capital. It examines the role capital markets can play in reducing the extent of the poverty trap, allowing poor countries at the margin to escape. The poverty trap in our model emerge only when the return to capital, widely defined as including both public and private capital stock, is increasing in capital.

The supply of capital from the global financial markets is homogenous. The capital is then differentiated by the end users. The end users in the model are the government and the private entrepreneurs, who use the capital for accumulating infrastructure[4] and private capital stock respectively. For poverty traps to exist, the returns at the margin need to be increasing in the total capital stock. At the margin, the returns to both, infrastructure and private capital, are decreasing on their own.

If returns to the total capital stock is increasing there is a *low stable steady-state* and a *high unstable steady-state*, in terms of per-capita income. The low stable steady state has a catchment area which we pin down. An economy that gets caught in the *deadly waters*, that is the catchment area surrounding the low steady-state, cannot escape and would converge to a low per-capita income. If the economy starts away from the catchment area, it grows perpetually. Conversely, if the returns to total capital stock is diminishing, there a unique *stable steady-state* and the economy converges to this steady state.

Papers like Galor and Zeira (1993) Aghion and Bolton (1997) and Matsuyama (2000) have previously modelled poverty traps as a result of credit frictions in presence of non-convexities technology. In this paper, the technology is convex. We attempt to explain the bi-modal distribution obtained by Quah (1997), while keeping the model as close as possible to the Solow model.

In Section 3.1, we model the economy in a co-operative environment under autarky. The government and the entrepreneurs cooperatively decide the proportion of output that should be allocated to infrastructure and the proportion that should be allocated to private investment. In Section 3.2, the agents in the economy have access to global capital markets that are perfect, i.e., they have no information or enforcement problem. The government and agents can borrow as much as they like. In Section 3.3 we model the economy in a non-co-operative environment under autarky. The government imposes a tax on entrepreneurs and funds the infrastructure investment in the economy through those tax revenues. In section 4 we model the economy in a non-co-operative environment with access to an imperfect global markets. Like Section 3.3, the infrastructure is financed through

[4]We make a strong assumption that the government is conscientious and invests all its resources into public goods and does not divert any resource.

tax revenues. There is an enforcement problem associated with debt contracts, which implies that the government and entrepreneurs are constrained in the amount they borrow.

Unsurprisingly, in all four specifications, there is a poverty trap or a catchment area of the low steady state, if the returns to total capital stock are increasing. With access to perfect capital markets, the catchment area is increasing in global interest rate. In autarky, the catchment area is decreasing in the saving rate and has a non-monotonic relationship with the tax rate. When the economy has access to imperfect capital market, as in the earlier specification, the catchment area is decreasing in the saving rate and has a non-monotonic relationship with the tax rate. The catchment area is also decreasing in both the government and the private entrepreneurs ability to borrow and increasing in global interest rate. This suggests that imperfections in the capital market may worsen the situation of the poor countries by increasing the catchment area but these frictions are not the fundamental cause of the poverty trap. The fundamental cause of the poverty trap remain the increasing returns to the total capital stock. Having said that, easing the borrowing restrictions for one category of end-users has a impact on both types of capital stock.

2. The Environment

The economy has a constant population that is normalised to 1. The agents consume s proportion of their income and consume the rest. There is no exogenous or endogenous technological progress in the economy. The output of the economy y is a function of the installed public capital $\bar{k}$ and private capital k in the economy.

$$y = \bar{k}^{\beta} \cdot k^{\alpha} \tag{1}$$

where $\beta \in (0, 1)$ and $\alpha \in (0, 1)$. We assume that private capital k is exclusively installed by the entrepreneurs and public capital $\bar{k}$ is exclusively installed by the government in the economy.

We normalise the production function so that $\bar{k} = 0$, $k = 0$ and $y = 0$ represents the point where production takes place without any capital and just using natural resources, i.e., the circumstances as they exist in the poorest countries in the world where there is almost no capital used in production process.

The production function may or may not have diminishing returns to capital depending on the value of $\alpha + \beta$. We assume that both $\bar{k}$ and k depreciate at an identical rate δ.

3. Benchmark Models

3.1. **Autarky with Cooperative Decision on Capital Allocation.** In an ideal world, the society collectively would choose how to allocate total capital stock between private and public. Let the proportion of the total capital k^T society *chooses* to allocate to infrastructure be $(1 - \lambda)$. This would imply that $\bar{k} = (1 - \lambda)k^T$. The rest of the capital stock is used for private production in the economy.[5] The production function of the economy is $y = \left[(1 - \lambda)k^T\right]^{\beta} \left[\lambda k^T\right]^{\alpha} = \phi(\lambda){k^T}^{\alpha+\beta}$

[5]Think of this as everyone in the society donating a λ proportion of their capital for public use.

which can be written as

$$y = \phi(\lambda) \cdot \left(k^T\right)^{\beta+\alpha}$$

where $\phi(\lambda) = (1-\lambda)^{\beta}\,\lambda^{\alpha}$. The growth of k^T is given by $\frac{\Delta k^T}{k^T} = \left[s\left(y/k^T\right) - \delta\right] = \left[s\phi(\lambda)\left(k^T\right)^{\alpha+\beta-1} - \delta\right]$. Proposition 1 gives us the steady-state conditions.

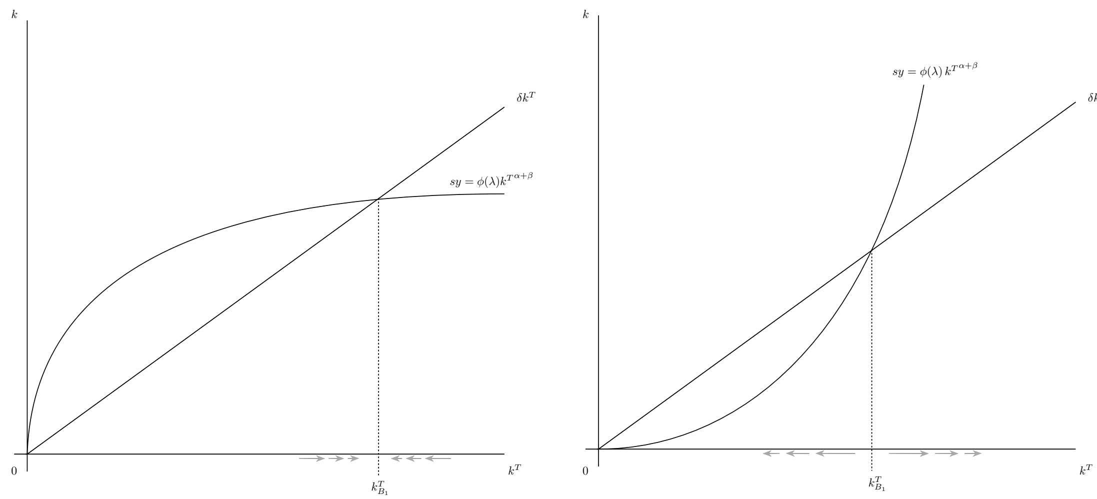

FIGURE 2. Capital Accumulation process in autarky and cooperation with $\alpha+\beta < 1$ and $\alpha+\beta > 1$ respectively

Proposition 1. *In autarky, if the agents collectively allocate capital between infrastructure and private investment, the economy would have following steady-state(s).*

i. *If $(\alpha+\beta) < 1$, there would be a unique stable steady-state at $k^T = k^T_{B_1}$ where*

$$k^T_{B_1} = \left[\frac{s\phi(\lambda)}{\delta}\right]^{\frac{1}{1-(\alpha+\beta)}} \tag{2}$$

ii. *If $(\alpha+\beta) > 1$, there is a stable steady at $k^T_A = 0$ and an unstable steady-state at $k^T_{B_1}$.*

iii. *If $(\alpha+\beta) = 1$, there is a unique steady at $k^T_A = 0$, which is stable if $s\phi(\lambda) < \delta$ and unstable if $s\phi(\lambda) > \delta$. If $s\phi(\lambda) = \delta$, there is a continuum of steady-states in the range $k \in [0, \infty)$.*

Proposition 1 holds if the conditions laid out in section A.1 are imposed on the production function.

