Endogenous Insurance and Informal Relationships
JOB MARKET PAPER

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Abstract

A rich literature seeks to explain the distinctive features of equilibrium institutions arising in risky environments which lack formal insurance and credit markets. I study the endogeneity of the matching between heterogeneously risk-averse individuals who, once matched, choose both the riskiness of the income stream they face (ex ante risk management) as well as how to share that risk (ex post risk management). I derive simple and empirically testable conditions for unique positive-assortative and negative-assortative matching in risk attitudes, and propose an experimental design to test the theory. Finally, I show that a policy which decreases aggregate risk improves welfare unambiguously when the matching is unchanged, but may hurt welfare when the endogenous network response is taken into account: the least risk-averse individuals abandon their roles as informal insurers in favor of entrepreneurial partnerships. This results in an increase in the risk borne by the most risk-averse agents, who must now match with each other on low-return investments.

JEL Classification Codes: O1, O13,O16, O17

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1 Introduction

The distinctive features of institutions and contract structures arising in the unique environments of developing economies have inspired a vast body of research. Informal insurance alone has received a great deal of attention.[1] The poor, especially the rural poor, persistently report risk as a serious problem (Alderman and Paxson (1992), Townsend (1994), Morduch (1995), Dercon (1996), Fafchamps (2008)). There is widespread consensus that people in developing countries live in more hazardous environments (in terms of climate, disease, and landscape, for example), and thus inherently face more risk (Dercon (1996), Fafchamps (2008)). Furthermore, they live close to minimal subsistence levels. This makes them particularly vulnerable to the risks they face, since even small negative shocks could have disastrous consequences.

Despite high levels of risk and vulnerability, however, the poor in developing countries often lack recourse to formal insurance and credit institutions. Consequently, they work with each other to manage risk. The pressures of costly state verification cause subsets of people ostensibly matched for other purposes to build risk management into their existing relationships. For example, two farmers working together to grow a harvest could smooth each other's consumption by agreeing to a sharing rule of their jointly-observed realized output, or they could try and establish sharing rules with outsiders. The difficulty of contracting on output with an outsider causes the farmers to prefer to adapt their working partnership to accommodate risk concerns. (See Townsend (1979) for more on the theory of costly state verification, and Townsend and Mueller (1998) for an extensive discussion of its relevance for informal insurance.)

Researchers have observed the poor incorporating risk management into their relationships in a variety of creative ways. Rosenzweig and Stark (1989) show that daughters are often strategically married to households in villages with highly dissimilar agroclimates, so that the two farming incomes will be minimally correlated. They show that the motivation for doing so is indeed insurance; households exposed to more income risk are more likely to invest in longer-distance marriage arrangements. Rosenzweig (1988) argues more generally that the formation of kinship networks between heterogeneously risk-averse individuals is motivated by insurance. Ligon et. al. (2002), Fafchamps (1999), and Kocherlakota (1996), among many others, analyze a pure risk-sharing relationship between two heterogeneously risk-averse households who perfectly observe each other's income. Ackerberg and Botticini (2002) study agricultural contracting in medieval Tuscany, and find evidence that heterogeneously risk-averse tenant farmers and landlords strategically formed sharecropping relationships based on differing risk attitudes, motivated by risk management concerns.

[1] For background and institutional details, see the classic papers of Alderman and Paxson (1992) and Morduch (1995), and the recent discussions provided by Dercon (2004) and Fafchamps (2008). Addressing the question of how well informal insurance works in practice is the seminal paper by Townsend (1994), with further papers on empirical tests of risk-sharing by Dercon and Krishnan (2000), Fafchamps and Lund (2003), Mazzocco and Saini (2012), and so on. Finally, many papers seek theoretical explanations for the observed limitations of informal insurance. The most prominent explanation is limited commitment, explored in a dynamic setting by Ligon et. al. (2002), and as a natural bound on group size by Genicot and Ray (2003), Bloch et. al. (2008).

This paper enriches our understanding of informal insurance by developing a rigorous framework of endogenous relationship formation between heterogeneously risk-averse individuals in risky environments, where these individuals may implement *ex ante* and *ex post* risk management strategies with each other in the absence of formal insurance and credit markets[2]. The existing risk-sharing analysis has largely focused on the insurance agreement reached by a single, isolated group of individuals. This approach provides insight into what behaviors to expect if a given group of individuals is matched, but does not provide insight into what groups could actually co-exist in the first place.

Therefore, the direct contribution of this theoretical understanding is an answer to the following question: how well-insured is a population of risk-averse individuals when they must rely only on interactions with each other to manage risk? Answering this question provides a characterization of the symbiosis between formal and informal insurance institutions, and enables us to assess how a change in one would affect the other. In general, without this theory, analysis of the welfare impacts of a policy may be very incomplete–the policy may trigger an unaccounted-for endogenous network response, and may help or hinder the ability of a network to respond endogenously in the first place. Finally, the theory contextualizes empirical observations–a variety of models may produce the data we observe. Developing these models is the only way to identify falsification tests which equip us to distinguish between them.

I study endogenous matching in a setting with the following key elements. First, a population of risk-averse individuals with CARA utility lacks access to formal insurance and credit institutions. Individuals belong to one of two groups, and members must partner up across groups in order to be productive.[3] For example, in an agricultural village, some individuals own land, while other, landless individuals possess farming expertise. No crops can be grown if landowners do not employ farmers. Alternatively, in a microentrepreneurship setting, some individuals might have skill A, while others have skill B, and a successful business venture requires the combination of both skills.

I introduce two key types of heterogeneity: heterogeneity of preferences, and heterogeneity of technology. Specifically, individuals vary in their degree of risk aversion, and have available to them an assortment of risky projects, which vary in their riskiness–a riskier project has a higher expected return, but also a higher variance of return. (In this model, I abstract from moral hazard concerns, in order to focus on the impact of the heterogeneous tradeoff in *ex ante* and *ex post* risk management strategies across partnerships of different risk compositions on the equilibrium matching. I model moral hazard explicitly in Wang (2012).) There is common knowledge of risk types and projects.

Since formal institutions are absent, individuals must rely on agreements with each other to

[2] This terminology derives from Morduch (1995), who also refers to ex ante risk management as "income-smoothing", and ex post risk management as "consumption-smoothing". Intuitively, an individual chooses both the riskiness of the action she takes, as well as how to smooth the riskiness of any given risky action.

[3] In fact, I show that this model nests a model in which individuals choose their own productive opportunities, and match to pool their incomes (see Appendix 2 for the proof). However, this version of the model (with joint productivity) is more tractable and natural for a study of endogeneity. Moreover, because it nests the first model, it addresses scenarios which do not nessarily involve joint produtivity, such as two farmers who grow their own crops but partner up to share risk.

manage the risks they face. A matched pair of individuals jointly chooses one project, and commits *ex ante* to a return-contingent sharing rule. The risk composition of the pair therefore determines both *ex ante* risk management (income-smoothing), that is, the riskiness of the income stream the pair chooses, as well as *ex post* risk management (consumption-smoothing), that is, the form of the sharing rule describing the split of each realized return. Hence, informal insurance motivations influence the composition of partnerships ostensibly formed to produce output, as well as the activities and contracts that are observed in equilibrium.

To fix the model in an example, consider sharecropping in Aurepalle, a village in Southern India which was among those sampled by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT). Sharecropping, or the practice of a farmer living on and cropping a plot belonging to a landowner, where the farmer pays rent as a *share* of realized profits (rather than as a fixed amount), continues to be dominant in parts of the world today, e.g. rice farming in Madagascar (Bellemare (2009)) and Bangladesh (Akanda et. al. (2008)).