If $(\alpha+\beta) < 1$, the per-capita production function is *concave* in total capital stock k and the marginal product of total capital stock is decreasing in k. $k^T_{B_1}$ is the only steady-state, which is also stable.[6] Conversely, if $(\alpha+\beta) > 1$, the per-capita production function is *convex* in k and the marginal product of capital is increasing in k. The economy would have a stable steady-state at $k^T_A = 0$ and an unstable steady-state at $k^T_{B_1}$.[7] The stable steady-state has a *catchment area* of

[6] $g_k > 0$ if $k < k_1^*$ and $g_k < 0$ if $k > k_1^*$.
[7] $g_{k^T} < 0$ if $0 < k' < {k^T}^*_3$ and $g_{k^T} > 0$ if ${k^T}^*_3 < k^T$.

$k^T \in [0, k^T_{B_1})$. In this area, the economy experiences negative growth till it converges to k^T_A. If the economy starts from $k^T \in [k^T_{B_1}, \infty)$, it experiences positive growth perpetually.

The objective of the society depends on whether the returns to capital are diminishing or not. If the returns to capital are diminishing, the per-capita output of the economy would converge to $y = \left(k^T_{B_0}\right)^{\alpha+\beta}$ and the society would like to maximise the per-capita output. Conversely, if the returns to capital was non-diminishing, the society would like to minimise the catchment area $k^T \in [0, k^T_{B_1})$.

Using Equation (2), we find that the output at $k^T_{B_1}$ is given the expression below.

$$y_{B_1} = y(k^T_{B_1}) = \left[\phi(\lambda)\left(\frac{s}{\delta}\right)^{\alpha+\beta}\right]^{\frac{1}{1-(\alpha+\beta)}}$$

λ through $\phi(\lambda)$ determines the per-capita output of the economy. $\phi^* = \phi\left(\frac{\alpha}{\alpha+\beta}\right)$ maximises the per-capita output of the economy if $\alpha + \beta < 1$ and minimises the catchment area if $\alpha + \beta > 1$. Under both conditions, the society should allocate $\frac{\beta}{\alpha+\beta}$ proportion of the their total capital stock k^T to infrastructure and use $\frac{\alpha}{\alpha+\beta}$ proportion of k^T for private investment in the economy. We summarise this discussion with Lemma 3.1.

Lemma 3.1. *If the society were to choose collectively and cooperatively to maximise the output per-capita if $\alpha+\beta < 1$, or minimise the catchment area if $\alpha+\beta > 1$, they would allocate $(1-\lambda) = \frac{\beta}{\alpha+\beta}$ proportion of capital to infrastructure and $\lambda = \frac{\alpha}{\alpha+\beta}$ proportion of capital to private investment.*

See section A.2 for a proof. The optimal $\phi(\lambda)$ is $\phi^* = \left(\frac{\beta}{\alpha+\beta}\right)^{\beta}\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}$. The total per capita capital stock and output with $\phi(\lambda) = \phi^*$ is given below.

$$k^T_{B_1}\Big|_{\lambda=\frac{\alpha}{\alpha+\beta}} = \left[\left(\frac{s}{\delta}\right)\cdot\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}$$

$$y_{B_1}\Big|_{\lambda=\frac{\alpha}{\alpha+\beta}} = \left[\left(\frac{s}{\delta}\right)^{\alpha+\beta}\cdot\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}$$

3.2. **Perfect Global Capital Markets.** In this section we assume that global capital markets are perfectly elastic at the prevailing global interest rate r and the demand for k and $\bar{k}$ is given by their respective net marginal products of capital. With perfect capital mobility, both the public capital stock $\bar{k}$ and the private capital stock k are determined by their respective market clearing conditions.

The entrepreneurs borrow for private investment and the government borrow to invest in infrastructure. Consequently, both the government and the private entrepreneurs choose simultaneously and non-cooperatively and take each other's actions as given.

Capital would flow in or out of the economy till the net marginal product of capital in both sectors are equal to the global interest rate r. It is useful to note that given r, k and $\bar{k}$ get set simultaneously as both the infrastructure and private markets for capital clear simultaneously.

Capital would flow in or out of the economy till the net marginal product of private investment is equal to r, i.e., $\frac{\partial y}{\partial k} = \frac{\alpha \bar{k}^{\beta}}{k^{1-\alpha}} = r + \delta$. This gives us the following market clearing condition.

$$k(\bar{k}) = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}} \tag{3}$$

In Equation (3), k is increasing in $\bar{k}$ and decreasing in r and it is concave in $\bar{k}$ if $\alpha + \beta < 1$ and convex if $\alpha + \beta > 1$.[8] Similarly, the market clearing condition for infrastructure sector gives us the following condition.

$$\bar{k}(k) = \left(\frac{\beta}{r+\delta}\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \tag{4}$$

In Equation (4), $\bar{k}$ is increasing in k and decreasing in r and it is concave in k if $\alpha + \beta < 1$ and convex if $\alpha + \beta > 1$. k and $\bar{k}$ are complementary factors. That is marginal product of private capital is increasing in $\bar{k}$[9] and marginal product of infrastructure capital is increasing in k.

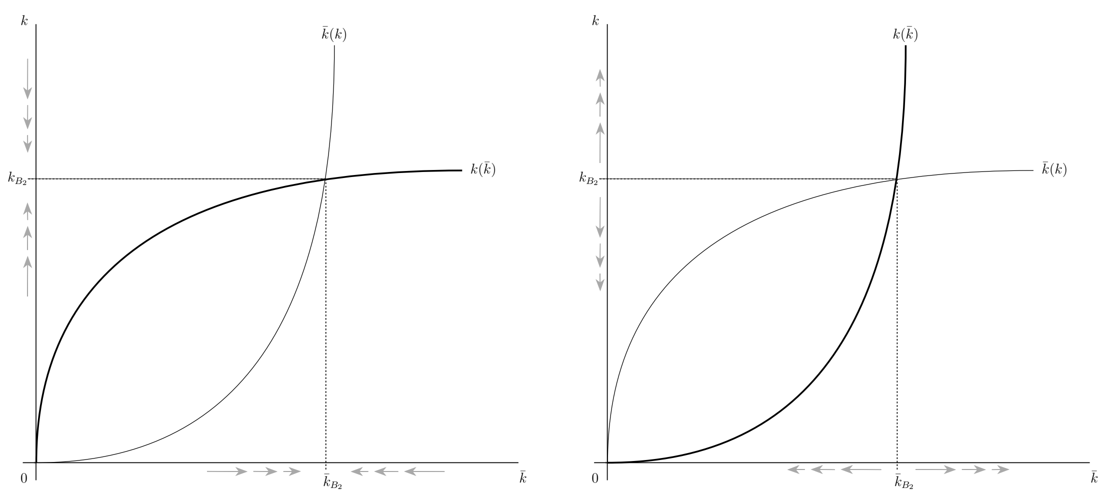

FIGURE 3. Perfect Capital Markets: $k(\bar{k})$ and $\bar{k}(k)$ reaction curves when $\alpha + \beta < 1$ and $\alpha + \beta > 1$

With perfect global capital markets, the economy would have following steady-state(s).

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[8]From equation (3), the market clearing condition for private capital, we get

$$\frac{\partial k(\bar{k})}{\partial \bar{k}} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \left(\frac{\beta}{1-\alpha}\right) \bar{k}^{\frac{\alpha+\beta-1}{1-\alpha}}$$

$$\frac{\partial^2 k(\bar{k})}{\partial \bar{k}^2} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \left(\frac{\beta}{1-\alpha}\right) \left(\frac{\alpha+\beta-1}{1-\alpha}\right) \bar{k}^{\frac{2\alpha+\beta-2}{1-\alpha}} \quad \begin{cases} > 0 & \alpha+\beta > 1 \\ < 0 & \alpha+\beta < 1 \end{cases}$$

Similarly, from equation (4), we get $\frac{\partial^2 k}{\partial \bar{k}(k)^2} k > 0$ if $\alpha + \beta > 0$ and $\frac{\partial^2 k}{\partial \bar{k}(k)^2} k < 0$ if $\alpha + \beta < 0$.

[9]k and $\bar{k}$ are complementary.

$$\frac{\partial^2 y}{\partial k \partial \bar{k}} = \frac{\alpha}{k^{1-\alpha}} \frac{\beta}{\bar{k}^{1-\beta}} > 0$$

Proposition 2 gives us the condition for the steady-state of the economy.

Proposition 2. *With perfect global capital markets, the economy would have following steady-state(s).*

i. *If* $\alpha + \beta > 1$ *the economy has a stable steady-state at* $(k_A, \bar{k}_A) = (0, 0)$ *and a unstable steady-state at* $(k_{B_2}, \bar{k}_{B_2})$ *where*

$$k_{B_2} = \left[\frac{\alpha^{(1-\beta)}\beta^{\beta}}{r+\delta}\right]^{\frac{1}{1-(\alpha+\beta)}}$$
$$\bar{k}_{B_2} = \left[\frac{\beta^{(1-\alpha)}\alpha^{\alpha}}{r+\delta}\right]^{\frac{1}{1-(\alpha+\beta)}} \tag{5}$$

ii. *If* $\alpha + \beta < 1$*, the economy has a unique stable steady-state at* $(k_{B_2}, \bar{k}_{B_2})$

iii. *If* $\alpha + \beta = 1$*, then there is stable steady-state at* $(k_A, \bar{k}_A) = (0, 0)$ *if* $r + \delta > \alpha^{1-\beta}\beta^{1-\alpha}$ *and unstable steady-state at* $(k, \bar{k}) = (0, 0)$ *if* $r+\delta < \alpha^{1-\beta}\beta^{1-\alpha}$. *If* $r+\delta = \alpha^{1-\beta}\beta^{1-\alpha}$*, then there is a continuum of steady-states at* $k = \bar{k}$*, where* $k = [0, \infty)]$.

If $\alpha + \beta < 1$, then per-capita output of the economy converges to an unique steady-state level $(k_{B_2}, \bar{k}_{B_2})$ where the economy experiences no growth. Conversely, if $\alpha + \beta > 1$, there are two steady-states $(k_A, \bar{k}_A)$ and $(k_B, \bar{k}_{B_2})$. If $k \in [0, k_{B_2})$ and $\bar{k} \in (0, \bar{k}_{B_2})$, then the economy converge to the stable steady-state at $(k_A, \bar{k}_A) = (0, 0)$. Conversely, if $k \in (k_{B_2}, \infty)$ and $\bar{k} \in (\bar{k}_{B_2}, \infty)$, then the economy will experience perpetual growth.

Thus, if $\alpha + \beta > 1$, $k \in [0, k_{B_2})$ and $\bar{k} \in (0, \bar{k}_{B_2})$ is the *catchment area* for the steady-state $(k_A, \bar{k}_A) = (0, 0)$. Beyond this catchment area, the economy experiences perpetual growth. We can summarise the discussion above with Corollary 3.1.

Corollary 3.1. *With perfect capital mobility,*

i. *if* $\alpha + \beta < 1$*, the economy will converge to a stable steady-state* $(k_{B_2}, \bar{k}_{B_2})$ *where*

$$y(\bar{k}_{B_2}, k_{B_2}) = \left[\frac{\beta^{\beta}\alpha^{\alpha}}{(r+\delta)^{\alpha+\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}$$

ii. *if* $\alpha + \beta > 1$*,*
   (a) k *and* $\bar{k}$ *will have perpetual negative growth till the economy converges to the stable steady-state* $(k_A, \bar{k}_A) = (0, 0)$ *if* $k \in [0, k_{B_2})$ *and* $\bar{k} \in [0, \bar{k}_{B_2})$
   (b) *there will be no growth in* k *and* $\bar{k}$ *if* $k = k_{B_2}$ *and* $\bar{k} = \bar{k}_{B_2}$
   (c) k *and* $\bar{k}$ *will perpetual positive growth if* $k \in (k_{B_2}, \infty)$ *and* $\bar{k} \in (\bar{k}_{B_2}, \infty)$

For the proof of the Corollary 3.1, see section A.3.

Corollary 3.2. *With perfect global capital markets,*

i. *if* $\alpha + \beta < 1$*, the steady-state output of the economy is decreasing in* r

ii. *if* $\alpha + \beta > 1$*, the catchment area for the economy is increasing in* r*.*

We find that both k_{B_2} and $\bar{k}_{B_2}$ are increasing in r if $\alpha+\beta > 1$ and decreasing in r if $\alpha+\beta < 1$.[10] This implies that if $\alpha + \beta < 1$, the steady state output of the economy decreases if the interest interest rate increases. Conversely, if $\alpha+\beta > 1$, the catchment area of the low steady-state $(k_A, \bar{k}_A)$ increases if the interest rate goes up. In Figure 4, a rise in interest rate increases the threshold (from A to B) beyond which there is perpetual growth in the economy.

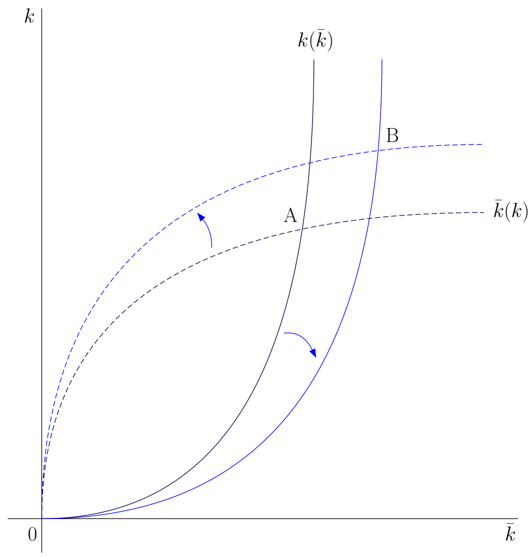

FIGURE 4. Perfect Capital Flow: Effect of increase in r when $\alpha + \beta > 1$

We can relate this to the recent experience of the BRIC countries. These countries seemed to have sustained a high growth rate during the great moderation, when the interest rates were low. These were also countries with better infrastructure than other low income countries to start with. The low sustained interest rates during the great moderation enabled these countries to attract both foreign direct investment and infrastructure investment. Once they have put sufficient infrastructure in place, an increase in r would not stop the perpetual growth if the infrastructure and private sector capital of the economy is beyond the threshold given by equation 5.