Townsend and Mueller (1998) conduct a detailed survey of sharecropping relationships in Aurepalle. The modal type of arrangement in their sample is a landowner who hires a group of tenant farmers to collectively work the land, where the landlord actively participates in the farming process. The paper explicitly notes that there are many standard single-tenancy arrangements which were not sampled; however, the authors felt that the collective tenancy group was a reasonable approximation to the single-tenancy case. There is also strong evidence that risk concerns played a major role in the sharecropping relationship, as evidenced by implicit and explicit risk contingencies in the contracts.[4]

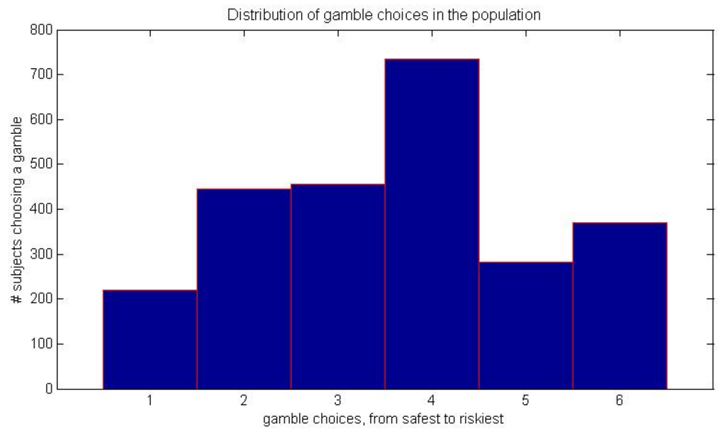
Notably, the paper finds that the *ex post* information sets of landlords and their tenants, along dimensions including realized harvest output, inputs, and reasons for crop failure, were found to coincide almost exactly. Communication and monitoring were found to be frequent and of high quality. Hence, although the abstraction away from moral hazard[5] serves the purpose of focusing on the tradeoff between use of *ex ante* and *ex post* insurance strategies, the model with this assumption is still a close fit for empirically observed relationships.

There is plenty of evidence that these landowners and farmers are risk-averse, and heterogeneous in their risk aversion. In a recent paper, Mazzocco and Saini (2012) develop a novel test and strongly reject a null hypothesis of homogeneous risk preferences in the ICRISAT dataset, for a very general class of preferences. Kurosaki (1990) finds evidence of substantial heterogeneity in risk aversion among ICRISAT landowners and farmers, and moreover does not find much evidence of heterogeneity in time preferences. Antle (1987) estimates the absolute Arrow-Pratt degrees of risk aversion for landowners and farmers in the Aurepalle data, and finds a mean risk aversion of 3.3 with a standard deviation of 2.6.

[4] For more detail on and further examples of the influence of risk concerns on relationship formation, as well as the documented importance of risk attitudes in building risk-sharing relationships, please refer to the final Appendix.

[5] Again, I model moral hazard explicitly in Wang (2012). Moral hazard has interesting implications which are essentially orthogonal to the key economic insight of this paper, which focuses on how the heterogeneous tradeoff of risk management strategies for different risk partnerships influences the equilibrium matching pattern.

Experiments which elicit risk attitudes by asking subjects to choose from a set of gambles differing in riskiness find much variation in gamble choice. For example, in a study of over 2,000 people living in 70 Colombian communities, where 66% live in rural areas, Attanasio et. al. (2012) find the following distribution (gamble 1 is the safest gamble, while gamble 6 is the riskiest):

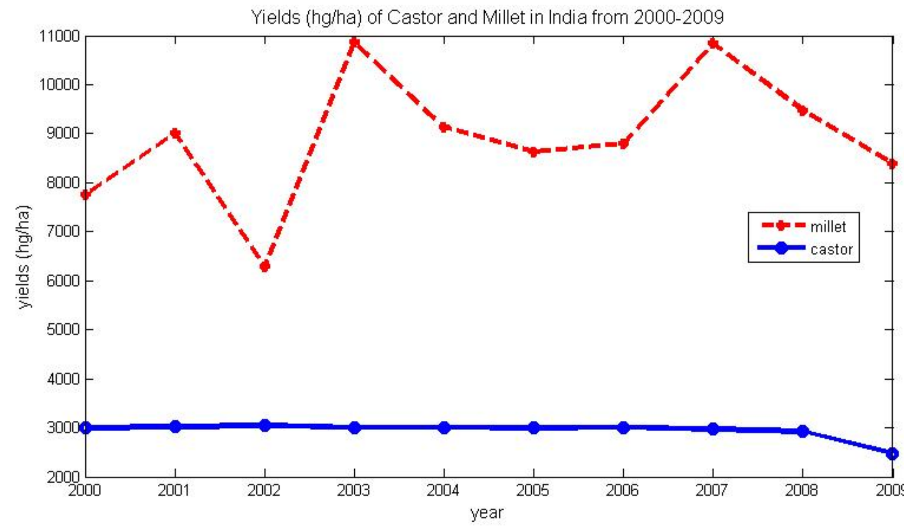


In addition to heterogeneity in risk attitude, individuals can choose to grow a variety of crops, where there is a great deal of heterogeneity in crop riskiness. The riskiness of crop income derives from two sources, yield risk and price risk, where different crops have different stochastic yield and price distributions. For example, grains and castor (oilseed) react differently to the agroclimactic characteristics of a given environment: level and fluctuation of rainfall, elevation and land slope, soil composition, and so on. Thus, in a given agroclimate, different crops have different yield distributions; in addition, the differences between yield distributions for a given set of crops varies across agroclimates. Grains and castor also react differently to inputs and farming methods: choice of irrigation system, timing of planting, fertilizer level and choice, and so on.

In Aurepalle, the dominant crops are castor, pearl millet, cotton, sorghum, pigeon pea intercrop, and paddy, where each crop is risky but the correlation of crop returns is low (Townsend (1994)).

Comparing castor yields to pearl millet yields, we can clearly see that castor (the solid line) is a safe crop, with low mean and low variance of yield, while pearl millet (the dashed line) is a risky crop, with high mean and high variance of yield[6]:

[6] Data from mongabay.com, a website maintaining worldwide crop yield and price information.



Landowners and farmers who are heterogeneous in their risk aversion are observed to choose different portfolios of crops, portfolios of land plots, inputs, and farming methods (see for example Lamb (2002)).

(For further examples of informal insurance relationships, the importance of risk attitudes in building risk-sharing relationships, heterogeneity in risk attitudes, and heterogeneity in technology, please see the last Appendix.)

The institutional feature of sharecropping which has received the most theoretical attention in the literature is the share contract. In a well-known paper, Stiglitz[7] in 1974 suggested that risk-averse landlords and tenant farmers adapt the incentive contracts of their employment relationships to accommodate their desire for risk protection, as a consequence of missing formal insurance. Townsend and Mueller (1998) explore the empirical relevancy of a wide variety of mechanism design theoretic ideas, such as monitoring and costly state verification. However, the question of which landlords match with which farmers, and why, is much less understood[8].

The main contributions of this paper are threefold: first, I find conditions on the fundamentals of the modeling environment for unique assortative matching, which are *independent* of the distributions of risk types in the economy. Equilibrium matching is driven by the interplay between *ex ante* and *ex post* risk management stratgies for a given partnership, where this interplay differs across partnerships of different risk compositions. I show that the fundamental of the model which determines this interplay is the marginal variance cost of taking up a riskier project–a riskier project has a higher mean but also a higher variance of return. The marginal variance cost describes the increase in variance incurred when moving from a project with some mean return to a project with a slightly higher mean return. A sufficient condition for unique negative-assortative matching in risk attitudes (that is, the i^{th} least risk-averse person is matched with the i^{th} most risk-averse

[7] Other papers which discussed similar ideas but differed in approach include Cheung (1968), Bardhan and Srinivasan (1971), and Rao (1971).

[8] I discuss the small body of literature on endogenous matching and informal insurance in detail later in this introduction.

person) is convexity of the marginal variance cost function, while concavity is sufficient for unique positive-assortative matching (that is, the i^{th} least risk-averse person is matched with the i^{th} least risk-averse person).

From this, I derive a simple falsifiability condition, which relies only on empirically observable data. I show that convexity of the marginal variance cost function is equivalent to the convexity of the mean returns of equilibrium projects in the representative rates of risk aversion of matched pairs. Similarly, concavity of the marginal variance cost funciton is equivalent to the concavity of the mean returns of equilibrium projects in the representative rates of risk aversion of matched pairs. Hence, as long as risk aversion is elicited or estimated (for instance, elicited using gamble choices as in Binswanger (1980) or Attanasio et. al. (2012), or estimated as in Ligon et. al. (2002)), network links are recorded (who works with whom), and some proxy of the mean income of each linked pair is observed, this falsifiability condition can be implemented. I also propose an experimental design to test this theory.

Finally, I show that this theory has substantial policy implications: a policy may trigger an endogenous network response. To account for this in our welfare analysis, we need a coherent and tractable model of endogeneity. For example, a policymaker may wish to stabilize the prices of the riskiest crops in an environment where crops with higher mean returns come at an increasingly steep marginal variance cost. (Crop price stabilization policies are very common in developing countries around the world; for instance, Chile has used a variable import tariff to maintain a price band around wheat.) The motivation behind such a policy is generally to reduce aggregate risk of a very risky environment, and to make riskier, higher mean crops more palatable.

I show that such a policy is unambiguously welfare-improving if we assume it has no effect on the existing network of relationships. However, the theory advanced in this paper allows us to characterize the equilibrium network response: I show that a decrease in aggregate risk, where the riskiest projects in particular become safer, may cause the least risk-averse agents to abandon their roles as "informal insurers" in favor of "entrepreneurial" pursuits. This forces the most risk-averse agents to pair with each other, leaving them strictly worse off as they lose the risk protection they had. Moreover, inequality is exacerbated: the least risk-averse individuals, who are now entrepreneurial partners, take advantage of the decrease in risk of higher mean projects, while the most risk-averse individuals, having only weak ability to smooth consumption together, rely heavily on income-smoothing to manage risk, and select especially low mean projects.

A few papers in the literature have modeled the introduction of formal insurance. Attanasio and Rios-Rull (2000) study the informal insurance agreement between two differently risk-averse individuals, and also argue that a decrease in aggregate risk may lead to the more risk-averse agent being worse off. This happens by way of limited commitment: the only punishment for reneging is the cutting off of all future ties. But a decrease in aggregate risk weakens this threat. Hence, lower levels of risk-sharing can be sustained, because the autarky value of each individual increases.

In my model, however, there are no commitment problems. The result would no longer hold in Attanasio and Rios-Rull (2000) if there were perfect commitment, as lowering the cost of autarky only matters through the threat of cutting off future ties. However, I show that, *even under perfect commitment*, reducing the riskiness of the environment, for instance by introducing formal insurance, can still reduce the welfare of the most risk-averse agents, because it changes the composition of the informal risk-sharing network. In my model, reducing the riskiness of the environment raises the value of autarky, but it *also* raises the value of being in a relationship, *heterogeneously* across partnerships of different risk compositions.

Furthermore, this intuition contrasts interestingly with the finding of a current working paper by Chiappori et. al. (2011), which estimates that the *least* risk-averse agents are hurt when formal insurance is introduced. The intuitive argument is that the least risk-averse agents are displaced as informal insurers. However, this exactly illuminates the need for a model of the *equilibrium network* of relationships–I show that the least risk-averse agents do leave their roles as informal insurers, but only because they prefer to undertake entrepreneurial pursuits instead.

This work unites the literature on institutions in risky environments with missing formal insurance and credit markets with the literature on endogeneous matching, primarily by introducing heterogeneity of risk aversion. A small body of literature has studied endogenous informal insurance relationships. Legros and Newman (2007) developed techniques to characterize stable matchings in nontransferable utility settings by generalizing the Shapley and Shubik (1972) and Becker (1974) supermodularity and submodularity conditions for matching under transferable utility. Under nontransferable utility, the indirect utility of each member of the first group given a partnership with each member of the second group can be calculated, fixing the second member's level of expected utility at some level v. Then, this indirect utility expression, which depends on both members' types and v, is analyzed for supermodularity and submodularity in risk types.

Chiappori and Reny (2006) and Schulhofer-Wohl (2006) both study a model of endogenous matching between heterogeneously risk-averse individuals under pure consumption-smoothing. That is, any matched pair faces the same exogenous risk, but a matched pair is able to commit to a return-contingent sharing rule given that risk.

Schulhofer-Wohl and Chiappori and Reny find that negative-assortative matching arises as the unique equilibrium. (Instead of using the approach of Legros and Newman, which is elegant but often intractably algebraic, Schulhofer-Wohl identifies a transferable utility representation of his model, and applies the standard Shapley and Shubik supermodularity conditions. I take this approach as well.) The key insight is that a less risk-averse man is differentially happier than a more risk-averse man to provide insurance for the most risk-averse woman, who is differentially happier than a less risk-averse woman to pay for it.

This paper nests models of endogenous matching under pure consumption-smoothing, and under pure income-smoothing. The direct theoretical innovation is to study the world where both types of risk management strategies are available, and to use the *interaction* of consumption-smoothing with income-smoothing to provide an understanding of the conditions under which negative-assortative matching arises as the unique equilibrium, and under which the "opposite extreme", positive-assortative matching, arises as the unique equilibrium.

This is not a trivial generalization, or a generalization made simply for realism. Identifying an environment in which both extremal types of matching patterns can arise, and understanding the differences in the environments which lead to one matching pattern versus the other, has two important consequences. First, in a recent experiment, Attanasio et. al. (2012) (discussed in more detail in the last Appendix) takes a rigorous experimental approach to the question of endogenous risk-sharing group formation. The key elements of the environment are limited commitment, project choice, and a sharing rule fixed at equal division. They find that individuals who are linked in kinship or friendship are more likely to form risk-sharing groups with each other than with strangers, most likely because of better quality information about risk attitudes as well as trust. Furthermore, they find that risk-sharing groups composed of individuals linked by a family or friendship tie are *positively-assorted* in risk attitude. This empirical observation of positive-assortative matching contrasts strikingly with the negative-assortative match predicted by the models of Schulhofer-Wohl and Chiappori and Reny. Additionally, these results suggest that family and friendship ties are important for identifying the pool of potential partners for a given individual (because an individual is unlikely to know the risk attitude of, or to trust a stranger), but the choice of partner from this pool for the purpose of insurance is primarily determined by risk attitudes.

This model is the first to my knowledge to provide a clear and economically insightful explanation for the unique emergence of each of the two extremal matching patterns. More generally, the falsifiability condition I derive allows researchers to check the validity of the "homogeneous household" assumption, which is fairly standard in work on empirical tests of risk-sharing, where data is often at the household level.

Even more importantly, this framework enables the kinds of policy analysis discussed previously (e.g. introduction of informal insurance). Analysis cannot account for an endogenous network response to a policy, or about the effect of a policy which restricts or enables a network to respond in the first place, if our models unambiguously predict negative-assortative matching, which we know to be contradicted by empirical evidence.