Perfect capital mobility determines the per-capita private and infrastructure capital stock of the economy, which in turn determines the per-capita output of the economy.

$$y_{B_2} = y(\bar{k}_{B_2}, k_{B_2}) = \left[\frac{\beta^{\beta}\alpha^{\alpha}}{(r+\delta)^{\alpha+\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}} \tag{6}$$

If $\alpha+\beta < 1$, then (6) gives us the steady state per-capita output level of the economy. Conversely, if $\alpha + \beta > 1$, then (6) gives us the per-capita output threshold beyond which the economy will

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$$\frac{\partial k_{B_2}(r)}{\partial r} = \frac{1}{(\alpha+\beta)-1}\left(\frac{1}{\alpha^{1-\beta}\beta^{\beta}}\right)^{\frac{1}{(\alpha+\beta)-1}}(r+\delta)^{\frac{2-(\alpha+\beta)}{(\alpha+\beta)-1}}\begin{cases} >0, & \text{if } \alpha+\beta>1 \\ <0, & \text{if } \alpha+\beta<1 \end{cases}$$

$$\frac{\partial \bar{k}_{B_2}(r)}{\partial r} = \frac{1}{(\alpha+\beta)-1}\left(\frac{1}{\beta^{1-\alpha}\alpha^{\alpha}}\right)^{\frac{1}{(\alpha+\beta)-1}}(r+\delta)^{\frac{2-(\alpha+\beta)}{(\alpha+\beta)-1}}\begin{cases} >0, & \text{if } \alpha+\beta>1 \\ <0, & \text{if } \alpha+\beta<1 \end{cases}$$

experience perpetual growth. Further, (6) is increasing in r if $\alpha+\beta>1$ and decreasing in r if $\alpha+\beta<1$.[11]

3.3. **Autarky with Taxes and Non-Co-operation.** In this section, we analyse the evolution of k and $\bar{k}$ in autarky. We continue with the dichotomy where the government only invests in infrastructure and entrepreneurs only invest in private private firms. The government and the private sector make their respective decisions non-co-operatively.

The government pays for infrastructure by imposing a proportional tax t on the entrepreneurs. The per-capita saving of entrepreneurs is given by $s_p = s(1-t)y$ and the government is able to garner resources to the tune of $s_g = ty$ to spend on infrastructure. To keep things simple, we keep the assumption that both $\bar{k}$ and k depreciates at the same rate δ.

The capital accumulation conditions for k and $\bar{k}$ are given by $\Delta k = s_p - \delta k = s(1-t)y - \delta k$ and $\Delta\bar{k} = s_g - \delta k = ty - \delta k$ respectively. Imposing $\Delta k = \Delta\bar{k} = 0$ gives us the following two conditions that describe the evolution of infrastructure and private capital stock in the economy.

$$\begin{aligned} k &= \left(\frac{s(1-t)}{\delta}\right)^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}} \\ \bar{k} &= \left(\frac{t}{\delta}\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \end{aligned} \tag{7}$$

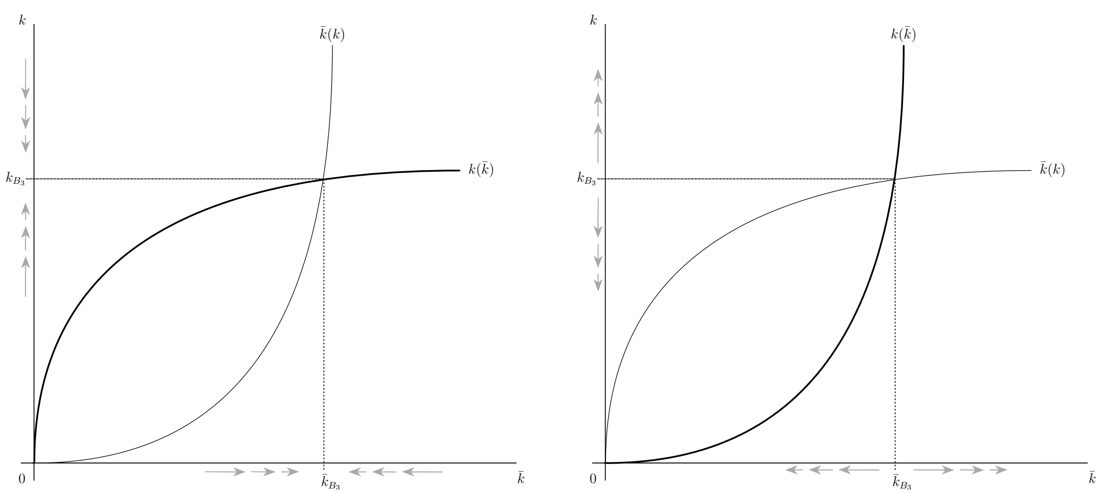

FIGURE 5. Autarky with Non-Co-operative Capital Allocation: $k(\bar{k})$ and $\bar{k}(k)$ reaction curves when $\alpha+\beta<1$ and $\alpha+\beta>1$

This economy has an unique stable steady-state if $\alpha+\beta<1$ and one stable and one unstable steady-state if $\alpha+\beta>1$. We summarise with Proposition 3.

Proposition 3. *In autarky with infrastructure funded by a proportional tax, the economy would have following steady-state(s).*

[11]The sign of $\frac{dy}{dr} = (\alpha^\alpha\beta^\beta)^{\frac{1}{(\alpha+\beta)-1}}\left[\frac{\alpha+\beta}{(\alpha+\beta)-1}\right] r^{\frac{1}{(\alpha+\beta)-1}}$ is determined by the sign of $(\alpha+\beta)-1$.

i. *If* $\alpha+\beta>1$ *the economy has a stable steady-state at* $(k_A,\bar{k}_A)=(0,0)$ *and a unstable steady-state at* $(k_{B_3},\bar{k}_{B_3})$ *where*

$$\begin{aligned} k_{B_3} &= \left[s^{1-\beta}\cdot\frac{(1-t)^{1-\beta}t^{\beta}}{\delta}\right]^{\frac{1}{1-(\alpha+\beta)}} \\ \bar{k}_{B_3} &= \left[s^{\alpha}\cdot\frac{(1-t)^{\alpha}t^{1-\alpha}}{\delta}\right]^{\frac{1}{1-(\alpha+\beta)}} \end{aligned} \tag{8}$$

ii. *If* $\alpha+\beta<1$*, the economy has a unique stable steady-state at* $(k_{B_3},\bar{k}_{B_3})$

iii. *If* $\alpha+\beta=1$*, then there is stable steady-state at* $(k_A,\bar{k}_A)=(0,0)$ *if* $\frac{t^{1-\alpha}}{(1-t)^{1-\beta}}<\delta^{\alpha-\beta}\left(\frac{1}{s}\right)^{1-\beta}$ *and unstable steady-state at* $(k,\bar{k})=(0,0)$ *if* $\frac{t^{1-\alpha}}{(1-t)^{1-\beta}}>\delta^{\alpha-\beta}\left(\frac{1}{s}\right)^{1-\beta}$*. If* $\frac{t^{1-\alpha}}{(1-t)^{1-\beta}}=\delta^{\alpha-\beta}\left(\frac{1}{s}\right)^{1-\beta}$*, then there is a continuum of steady-states at* $k=\bar{k}$*, where* $k=[0,\infty)]$*.*

Corollary 3.3 summarises the growth and convergence properties of this economy.