The rest of the paper proceeds as follows. In the next section, I describe the model, state and discuss the main results, provide support for the theory in existing literature, and discuss key elements of the model. I then show that a policy which decreases aggregate risk unambiguously improves welfare when status quo relationships are assumed to remain unchanged, but may in fact increase the individual risk borne by the most risk-averse agents and decrease aggregate welfare, once the endogenous network response is taken into account. Following this, I describe an experiment to test the theory. Finally, I conclude. All technical details are relegated to the Appendices.

2 The Model

In this section, I introduce a framework designed to analyze the equilibrium composition of risk-sharing pairs in a risky environment where both income-smoothing and consumption-smoothing risk management tools are available to agents with heterogeneous risk attitudes.

The framework consists of the following elements:

The population of agents: the economy is populated by two groups of agents, $G1$ and $G2$, where $|G1| = |G2| \in \{2, 3, ..., Z\}$, $Z < \infty$. All agents have CARA utility $u(x) = -e^{-rx}$ and are identical in every respect except for their Arrow-Pratt coefficient r (and their group membership).[9] Assume that members r_1 of $G1$ are distinct in some unmodeled way from members r_2 of $G2$ (e.g. they differ in type of human capital, time constraints, status, sex, etc.). There are no distributional assumptions on the risk types in the economy.

The risky environment: a spectrum of risky projects $p > 0$ is available in the economy, where the returns of project p are distributed $R_p \sim N(p, V(p))$, $V(0) = 0$ and $V(p) > 0$ for $p > 0$, where the function $V(p)$ has the following properties: $V'(0) = 0$, $V'(p) > 0$ for $p > 0$, and $V''(p) > 0$[10]. That is, riskier projects have higher mean but also higher variance, where $V(p)$ describes the "variance cost" of a project with mean return p. (See the section following the results for a discussion of the assumption of normally-distributed returns, and Appendix 1 for an analysis of this model with a general symmetric distribution of returns.)

Any project p requires two agents, one from $G1$ and one from $G2$[11]. For example, a landlord must contribute land and a farmer, expertise, in order for any crop to grow. A given matched pair (r_1, r_2) jointly selects a project[12]. Assume that staying unmatched is disastrous for any agent (see Appendix 2 for a proof that the framework with joint project choice and infinite disutility from being unmatched is equivalent to the framework where each partner individually chooses a project and the returns are pooled, and individual rationality is a constraint that must be satisfied). All projects are equally available to each possible pair, an agent can be involved in at most one project, and there are no "project externalities". That is, one pair's project choice does not affect availablity or returns of any other pair's project.

There are no moral hazard considerations in this model. Please refer to Wang (2012) for an

[9] Of course, in reality, types are multidimensional, and matching decisions are not exclusively based on risk attitudes. It is worth noting that the model can account for this. For example, kinship and friendship ties are important, in large part because of information (they know each other's risk types), and commitment (they trust each other, or can discipline each other). Kinship and friendship ties would enter into this theory in the following way: an individual would first identify a pool of feasible risk-sharing partners. This pool would be determined by kinship and friendship ties, because of good information and commitment. Following this, individuals would choose risk-sharing partners from these pools. This choice would be driven by risk attitudes, as addressed in this benchmark with full information and commitment.

Thus, this theory can be thought of as addressing the stage of matching that occurs after pools of feasible partners have been identified.

[10] Note that $V(p) > 0$ is simply a requirement that variance be positive. Moreover, the assumption that $V'(p) > 0$ is without loss of generality–we could begin by considering the entire space of project returns $N(p, V(p))$, where $V(p)$ is unconstrained. Then, any agent with monotonic utility would choose the project with lower variance, between two projects with the same mean. Tracing out the set of projects that agents would actually undertake leads us to $V'(p) > 0$. The assumption $V''(p) > 0$ is to ensure global concavity of an agent r's expected utility in p and hence an interior solution for project choice.

[11] The fact that group size is bounded can be justified by, for example, costly state verification considerations. Specifying that the group is of size two allows for a richer action space (endogenous project and contract choice).

[12] It is known from Wilson (1968) that, becasue of CARA utility, any pair acts as a syndicate and "agrees" on project choice.

explicit treatment of moral hazard in an endogenous matching problem.

Information and commitment: there is no informational problem. All agents know each other's risk types.

A given matched pair (r_1, r_2) observes the realized output of their partnership (but not the outputs of other partnerships), and is able to commit ex ante to a return-contingent sharing rule $s(R_{p_{12}})$, where $R_{p_{12}}$ is the realized return of (r_1, r_2)'s joint project p_{12}.

The equilibrium: An equilibrium consists of a match function $\mu(r_1) = r_2$ such that a distinct r_1 is matched with a distinct r_2, and nobody is matched to more than one person, with the additional property that no agent is able to propose a project and sharing rule to an agent to whom she is not matched under μ, such that both of them are happier than they are under μ (stability).

Further, there is a return-contingent sharing rule for each matched pair $(r_1, \mu(r_1))$, such that no pair could set a different sharing rule and both agents would be better off, and there is a project for each matched pair, such that no pair could choose a different project and both be better off given the sharing rule.

Matching patterns: It will be helpful to introduce some matching terminology. Suppose the people in $G1$ and in $G2$ are ordered from least to most risk-averse: $\{r_1^j, r_2^j, ..., r_Z^j\}$, $j \in \{1, 2\}$. Then "positive-assortative matching" (PAM) refers to the case where the i^{th} least risk-averse person in $G1$ is matched with the i^{th} least risk-averse person in $G2$: $\mu(r_i^1) = r_i^2$, $i \in \{1, ..., Z\}$. On the other hand, "negative-assortative matching" (NAM) refers to the case where the i^{th} least risk-averse person in $G1$ is matched with the i^{th} *most* risk-averse person in $G2$: $\mu(r_i^1) = r_{Z-i+1}^2$, $i \in \{1, ..., Z\}$. To say that the unique equilibrium matching pattern is PAM, for example, is to mean that the only μ which can be stable under optimal within-pair sharing rules and projects is the match function which assigns agents to each other positive-assortatively in risk attitudes.

A more detailed discussion of the elements of the framework follows the statement of results (with technical details relegated to the Appendix). The discussed elements include the decision to model pairs as jointly selecting a project, the assumption of normally-distributed returns, the focus on partnerships, and two-sided versus one-sided matching.

3 Results

The heterogeneity of risk-aversion in agents makes this a model of matching under nontransferable utility. That is, the amount of utility received by an agent with risk aversion r_1 from one unit of output differs from the amount of utility an agent with risk aversion r_2 receives from one unit of output. Thus, we cannot directly apply the Shapley and Shubik (1962) result on sufficient conditions for assortative matching in transferable utility games.

It will be helpful to review briefly that environment and result. Consider a population consisting of two groups of risk-neutral workers, where all workers have utility $u(c) = c$. Let x denote the ability of workers in one group, and y denote the ability of workers in the other group. The production function is given by $f(x, y)$, which can be thought of as: "the size of the pie generated by matched workers x and y". Then, $\frac{df}{dxdy} > 0$ is a sufficient condition for unique positive-assortative matching, and $\frac{df}{dxdy} < 0$ is a sufficient condition for unique negative-assortative matching.

My approach here will be to find the analogy to the Shapley and Shubik function $f(x, y)$ for this model where utility is *not* transferable. In Proposition 1 below, I prove that *expected utility* is transferable in this model–instead of thinking about moving "*ex post*" units of output between agents, we should instead think about moving "*ex ante*" units of expected utility. I show that the sum of the certainty-equivalents $CE(r_1, r_2)$ of a given matched pair (r_1, r_2) is the analogy to the joint output production function in the transferable utility problem. The sum of the certainty-equivalents of a matched pair is "the size of the expected utility pie generated by matched agents r_1 and r_2", and sufficient conditions for positive-assortative and negative-assortative matching now come from the supermodularity and submodularity of $CE(r_1, r_2)$ in r_1, r_2. More technically, expected utility is transferable here because the expected utility Pareto possibility frontier for a pair (r_1, r_2) is a line with slope -1 under some monotonic transformation.