Corollary 3.3. *In autarky with infrastructure funded by a proportional tax,*

i. *If* $\alpha+\beta<1$*, the economy will converge to a stable steady-state* $(k_{B_3},\bar{k}_{B_3})$ *where*

$$y_{B_3}=y(\bar{k}_{B_3},k_{B_3})=\left(\frac{s^{\alpha}}{\delta^{\alpha+\beta}}\cdot(1-t)^{\alpha}t^{\beta}\right)^{\frac{1}{1-(\alpha+\beta)}} \tag{9}$$

ii. *if* $\alpha+\beta>1$*,*
   (a) k *and* $\bar{k}$ *will have perpetual negative growth till the economy converges to the stable steady-state* $(k_A,\bar{k}_A)=(0,0)$ *if* $k\in[0,k_{B_3})$ *and* $\bar{k}\in[0,\bar{k}_{B_3})$
   (b) *there will be no growth in* k *and* $\bar{k}$ *if it starts from* $k=k_{B_3}$ *and* $\bar{k}=\bar{k}_{B_3}$
   (c) k *and* $\bar{k}$ *will perpetual positive growth if it starts from* $k\in(k_{B_3},\infty)$ *and* $\bar{k}\in(\bar{k}_{B_3},\infty)$

y_{B_3} if the steady state level of per-capita output for the economy if $\alpha+\beta<1$ and increasing in s and $\phi(1-t)$ if $\alpha+\beta<1$. y_{B_3} is the threshold beyond which the economy grows perpetually if $\alpha+\beta>1$ and decreasing in s and $\phi(1-t)$ if $\alpha+\beta<1$

Corollary 3.4. *In autarky,*

i. *if* $\alpha+\beta<1$*, the steady-state output of the economy is increasing in* s
ii. *if* $\alpha+\beta>1$*, the catchment area for the economy is decreasing* s*.*

4. Capital Markets with Enforcement Problem

The credit market has an *enforcement problem*, where the lenders find it difficult to enforce contracts and extract the full repayment from the borrower. If the borrower chooses to default, then lender can only wrench μ proportion of the output from the borrower. We assume that the borrower can choose to default but cannot choose not to pay their taxes.

If an entrepreneur borrows b at interest rate r and the lender can only wrench μ_p proportion of her output, she will only repay if $rb\leqslant\mu_p(1-t)y$. b_p, the maximum amount that the borrower can borrow from the capital markets, is given by

$$b_p = \frac{\mu_p(1-t)y}{r} \tag{10}$$

Similarly, if the government borrows b at interest rate r and the lender can only wrench μ_g proportion of the government's tax revenues, it will only repay if $rb \leqslant \mu_g ty$. b^g, the maximum amount that the borrower can borrow from the capital markets, is given by

$$b_g = \frac{\mu_g ty}{r} \tag{11}$$

If private entrepreneur's borrow b_p or less and government borrows b_g or less, it is in their interest to repay back and there would be no defaults on loans in equilibrium.

We continue with the assumption that there is a clear dichotomy where only entrepreneurs invest in private capital and government invests only in infrastructure. The government taxes the individual entrepreneur at the rate t and the entrepreneurs are able to keep $(1-t)$ for themselves. Thus, the entrepreneur's savings are given by $s_p = s(1-t)y$ and government's available resources for infrastructure investment are $s_g = ty$. Both infrastructure and private capital stock depreciate at the same rate δ.

Evolution of k and $\bar{k}$ are determined by $\Delta k = \left(s + \frac{\mu_p}{r}\right)(1-t)y - \delta k$ and $\Delta \bar{k} = \left(1 + \frac{\mu_g}{r}\right) ty - \delta \bar{k}$. Imposing the condition that $\Delta k = \Delta \bar{k} = 0$ gives us the following conditions for k and $\bar{k}$.

$$\begin{aligned} k &= \left[\left(s + \frac{\mu_p}{r}\right) \frac{(1-t)}{\delta}\right]^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}} \\ \bar{k} &= \left[\left(1 + \frac{\mu_g}{r}\right) \cdot \frac{t}{\delta}\right]^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \end{aligned} \tag{12}$$

For a given $\bar{k}$, k is higher in (12) than in (7) if $s + \frac{\mu_p}{r} > 1$ or $\mu_p > r(1-s)$. Access to global capital markets stimulates the private investment if μ_p is higher than the threshold that is increasing in r and decreasing in s.

Similarly, for a given k, $\bar{k}$ is higher in (12) than in (7) if $1 + \frac{\mu_g}{r} > 1$ or $\mu_g > 0$. Access to global capital markets stimulate the infrastructure investment if $\mu_g > 0$.

Proposition 4 describes the steady-state of the economy.

Proposition 4. *With an enforcement problem in the global capital markets and infrastructure funded by a proportional tax,*

i. *if $\alpha + \beta > 1$, the economy has a stable steady-state at $(k_A, \bar{k}_A) = (0, 0)$ and a unstable steady-state at $(k_{B_4}, \bar{k}_{B_4})$ where*

$$\begin{aligned} k_{B_4} &= \left[\left(s + \frac{\mu_p}{r}\right)^{1-\beta} \left(1 + \frac{\mu_g}{r}\right)^{\beta} \frac{(1-t)^{1-\beta} t^{\beta}}{\delta}\right]^{\frac{1}{1-(\alpha+\beta)}} \\ \bar{k}_{B_4} &= \left[\left(s + \frac{\mu_p}{r}\right)^{\alpha} \left(1 + \frac{\mu_g}{r}\right)^{1-\alpha} \frac{(1-t)^{\alpha} t^{1-\alpha}}{\delta}\right]^{\frac{1}{1-(\alpha+\beta)}} \end{aligned} \tag{13}$$

ii. *if $\alpha + \beta < 1$, the economy has a unique stable steady-state at $(k_{B_4}, \bar{k}_{B_4})$*

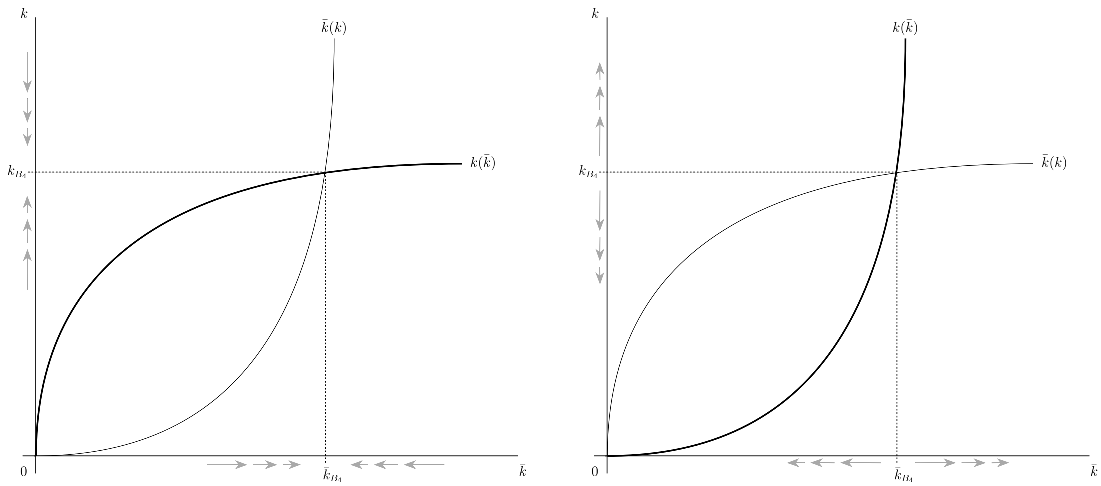

FIGURE 6. Capital Markets with Enforcement Problem: $k(\bar{k})$ and $\bar{k}(k)$ reaction curves when $\alpha + \beta < 1$ and $\alpha + \beta > 1$

iii. *if* $\alpha + \beta = 1$, *then there is stable steady-state at* $(k_A, \bar{k}_A) = (0, 0)$ *if* $t < \frac{sr+\mu_p}{(1+s)r+2\mu_g}$ *and unstable steady-state at* $(k, \bar{k}) = (0, 0)$ *if* $t > \frac{sr+\mu_p}{(1+s)r+2\mu_g}$. *If* $t = \frac{sr+\mu_p}{(1+s)r+2\mu_g}$, *then there is a continuum of steady-states at* $k = \bar{k}$, *where* $k = [0, \infty)]$.

If there is no access to credit, then the whole problem reduces to the non-co-operative problem in autarky. To see this, if we set $\mu_p = \mu_g = 0$, Condition 12 and Condition 13 reduce to Condition 7 and Condition 5 in section 3.3. From Equation (13) we can see that $t = \beta$ maximises k_{B_4} if $\alpha + \beta < 1$ and minimises it if $\alpha + \beta > 1$. Similarly, $t = 1 - \alpha$ maximises k_{B_4} if $\alpha + \beta < 1$ and minimised it if $\alpha + \beta > 1$.

Corollary 4.1 describes the convergence properties of the economy.

Corollary 4.1. *With an enforcement problem in the global capital markets and infrastructure funded by a proportional tax,*

i. *if* $\alpha + \beta < 1$, *the economy will converge to a stable steady-state* $(k_{B_4}, \bar{k}_{B_4})$ *where*

$$y_{B_4} = y(\bar{k}_{B_4}, k_{B_4}) = \left[\left(s + \frac{\mu_p}{r}\right)^{\alpha} \left(1 + \frac{\mu_g}{r}\right)^{\beta} \cdot \frac{(1-t)^{\alpha} t^{\beta}}{\delta^{\alpha+\beta}} \right]^{\frac{1}{1-(\alpha+\beta)}} \tag{14}$$

ii. *if* $\alpha + \beta > 1$,

(a) k *and* $\bar{k}$ *will have perpetual negative growth till the economy converges to the stable steady-state* $(k_A, \bar{k}_A) = (0, 0)$ *if* $k \in [0, k_{B_4})$ *and* $\bar{k} \in [0, \bar{k}_{B_4})$

(b) *there will be no growth in* k *and* $\bar{k}$ *if it starts from* $k = k_{B_4}$ *and* $\bar{k} = \bar{k}_{B_4}$

(c) k *and* $\bar{k}$ *will perpetual positive growth if it starts from* $k \in (k_{B_4}, \infty)$ *and* $\bar{k} \in (\bar{k}_{B_4}, \infty)$

μ_g and μ_p and can be considered leverage factors. Given a level of output, interest rate and tax rate, increase in μ_p and μ_g increase the amount that the government and the private entrepreneurs can borrow.

Using Equation (14), we show in Section A.4 that $\frac{\partial y_{B_4}}{\partial \mu_p} > 0$ if $\alpha+\beta < 1$ and $\frac{\partial y_{B_4}}{\partial \mu_p} < 0$ if $\alpha+\beta > 1$. We also show that $\frac{\partial^2 y_{B_4}}{\partial \mu_p \partial \mu_g} > 0$ holds irrespective of the value of $\alpha + \beta$. This show that steady state output is increase in μ_g and μ_p and the catchment area is decreasing in μ_g and μ_p. Further, the marginal impact each leverage factor has on y_{B_4} is increases if the other leverage factor increases. Thus, an increase in μ_p makes the effect the μ_g has on y_{B_4} more effective and vice-versa.

Corollary 4.2. *When the global capital markets are imperfect,*

i. *if $\alpha+\beta < 1$, the steady-state output of the economy is increasing in μ_p, μ_g, s and decreasing in r.*
ii. *if $\alpha + \beta > 1$, the catchment area for the economy is decreasing in μ_p, μ_g, s and increasing in r.*

See Section A.4 for the derivations.

4.1. **Impact of Access to International Capital Markets.** In this section we evaluate the effect having access to international capital markets has on an economy. In section 4.1 we will see that starting from autarky, gaining access to imperfect capital markets *always increases* the output levels of the an economy if $\alpha + \beta < 1$ and *decreases* the catchment area if $\alpha + \beta > 1$. This is because the supply of capital in autarky is inelastic and access to imperfect capital markets, where the government and the entrepreneurs can leverage their output by leverage factors μ_g and μ_p, augments the supply of capital.

Access to perfect capital markets implies that supply of capital is stock is entirely elastic at r. In autarky, the supply of capital is inelastic and determined by the previous period's output. In section 4.1, when an economy moves from autarky to perfect global capital market, economy's steady state per-capita output increases and the catchment area decreases only if the combination of s and r are below a constant value. Thus, for a given global interest rate, access to perfect capital markets increase the steady output and decreases the catchment area only if the economy saving rate is below a threshold.

In section 4.1, we look at the impact removing imperfection in the capital markets would have on the economy. With imperfect capital market, the supply of capital is inelastic beyond a certain point determined the the tax revenues and the output of the economy and the leverage factors of the government and the entrepreneurs. When we move from an imperfect to perfect global capital market, the economy's steady state per-capita output increases and the catchment area decreases only if if condition in (16) are met, i.e., the combination of s, r, μ_g and μ_p are below a constant value.

Moving from Autarky to having Access to Imperfect Global Capital Markets. By comparing y_{B_4} with y_{B_3}, we can look at the impact access to imperfect global capital markets has on the economy which was previously in autarky.[12] y_{B_3} is the output of the economy in autarky. y_{B_4} is the output of the economy when it has access to global capital markets which have a problem enforcing debt contracts with government and entrepreneurs in the country.

$$\left[\frac{y_{B_4}}{y_{B_3}}\right] = \left[\frac{\left(s+\frac{\mu_p}{r}\right)^{\alpha}\left(1+\frac{\mu_g}{r}\right)^{\beta}}{s^{\alpha}}\right]^{\frac{1}{1-(\alpha+\beta)}} \begin{cases} >1, & \text{if} \quad \alpha+\beta<1 \\ <1, & \text{if} \quad \alpha+\beta>1 \end{cases}$$

For an economy in autarky, access to perfect capital market always increases the per-capita output if $\alpha+\beta<1$ and decreases the catchment area if $\alpha+\beta>1$.[13]

Moving from Autarky to having Access to Perfect Global Capital Markets. By comparing y_{B_2} with y_{B_3}, we can gauge the impact access to perfect global markets has on an economy that previously existed in autarky. y_{B_2} is the output of the economy when it has access to perfect global capital markets.

$$\frac{y_{B_2}}{y_{B_3}} = \left(\frac{\alpha^{\alpha}\beta^{\beta}}{s^{\alpha}(1-t)^{\alpha}t^{\beta}}\frac{1}{\left(\frac{r}{\delta}+1\right)^{\alpha+\beta}}\right)^{\frac{1}{1-(\alpha+\beta)}} \begin{cases} >1, & \text{if} \quad \alpha+\beta<1 \quad \text{and} \quad s^{\frac{\alpha}{\alpha+\beta}}(r+\delta)<\delta\left[\frac{\alpha^{\alpha}\beta^{\beta}}{(1-t)^{\alpha}t^{\beta}}\right]^{\frac{1}{\alpha+\beta}} \\ <1, & \text{if} \quad \alpha+\beta>1 \quad \text{and} \quad s^{\frac{\alpha}{\alpha+\beta}}(r+\delta)<\delta\left[\frac{\alpha^{\alpha}\beta^{\beta}}{(1-t)^{\alpha}t^{\beta}}\right]^{\frac{1}{\alpha+\beta}} \end{cases}$$