Proposition 1 *Expected utility is transferable in this model.*

The proof is in Appendix 3.

Thus, conditions for the supermodularity and submodularity of $CE(r_1, r_2)$ in r_1, r_2 are sufficient conditions for unique PAM and NAM, respectively[13]. Finding these conditions requires characterizing the sum of certainty-equivalents generated by a matched pair. I sketch the important steps here (details can be found in the proof of Proposition 1).

To understand which individuals match in equilibrium and which project and sharing rule each of these equilibrium partnerships chooses, it is necessary first to understand the project and sharing rule chosen by a given matched pair (r_1, r_2). Suppose r_1 and r_2 had already selected a project p. Then, conditional on r_1 ensuring r_2 some level of expected utility $-e^{-v}$, the optimal sharing rule solves:

$$\max_{s(R_p)} E\left[-e^{-r_1[R_p - s(R_p)]}|p\right] \ s.t.$$

$$E\left[-e^{-r_2 s(R_p)}|p\right] \quad \geq -e^{-v}$$

Solving this program yields the following form for the optimal sharing rule:

$$s^*(R_p) = \frac{r_1}{r_1 + r_2} R_p - \frac{r_1}{r_1 + r_2} p + \frac{1}{2} \frac{r_1^2 r_2}{(r_1 + r_2)^2} V(p) + \frac{v}{r_2}$$

[13] Note that an alternative approach to characterizing sufficient conditions for assortative matching in models of nontransferable utility can be found in Legros and Newman (2007). Consider any four types $t_1, t_1' \in G1, t_2, t_2' \in G2$ in the economy, where $t_1 > t_1'$ and $t_2 > t_2'$. If, whenever agent t_1' is indifferent between ensuring agent t_2 expected utility v and ensuring agent t_2' expected utility u, agent t_1 *strictly prefers* (*strictly dislikes*) ensuring agent t_2 expected utility v to ensuring agent t_2' expected utility u, then positive-assortative (negative-assortative) matching is the unique equilibrium.

These conditions are intuitive and an elegant generalization of the Shapley-Shubik result, but are often algebraically messy to use. The transferable expected utility approach proves to be much cleaner.

Note that it is linear (unsurprising, given there is no moral hazard)[14], and the less risk-averse member of the partnership faces an income stream which is more highly dependent on realized project return.

Given the optimal sharing rule, and v, we can solve for the project a pair would choose. For any v, the pair agrees on project choice, since r_2 is ensured a fixed level of expected utility and is therefore indifferent over all projects. Hence, r_1 simply chooses the project that would make her the happiest, conditional on her having to ensure r_2 expected utility $-e^{-v}$. Furthermore, the pair agrees on the same project, *independent* of the "split of utility", v.

Define a function $M(p)$, which describes the marginal variance cost of a small increase in mean return from level p. That is:

$$M(p) = V'(p)$$

Then a partnership chooses a project by equating the marginal cost of increased variance with the marginal benefit of increased mean, weighted by their risk attitudes:

$$\begin{aligned} M(p^*) &= \frac{2(r_1+r_2)}{r_1 r_2} \Leftrightarrow \\ p^* &= M^{-1}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right] \end{aligned}$$

Now that we have characterized the project $p^*(r_1, r_2)$ and sharing rule $s(R_p|p^*(r_1, r_2), v)$ chosen by a matched pair (r_1, r_2), we can characterize the partnership's sum of certainty-equivalents:

$$\begin{aligned} CE(r_1, r_2) &= p^*(r_1, r_2) - \frac{r_1 r_2}{2(r_1+r_2)} V(p^*(r_1, r_2)) \\ &= M^{-1}\left(\frac{2(r_1+r_2)}{r_1 r_2}\right) - \frac{r_1 r_2}{2(r_1+r_2)} V\left(M^{-1}\left(\frac{2(r_1+r_2)}{r_1 r_2}\right)\right) \end{aligned}$$

It remains only to identify conditions on this expression for its supermodularity and its submodularity in r_1, r_2.

This leads to the main matching result.

Proposition 2 *1. A sufficient condition for PAM to be the unique equilibrium matching pattern is $M''(p) < 0$ for $p > 0$ (the marginal variance cost function is concave).*

2. A sufficient condition for NAM to be the unique equilibrium matching pattern is $M''(p) > 0$ for $p > 0$ (the marginal variance cost function is convex).

[14] Clearly, if moral hazard were modeled here, incentive provision would also enter into the sharing rule–a more risk-averse individual is willing to pay a higher premium for insurance, but is harder to incentivize. In Wang (2012), I identify a framework in which the equilibrium sharing rule is piecewise linear, and neatly separates risk protection and incentive provision.

3. A sufficient condition for any matching pattern to be sustainable as an equilibrium is $M''(p) = 0$ for $p > 0$ (the marginal variance cost function is linear).

(Technical details of the proof can be found in Appendix 4.)

Note that this result shows that the equilibrium matching pattern is independent of the distributions of risk types in the economy, which makes its application very flexible.[15]

Let's think further about the following expression, which appears in the expression for optimal project choice and $CE(r_1, r_2)$:

$$\hat{H}(r_1, r_2) \equiv \frac{r_1 r_2}{r_1 + r_2}$$

We can think of this expression $\hat{H}(r_1, r_2)$ as being the "representative risk type" of a matched pair (r_1, r_2)–that is, the matched pair (r_1, r_2) acts as a single CARA utility agent with risk attitude $\hat{H}(r_1, r_2)$.

This means that we can also define the "representative rate of risk aversion" for a matched pair (r_1, r_2):

$$\begin{aligned} \tilde{H}(r_1, r_2) &\equiv \frac{1}{\hat{H}(r_1, r_2)} \\ &= \frac{r_1 + r_2}{r_1 r_2} \\ &= \frac{1}{r_1} + \frac{1}{r_2} \end{aligned}$$

Note that $\frac{1}{r}$ captures the rate at which an individual r's valuation for an additional unit of output falls.

Our expression for optimal project choice is then:

$$p^*(r_1, r_2) = M^{-1}\left[2\tilde{H}(r_1, r_2)\right]$$

We can see that the size of the expected utility pie of a matched pair (r_1, r_2) is influenced by the risk types of the agents in two ways: the more risk-averse the "representative type" of the matched pair (equivalently, the smaller the representative rate of risk aversion), the safer the equilibrium project will be (lower mean and lower variance), and the harsher will be the adverse impact of an increase in variance on $CE(r_1, r_2)$ (since $V(p^*(r_1, r_2))$ is multiplied by $\hat{H}(r_1, r_2)$).

What is the intuition behind the matching result? We know from the literature the intuition behind negative-assortative matching when agents lack income-smoothing tools–in that case, it is the least risk-averse guy in $G1$ who is "willing to place the highest bid" for the most risk-averse

[15] These sufficient conditions could be weakened slightly by introducing slight dependence on the distributions of risk types, only through the supports: the concavity, convexity, and linearity of $M(\cdot)$ need only to hold on the domain $\left[\frac{2}{\max(r_1)} + \frac{2}{\max(r_2)}, \frac{2}{\min(r_1)} + \frac{2}{\min(r_2)}\right]$, and not on $\mathbb{R}^+$.

agent in $G2$. This is because the least risk-averse agent is the most willing to provide insurance, and subsequently she provides the highest level of insurance, while at the same time the most risk-averse agent has the highest willingness to pay for good insurance. This "bidding" turns out to have a monotonic property, in that once the least risk-averse agent in $G1$ and the most risk-averse agent in $G2$ are matched and are "removed from the pool", the least risk-averse of the remaining agents in $G1$ is then matched with the most risk-averse of the remaining agents in $G2$, and so on.