For an economy in autarky, gaining access to perfect capital market, increases the per-capita output (if $\alpha+\beta<1$) and decreases the catchment area (if $\alpha+\beta>1$) if the combination of interest rate and saving rate satisfies the following condition.

$$s^{\frac{\alpha}{\alpha+\beta}}(r+\delta)<\delta\left[\frac{\alpha^{\alpha}\beta^{\beta}}{(1-t)^{\alpha}t^{\beta}}\right]^{\frac{1}{\alpha+\beta}} \tag{15}$$

Access to Perfect Global Capital Markets with $t=\frac{\alpha}{\alpha+\beta}$.

$$\left[\frac{y_{B_2}}{y_{B_3}}\right]_{t=\frac{\beta}{\alpha+\beta}} = \left[\left(\frac{\delta(\alpha+\beta)}{(r+\delta)}\right)^{\alpha+\beta}\frac{1}{s^{\alpha}}\right]^{\frac{1}{1-\alpha+\beta}} \begin{cases} >1, & \text{if} \quad \alpha+\beta<1 \quad \text{and} \quad s^{\frac{\alpha}{\alpha+\beta}}(r+\delta)<\delta(\alpha+\beta) \\ <1, & \text{if} \quad \alpha+\beta>1 \quad \text{and} \quad s^{\frac{\alpha}{\alpha+\beta}}(r+\delta)<\delta(\alpha+\beta) \end{cases}$$

s and r now have to satisfy the following condition.

$$s^{\frac{\alpha}{\alpha+\beta}}(r+\delta)<\delta(\alpha+\beta)$$

The new condition is obtained by replacing $(1-t)^{\alpha}t^{\beta}$ with $\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}$ in equation (15).

Impact of the Enforcement Problem. We can look at the impact the enforcement problem has on the economy comparing by y_{B_4} with y_{B_2}.

[12] Assuming that the tax rate does not change.

[13] This is because $\left(1+\frac{\mu_p}{sr}\right)^{\alpha}\left(1+\frac{\mu_g}{r}\right)^{\beta}>1$ holds if μ_p, μ_g, s and r are greater than zero.

$$\left[\frac{y_{B_2}}{y_{B_4}}\right] = \left[\frac{\alpha^\alpha \beta^\beta}{(s+\frac{\mu_p}{r})^\alpha\,(1+\frac{\mu_g}{r})^\beta\,(1-t)^\alpha t^\beta\,\left(\frac{r}{\delta}+1\right)^{\alpha+\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}$$

$$\begin{cases} >1, & \text{if} \quad \alpha+\beta<1 \quad \text{and} \quad \delta\left[\frac{\alpha^\alpha\beta^\beta}{\left(1+\frac{\mu_g}{r}\right)^\beta(1-t)^\alpha t^\beta}\right]^{\frac{1}{\alpha+\beta}} > \left(s+\frac{\mu_p}{r}\right)^{\frac{\alpha}{\alpha+\beta}}(r+\delta) \\ <1, & \text{if} \quad \alpha+\beta>1 \quad \text{and} \quad \delta\left[\frac{\alpha^\alpha\beta^\beta}{\left(1+\frac{\mu_g}{r}\right)^\beta(1-t)^\alpha t^\beta}\right]^{\frac{1}{\alpha+\beta}} > \left(s+\frac{\mu_p}{r}\right)^{\frac{\alpha}{\alpha+\beta}}(r+\delta) \end{cases}$$

For an economy in autarky, gaining access to perfect capital markets, increases the per-capita output (if $\alpha+\beta<1$) and decreases the catchment area (if $\alpha+\beta>1$) if the combination of interest rate and saving rate satisfies the following condition.

$$\delta\left[\frac{\alpha^\alpha\beta^\beta}{(1+\frac{\mu_g}{r})^\beta\cdot(1-t)^\alpha t^\beta}\right]^{\frac{1}{\alpha+\beta}} > \left(s+\frac{\mu_p}{r}\right)^{\frac{\alpha}{\alpha+\beta}}(r+\delta) \tag{16}$$

If $t=\frac{\alpha}{\alpha+\beta}$, then the condition becomes

$$\frac{\delta(\alpha+\beta)}{(1+\frac{\mu_g}{r})^{\frac{\beta}{\alpha+\beta}}} > \left(s+\frac{\mu_p}{r}\right)^{\frac{\alpha}{\alpha+\beta}}(r+\delta)$$

5. Conclusion

The paper attempts to understand why the poorest countries in the world are not able to catch up with the richest countries in the world. We explore the role capital markets could potentially play in facilitating this catch up process.

We modelled the output as a function of public and private capital stock. The government's role in the model is to impose a tax on private entrepreneurs and use the tax revenues to install the infrastructure or public capital stock. The government of the poor countries are able to access the capital markets to borrow and invest in public capital. We make a strong assumption here and assume that the government is always efficient and conscientious, that is, it invests all the tax revenues into the installing public capital.

We were able to show that if returns to total capital stock in the economy was increasing, at low level of per capita income, the economy would be caught in a poverty trap. The size of poverty traps or the catchment area in autarky is decreasing in the saving rate of the economy and with access to perfect capital markets, increasing in global interest rate.

We also analysed the situation where the global capital markets were imperfect due to an enforcement problem and borrowers could leverage their income by a certain leverage factor. In this case, the size of the poverty trap depends on the both the government and entrepreneur's leverage factor. Further, the two leverage factors are complementary. This implies that constraints on government's ability to borrow in the capital markets constrains the entrepreneur's ability to borrow. Similarly, constraints on the entrepreneur's ability to borrow in the capital markets constrains the government's ability to borrow.

Conversely, if the returns to total capital stock in the economy were decreasing, there would be no poverty trap. The economy would converge to the unique steady-state. We find that the steady state per-capita output level is increasing in the saving rate in autarky and decreasing in global interest rate if the global capital markets are perfect. With imperfect interest rate, the steady-state output level is increasing in the two leverage factors.

This paper attempts to explain why middle income economies like Brazil, China and India seem on track to converge with the rest of the OECD countries, where are countries like sub-saharan Africa do not seem to be converging at all. Burgess and Besley (2003) show that poverty alleviation in China and India has far exceeded expectation of the Millennium Development Goal and Sub-Saharan Africa is lagging behind. Similarly, Collier (2007) bemoans the fact that growth in Indian and China has moved the focus away from the Africa, where things have not changed. The model suggests that if returns to the widely defined capital stock is increasing, access to capital global markets only helps the economy has once it has crossed a threshold in terms of both public and private capital.

Appendix A. Proofs

A.1. **Conditions for Proposition 1.** The results in the Proposition 1 hold because y is continuous, differentiable and

i. $\lim_{k^T\to 0} \frac{dy}{dk^T} = \infty$ and $\lim_{k^T\to 0} \frac{dy}{dk^T} = 0$ if $\alpha+\beta<1$
ii. $\lim_{k^T\to 0} \frac{dy}{dk^T} = 0$ and $\lim_{k^T\to 0} \frac{dy}{dk^T} = \infty$ if $\alpha+\beta>1$
iii. $\lim_{k^T\to 0} \frac{dy}{dk^T} = \phi^*(\alpha+\beta)$ and $\lim_{k^T\to 0} \frac{dy}{dk^T} = \phi^*(\alpha+\beta)$ if $\alpha+\beta=1$

A.2. **Optimal λ.**

Optimise $\phi = (1-\lambda)^\beta\lambda^\alpha$ where $\alpha>0$ and $\beta>0$.

$$\frac{d\phi}{d\lambda} = (1-\lambda)^\beta\lambda^\alpha\left[\frac{\alpha}{\lambda}-\frac{\beta}{1-\lambda}\right] = \phi\left[\frac{\alpha}{\lambda}-\frac{\beta}{1-\lambda}\right]=0$$

$$\begin{aligned}\frac{d^2\phi}{d\lambda^2} &= \phi\left[-\left(\frac{\alpha}{\lambda^2}+\frac{\beta}{(1-\lambda)^2}\right)\right]+\left[\frac{\alpha}{\lambda}-\frac{\beta}{1-\lambda}\right]\frac{d\phi}{d\lambda}\\ &= \phi\left[\frac{\alpha(1-\alpha)}{\lambda^2}+\frac{\beta(\beta-1)}{(1-\lambda)^2}-\frac{2\alpha\beta}{\lambda(1-\lambda)}\right]<0\end{aligned}$$

This implies that $\lambda = \frac{\alpha}{\alpha+\beta}$ is a global maxima.

Similarly, for the function $\phi(1-t) = (t)^\beta(1-t)^\alpha$, obtaining $\frac{d\phi(1-t)}{dt}=0$ gives us $(1-t)=\frac{\alpha}{\alpha+\beta}$, which is a global maxima for this function given that $\frac{d^2\phi(1-t)}{dt^2}<0$.

A.3. **Unstable and Stable Steady States with Perfect Global Capital Markets.** To show that there is an unstable equilibrium at $(k_{B_2}, \bar{k}_{B_2})$, we basically have to show that for a particular infrastructure capital stock $\bar{k}' > \bar{k}_{B_2}$, the following holds if $\alpha + \beta > 1$.

$$\bar{k}(k(\bar{k}')) > \bar{k}' \qquad\qquad \text{where} \quad k(\bar{k}) = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}}$$

$$\bar{k}(k) = \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}}$$

This gives us

$$\left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left[\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \left(\bar{k}'\right)^{\frac{\alpha}{1-\beta}}\right] > \bar{k}'$$

$$\left(\bar{k}'\right)^{\frac{(\alpha+\beta)-1}{(1-\alpha)(1-\beta)}} > \left(\frac{r}{\beta}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{(1-\alpha)(1-\beta)}}$$

$$\bar{k}' > \left(\frac{r}{\beta^{1-\alpha}\alpha^{\alpha}}\right)^{\frac{1}{(\alpha+\beta)-1}} \equiv \bar{k}_C$$

Similarly, we can show that $k\big(\bar{k}(k')\big) > k'$ if $k > k_{B_2}$. Thus, if $k > k_{B_2}$ and $\bar{k} > \bar{k}_{B_2}$, the k and $\bar{k}$ will have perpetual positive growth. Conversely, if $k < k_{B_2}$ and $\bar{k} < \bar{k}_{B_2}$, the k and $\bar{k}$ will have perpetual negative growth till the economy converges to the steady state $(k_A, \bar{k}_A) = (0, 0)$.

We can use the same approach to show that if $\alpha + \beta < 1$, k and $\bar{k}$ converge to k_{B_2} and $\bar{k}_{B_2}$ respectively.

We can also show that for a generic problem with $k(\bar{k}) = \theta^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}}$ and $\bar{k}(k) = \bar{\theta}^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}}$ where $\theta > 0$ and $\bar{\theta} > 0$ are constants, there would be a unique stable steady state at k and $\bar{k}$ given by Equation (17) if $\alpha + \beta < 1$. Conversely, there would be a stable steady state at $(0, 0)$ and unstable steady state at k and $\bar{k}$ given by Equation (17) if $\alpha + \beta > 1$.

$$\begin{aligned} k &= \left(\theta^{1-\beta}\bar{\theta}^{\beta}\right)^{\frac{1}{1-(\alpha+\beta)}} \\ \bar{k} &= \left(\theta^{\alpha}\bar{\theta}^{1-\alpha}\right)^{\frac{1}{1-(\alpha+\beta)}} \end{aligned} \tag{17}$$

A.4. **Signs for $\frac{\partial y_{B_4}}{\partial \mu_p}$ and $\frac{\partial^2 y_{B_4}}{\partial \mu_p \partial \mu_g}$.** Equation (14) can be written as

$$y_{B_4} = (\psi K)^{\frac{1}{1-\alpha-\beta}} \tag{18}$$

where $K = \frac{(1-t)^{\alpha} t^{\beta}}{\delta^{\alpha+\beta}}$, $\psi = \left(s + \frac{\mu_p}{r}\right)^{\alpha} \left(1 + \frac{\mu_g}{r}\right)^{\beta}$ and $\ln(\psi) = \alpha \ln\left(s + \frac{\mu_p}{r}\right) + \beta \ln\left(1 + \frac{\mu_g}{r}\right)$ We can take logs and write equation (18) as $\ln(y_{B_4}) = \frac{\ln(\psi) + \ln(K)}{1-(\alpha+\beta)}$. This gives us

$$
\begin{aligned}
\frac{\partial \ln(\psi)}{\partial r} &= \left(\frac{-1}{r^2}\right)\left[\frac{\alpha\mu_p}{s + \frac{\mu_p}{r}} + \frac{\beta\mu_g}{s + \frac{\mu_g}{r}}\right] < 0 \\
\frac{\partial \ln(\psi)}{\partial s} &= \frac{\alpha}{\left(s + \frac{\mu_p}{r}\right)} > 0 \\
\frac{\partial \ln(\psi)}{\partial \mu_p} &= \frac{\alpha}{\left(s + \frac{\mu_p}{r}\right)}\left(\frac{1}{r}\right) > 0 \\
\frac{\partial^2 \ln(\psi)}{\partial \mu_p \partial \mu_g} &= 0
\end{aligned}
\tag{19}
$$

Using (14) and (19) we can show that

$$
\begin{aligned}
\frac{\partial y_{B_4}}{\partial r} &= \frac{y}{1-(\alpha+\beta)}\left(\frac{\partial \ln(\psi)}{\partial r}\right) \quad \begin{cases} < 0, & if \quad \alpha+\beta < 1 \\ > 0, & if \quad \alpha+\beta > 1 \end{cases} \\
\frac{\partial y_{B_4}}{\partial s} &= \frac{y}{1-(\alpha+\beta)}\left(\frac{\partial \ln(\psi)}{\partial s}\right) \quad \begin{cases} > 0, & if \quad \alpha+\beta < 1 \\ < 0, & if \quad \alpha+\beta > 1 \end{cases} \\
\frac{\partial y_{B_4}}{\partial \mu_p} &= \frac{y}{1-(\alpha+\beta)}\left(\frac{\partial \ln(\psi)}{\partial \mu_p}\right) \quad \begin{cases} > 0, & if \quad \alpha+\beta < 1 \\ < 0, & if \quad \alpha+\beta > 1 \end{cases} \\
\frac{\partial^2 y_{B_4}}{\partial \mu_p \partial \mu_g} &= \frac{y}{1-(\alpha+\beta)}\left[\frac{\partial^2 \ln(\psi)}{\partial \mu_p \partial \mu_g} + \frac{1}{1-(\alpha+\beta)}\left(\frac{\partial \psi}{\partial \mu_p}\right)^2\right] \\
&= y\left[\frac{1}{1-(\alpha+\beta)}\left(\frac{\partial \psi}{\partial \mu_p}\right)\right]^2 > 0
\end{aligned}
$$

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