My result shows that accounting for the interaction of income-smoothing with consumption-smoothing leads to the emergence of *both* extremal matching patterns. Loosely, a risk-averse individual prefers a partner unlike herself for consumption-smoothing purposes (a "gains from trade" matching motivation), but prefers a partner like herself for income-smoothing purposes (a "similarity of perspective" matching motivation). In a model with both consumption- and income-smoothing, playing the role of informal insurer isn't as straightforward for the less risk-averse agents–the consumption-smoothing that a less risk-averse agent offers to a more risk-averse partner comes at a cost. Namely, the insurance that a less risk-averse agent provides within the relationship affects the choice of income stream–if the less risk-averse agent did not have to bear most of the consumption risk, she would have preferred to choose a riskier income stream with higher mean return.

When the marginal variance cost is increasing in expected return, and increasing more rapidly for higher levels of expected returns, the benefit of playing the role of informal insurer dominates the costs for the less risk-averse agents, because optimal consumption-smoothing does not come at much of a cost–it does not really involve a sacrifice in the choice of income stream, since the less risk-averse agents also prefer to stick to safer projects. In this environment, there is little friction between consumption-smoothing and income-smoothing for all agents across all partner types, and thus the "gains from trade" consumption-smoothing motivation dominates, resulting in unique negative-assortative matching.

However, when the marginal variance cost is increasing in expected return, but increasing more *slowly* for higher levels of expected returns, the role of informal insurer becomes less appealing for a less risk-averse agent. Now, when the less risk-averse agent shoulders most of the risk in a relationship with a more risk-averse agent, she must make a sacrifice in the choice of income stream–were she to be slightly more insured herself, she would start a project with a much higher expected return, since marginal variance cost is concave in expected returns. Hence, in this environment, the less risk-averse agents experience friction between consumption-smoothing and income-smoothing when partnered with more risk-averse agents, and this friction diminishes when they pair up with fellow less risk-averse agents instead. This causes the less risk-averse agents to choose entrepreneurial pursuits with other less risk-averse agents, over informal insurance relationships with more risk-averse agents, and unique positive-assortative matching results.

Finally, when the marginal variance cost is increasing linearly in expected return, the total sum of certainty-equivalents is the same across all possible matching patterns. Partnerships of different risk compositions do generate different pair-specific certainty-equivalents, but all risk types in $G1$

have the same *strength of preference* between any two individuals in $G2$ (and vice versa). Hence, any matching pattern is sustainable as an equilibrium.

It is worth asking what happens in the model when the income-smoothing channel is shut down, and what happens when the consumption-smoothing channel is shut down. Shutting down the income-smoothing channel effectively corresponds with the Chiappori and Reny (2006) and the Schulhofer-Wohl (2006) settings, so it is reassuring that in this setting, in the absence of project choice, negative-assortative matching arises as the unique equilibrium. Furthermore, when instead the consumption-smoothing channel is shut down (by holding the division of output fixed at a constant across all possible pairs), the unique equilibrium matching pattern is positive-assortative. Please see Appendices 5 and 6 for details.

Can this theory be falsified empirically? An obvious challenge of taking this result to the data is that we may not easily observe this $V(p)$ object. The following two results will be helpful for more concretely linking the intuition of the matching equilibrium to economic concepts.

Proposition 3 *The mean return of optimal project choice $p^*(r_1, r_2)$ if (r_1, r_2) are matched is supermodular ($\frac{dp}{dr_1 dr_2} > 0$) in r_1, r_2 iff $M''(p) < 0$, and is submodular ($\frac{dp}{dr_1 dr_2} < 0$) in r_1, r_2 iff $M''(p) > 0$.*

Proposition 4 *The mean return of optimal project choice $p^*(r_1, r_2)$ for a pair (r_1, r_2) is convex in $\tilde{H}(r_1, r_2)$ iff $M''(p) < 0$, and is concave in $\tilde{H}(r_1, r_2)$ iff $M''(p) > 0$.*

(Recall that $\tilde{H}(r_1, r_2) \equiv \frac{1}{r_1} + \frac{1}{r_2}$.)

[See Appendix for the proofs.]

In words, the first corollary tells us that the sufficient condition for unique positive-assortative matching corresponds exactly to the supermodularity of the mean return of a matched pair's optimal choice of project in the risk types of the matched pair, while the sufficient condition for unique negative-assortative matching corresponds exactly to the submodularity of the mean return of a matched pair's optimal choice of project in the risk types of the pair.

However, we cannot directly test for this relationship, because we only observe *equilibrium* projects.

To be more clear about this, consider the following example. Suppose $G1 = \{r_1^A, r_2^A, r_3^A\}$ and $G2 = \{r_1^B, r_2^B, r_3^B\}$, where $r_1^{A,B} < r_2^{A,B} < r_3^{A,B}$.

Then, suppose the underlying marginal variance cost function is such that $M''(p) < 0$. Then we observe agents positively-assorting along risk attitudes. But we do not observe the marginal variance cost function $M(p)$, or the variance function $V(p)$. We only observe mean project returns in equilibrium, $p(r_1^A, r_1^B), p(r_2^A, r_2^B)$, and $p(r_3^A, r_3^B)$, as well as the risk types of agents in $G1$ and $G2$, and the positive-assortative linkages between $G1$ and $G2$.

If we could show that the mean of *optimal* joint project choice were supermodular in r_i^A, r_j^B, then this would be evidence supportive of the theory. But we do not observe $p(r_i^A, r_j^B)$ for $i \neq j$– that is, we do *not* observe the projects that individuals who are unmatched in equilibrium *would* *have chosen* had they been forced to match. So, we cannot use this result to see test whether or not the data supports the theory.

Fortunately, this result leads to a second result, which *is* testable with data we are likely to have. We can use our observed matchings (r_1^A, r_1^B), (r_2^A, r_2^B), and (r_3^A, r_3^B) to create a variable $H_i \equiv \tilde{H}(r_i^A, r_i^B)$ for each matched pair. Then, we can regress $p_i \equiv p(r_i^A, r_i^B)$ on r_i^A, r_i^B, and $\tilde{H}_i^2$. If the regression shows that p_i is convex in $\tilde{H}_i$, then this is evidence supportive of the theory, since we have established the following: (a) individuals are matched positive-assortatively in risk attitudes in the data, and (b) the mean returns of equilibrium projects are convex in the representative rates of risk aversion of the matched pairs. We know from the theory that (b) is equivalent to the original matching condition of concavity of the marginal variance cost function $M''(p) < 0$.

Hence, it is clear that there are concrete, observable differences between environments in which PAM arises as the unique matching equilibrium, and environments in which NAM arises as the unique matching equilibrium. The falsifiability condition is constructed by exploiting these differences. Loosely, equilibrium project choice varies less and representative risk types are more clustered in environments where NAM arises than in environments where PAM arises. This captures the sense in which differences in risk attitude cause far less conflict about preferred income stream in NAM environments, while differences in risk attitude can cause substantial conflict about preferred income stream in PAM environments.

Finally, can we say anything about efficiency?

Proposition 5 *The equilibrium maximizes the sum of certainty-equivalents, and is Pareto efficient.*

Proposition 6 *The equilibrium maximizes total mean project returns (conditional on a matched pair choosing project optimally).*

Both of these propositions follow straightforwardly from the conditions for positive- and negative-assortative matching.

Since the sum of certainty-equivalents is a social welfare function, and the equilibrium maximizes this sum, it must be Pareto efficient. Intuitively, if an agent could be made better off without making his partner worse off via a change in sharing rule or project choice, then that sharing rule or project choice was not optimal in the first place and thus not an equilibrium. If an agent could be made better off without making anyone worse off by switching partners, then that switch would have occurred.

The first proposition suggests that the natural social welfare function to consider in this model is the total sum of certainty-equivalents[16]. This choice of social welfare function smoothly accommodates the idiosyncratic and the aggregate risk in this model, as well as the nonuniqueness of individual utility (the nonunique equilibrium split of utility in each pair) due to endogenous matching, by quantifying social welfare in units of pairwise expected utility.

[16] Other papers have discussed the total sum of certainty-equivalents as a natural social welfare function in a setting where households share risk. See for example Chambers and Echenique (2012).

The second proposition tells us that, in a given environment, any movement of the matching pattern away from the equilibrium results in a decrease in total expected project returns, as long as a matched pair chooses its project optimally.

3.1 Support in the Literature

Is there support for this theory in existing data? The data from the experiment of Attanasio et. al. (2012) and the sharecropping data from Ackerberg and Botticini (2002) suggest that the answer is yes. (In Section 5, I describe the ideal experiment to test this theory.)

Attanasio et. al. (2012) run a unique experiment with 70 Colombian communities. They elicit risk attitudes by privately offering subjects a choice of gambles in the first round, which vary in riskiness (higher mean projects come at the cost of higher variance). In addition, they gather data on pre-existing kinship and friendship networks. Kinship and friendship ties matter for two important reasons: first, individuals are likely to know the risk attitudes of family and friends, and unlikely to know the risk attitudes of strangers. Second, individuals are likely to trust family and friends over strangers (regeners cannot be detected in this experiment).

Attanasio et. al. find that family and friends are more likely to group together, and conditional on this, have a strong tendency to assort positively in risk attitudes, while groups composed of strangers did not appear to match with respect to risk attitudes. The intuitive explanation for this is that family and friends knew each other's risk attitudes, while strangers did not.

My model focuses on the case where individuals know each other's risk attitudes. Note that family and friendship ties can be thought of as entering the matching process in a "period 0" stage, where individuals first identify pools of feasible risk-sharing partners. Family and friends tend to fall into this pool for informational and commitment reasons. Then, the choice of partner from this pool is driven by risk attitudes, and is determined by the forces described in this paper. Thus, the subset of the Attanasio et. al. data which is relevant for this paper is the data pertaining to matched groups reporting at least one family/friendship tie.

The choice of gambles is described in the paper, so the function $V(p)$ could be characterized, but recall from the results section that when the channel of consumption-smoothing is shut down (in this experiment, the sharing rule is fixed at equal division for all partnerships), positive-assortative matching arises uniquely. So, the task is to check the falsifiability condition of Proposition 4: were the mean returns of equilibrium project choices convex in rate of risk aversion?

To answer this question, I use the data the authors have provided online. (I focus only on risk-averse people–there were a small number of risk-loving individuals who chose the riskiest gamble, which had the same mean as the second-riskiest gamble, but a higher variance.) First, I confirm that members of groups with at least one friendship tie chose similar projects. This bears out in the data: the mean difference in second-round gamble choice within familiar groups is 0, and the modal difference is also 0. Following the falsifiability condition of Proposition 4, I regress second round gamble choice on the degree of risk aversion (proxied by first round gamble choice), and the squared *rate* of risk aversion, to check for convexity of project choice (and to confirm that more risk-averse agents choose safer gambles)[17]: i indexes individuals who partnered with a friend:

$$p_i = \beta_1 + \beta_2 \left(\frac{1}{r_i}\right)^2 + \varepsilon_i$$

The OLS estimate is $\beta_2 = 0.03^{***}$ (standard error 0.003)[18]. But this positive sign ($\beta_2 > 0$, significant at 1%) precisely implies that project choice is convex in rate of risk aversion.

Hence, the experimental results support the theory: positive-assortative matching in risk attitudes is observed among individuals who know each other's risk attitudes and are able to trust and commit, as is predicted by the model, since individuals are able to income-smooth but are not able to consumption-smooth. Moreover, as predicted by the model (see Proposition 4), the mean returns of chosen gambles are convex in the rates of risk aversion.

A second paper providing empirical support for this theory is Ackerberg and Botticini (2002). They provide evidence of endogenous matching motivated by risk between landowners and sharecroppers in medieval Tuscany. They assume that crops of varying riskiness (the safe crop of cereal, the risky crop of vines, and mixtures of the two) are exogenously assigned to landowners, and tenants of differing risk aversion were matched to different crop plots/landowners. That is, they assume that individuals were able to consumption-smooth, but were restricted in their ability to income-smooth. In their data, they observe that share contracts were associated with the safer crop of cereal, while fixed rent (residual claimancy) contracts were associated with the riskier crop of vines. To explain this observation, they provide empirical evidence that risk-neutral tenants were assigned to the riskier crops, resulting in fixed rent contracts for vines, while risk-averse tenants were assigned to the safer crops, resulting in share contracts on cereals.

But the data also shows that the types of crops cultivated differed starkly across two regions of Tuscany they studied: San Gimignano and Pescia. The following table shows the number of pure vine plots, pure cereal plots, and mixed plots cultivated in Pescia and in San Gimignano:

	Pescia	*San G.*
	# plots	# plots
Vines (risky)	47	6
Mixed	4	127
Cereals (safe)	178	17

The difference in the equilibrium crop mix is striking. In particular, "mixed" is not a third crop in and of itself, but a mixture of the risky vines crop and the safer cereals crop (likely differing mixtures; the degree of mixing is not recorded). This suggests that individuals may have had greater ability to income-smooth than previously assumed. How can this setting be analyzed within the framework of this paper?

[17] In the experiment, gamble 1 is the safest and gamble 5 is the riskiest (I drop gamble 6, the risk-loving gamble). A gamble choice of a higher denomination corresponds with a lower degree of risk aversion. Hence, I proxy $\frac{1}{r_i}$, the rate of risk-aversion, with g_i, that is, i's gamble choice.

[18] The OLS estimate of the constant is 3.56^{***} (standard error 0.07).

The theory suggests that the agroclimactic differences between Pescia and San Gimignano may be important. These two regions are located fairly close together (about 100 km apart, according to Google maps), so they shared the same governance and were unlikely to differ in norms. However, Pescia receives higher mean rainfall, while San Gimignano has lower mean rainfall (Dalla Marta et. al. (2010)). A primary reason higher mean crops have higher variance of yield is that they are more sensitive to rainfall–given enough rainfall, the crops can yield a substantial harvest, but any shortage will lead to blight. Safer crops can produce a positive yield even with low levels of rainfall, but added rainfall is unlikely to increase this yield significantly.

Hence, crops which were less sensitive to rain (safer crops) may have had similar distributions of yields in both Pescia and San Gimignano, while crops which were more sensitive to rain were likely to have had variance of yield in San Gimignano than in Pescia. In the language of this model, the marginal variance cost function of crops with mean p in Pescia was likely to have been concave in p ($M''(p) < 0$), while the marginal variance cost function of crops with mean p in San Gimignano was likely to have been convex in p ($M''(p) > 0$).

Thus, while the equilibrium choice of crop portfolio in both regions seems to have encompassed a spectrum between cereal crops (the safer, more rain-robust crops), and vines (the riskier, more rain-sensitive crop), the theory suggests that landowners and farmers in Pescia matched positive-assortatively and chose the endpoints of this spectrum (pure cereals and pure vines), while landowners and farmers in San Gimignano matched negative-assortatively and chose the middle points of this spectrum (mixtures of crops and vines). This is in fact the pattern described by the table.

3.2 Discussion of the Model

The specification that agents jointly start a project and hence never wish to remain unmatched may seem restrictive, but in fact nests the following framework. Suppose each agent chooses her own risky income stream. Subsequently, risk-sharing partnerships are formed, and a matched pair pools their realized incomes and splits the pooled amount according to some agreed-upon sharing rule. An equilibrium matching must satisfy both the stability requirement, and an individual-rationality (IR) requirement: the expected utility any agent receives in her partnership must exceed her utility from being alone.

I show rigorously that my model nests this framework in Appendix 2. The idea behind the equivalence is the following: in this framework where individuals choose their own risky income streams, the autarkic sharing rule is always an option for a matched pair. Hence, IR is always satisfied–two individuals can always match and agree to a sharing rule of the form, "Each person keeps her own realized income." There are no explicit costs to matching, so no one remains unmatched.

Furthermore, since a pair of agents pools their realized incomes, couples are essentially choosing their preferred pooled income stream when they match in equilibrium (if income streams are independent, this is simply the convolution of individual income distributions). But this is equivalent to couples explicitly jointly choosing a risky project from some spectrum, and assuming that no agent stays unmatched. This latter specification is much more natural for an analysis of income-smoothing, consumption-smoothing, and endogenous matching.

In this model, I also assume that returns are normally-distributed. The normal distribution has a "representational convenience", in that it has the feature that the only nonzero cumulants[19] are the mean and the variance. A natural concern is that this "representational convenience" is driving the result.

I solve a generalization of the model under the assumption that returns follow an arbitrary symmetric distribution, and show that normality is not driving the result. The normal distribution does generate unusual tractability: the mean-variance characterization yields a natural parameterization of the risky project space, so that the function $V(p)$ captures entirely the "cost" of gains in expected return in terms of variance, since there are no higher order cumulants. But consideration of a generalized model shows that the key underlying determinant of equilibrium matching is still a cost-benefit tradeoff across the spectrum of risky projects, where this tradeoff is simply less concise than it was before: the "cost" of gains in expected return is now captured by the sum of all the higher order cumulants, not just the variance. But the sum of all the higher-order cumulants essentially acts as a "generalized variance" in this context–the sufficient condition for matching is still a parametric condition which can be calculated. Thus, there is no loss in economic insight by specifically assuming normally-distributed returns. Please refer to Appendix 1 for more details and results for the generalized model.

A third piece of the model is the restriction to groups of size two. Costly state verification does provide a logical upper bound on group size. However, this assumption plays a key role in the theory. It is indeed a restriction, but there is a very sharp modeling tradeoff between richness of action space and richness of link formation. Focusing on a network of pairs allows me to optimize over the whole space of sharing rules and projects, so I can analyze how the tradeoff between income-smoothing and consumption-smoothing within a pair influences the composition of the equilibrium network. This would not be possible if I allowed for any kind of match formation.

A final piece of the model is the two-sidedness of the matching problem. Why not one-sided matching? The primary reason for this is technical: under one-sided matching, "negative-assortative" and "positive-assortative" no longer uniquely identify matching patterns. That is, when there are two distinct groups of agents, e.g. $G1 = \{1, 2, 3\}$ and $G2 = \{3, 4, 5\}$, the positive-assortative matching is clearly $(1, 3), (2, 4), (3, 5)$. However, if there were only one group of agents, $G = \{1, 2, 3, 3, 4, 5\}$, then $(1, 3), (2, 3), (4, 5)$ is also a positive-assortative matching pattern. Moreover, theoretically there may be problems with existence in one-sided matching problems. These reasons are clearly superficial–it is clear that the assumption of two-sided matching is merely for convenience, and does not detract from any deeper economic insight.

[19] Recall that the cumulant-generating function is the log of the moment-generating function. The first and second cumulants of any distribution are the mean and the variance.

4 Policy

In this section, I show how to use the theory to analyze the aggregate and distributional welfare imapcts of a hypothetical policy. The natural social welfare function in this setting is the total sum of certainty-equivalents (see the discussion at the end of the Results section).

Using the theory concretely requires consideration of a few practical issues. For example, the practical issue of the potential for the two groups in a population to be of different sizes (e.g. there may be more tenant farmers than landowners) is addressed in the following subsection.

4.1 Differently-sized groups

The assumption of equally-sized groups is unlikely to hold in real-world settings. In the benchmark model, it is a convenient and relatively harmless assumption to make, because the focus of the theory is on the relationship between the scope of income-smoothing and consumption-smoothing informal insurance strategies in a partnership, and the risk composition of equilibrium partnerships, and this insight does not depend on group size.

However, if we want to use this framework to evaluate policies, we need to account for the possibility of differently-sized groups, because this could conceivably affect, among other things, the magnitude of the policy impact. To consider an extreme example, if one group consists only of one person, while the other group consists of some large number of people, then the individual in the first group may change partners in response to the policy, but most people will be unaffected. Therefore, to realistically think about policy, we need to understand what would happen in this model if the two groups were of different sizes.

Lemma 7 *If $|G1| < |G2|$ ($|G1| > |G2|$) the most risk-averse excess agents of $G2$ ($G1$) will be unmatched. The remaining agents are matched according to the main result (Proposition 2).*

A proof is provided in Appendix 7.

Hence, we see that if there are more tenant farmers than landlords, for example, the most risk-averse tenant farmers will be unmatched, and we can use the conditions from Proposition 2 to think about the endogenous matching between the remaining farmers and landlords.

4.2 Crop price stabilization/Introduction of formal insurance

Price stabilization, particularly of crops, is frequently proposed by governments as a tool for alleviating the substantial risk burden shouldered by the poor (Knudsen and Nash (1990), Minot (2010)). Farmers face a large amount of yield and price risk. Crop price stabilization should therefore reduce the income risk of farmers, and should particularly benefit the most risk-averse, poorest agents (Dawe (2001)).

A key contribution of my framework is to illuminate the importance of accounting for the endogenous informal risk-sharing network response when analyzing the costs and benefits of such a policy, a point that, to my knowledge, has not yet been made in the literature.

How should crop price stabilization be modeled here? Some crops have higher price risk than others. In particular, one channel leading to differences in price risk is differences in yield risk. When yield is more uncertain (because the crop is more sensitive to weather shocks and growing conditions), the world price is more uncertain and expectations about what that price will be at the start of the growing season are correspondingly noisier. On the other hand, when a crop is very robust to weather shocks and growing conditions, so that yield fluctuates very little, the world price also fluctuates less, and expectations about that price are much sharper at the start of the growing season. Hence, in this exercise, we can think of risky crops as having high mean and variance of yield, and noisy expectations about price, while safe crops have low mean and variance of yield, and sharp expectations about price.

Now, consider a setting where higher mean crops come at an incredibly high cost of price and yield risk. In particular, the marginal variance cost is convex in mean returns. Suppose the government wishes to encourage producers to grow higher mean crops, and implements a variable tariff that sets a price band around the prices of the riskiest (highest mean) crops. This policy causes aggregate risk to fall, since $V(p)$ is weakly smaller for all $p > 0$, and decrease the variance $V(p)$ by a larger amount for higher p crops.

It will be helpful to employ a simple functional form characterization to analyze this effect. Specifically, suppose that in the status quo, the profits of a crop with mean p followed this distribution: $\pi_p \sim N(p, p^{N_1})$. Following the stabilization policy, the distribution of profits is $\pi_p \sim N(p, p^{N_2})$, where $N_1 > N_2$: that is, $V(p)$ fell for each p, and became less convex.

We know from Proposition 2 that if $R_p \sim N(p, p^N)$, the unique equilibrium matching pattern is NAM if $N > 2$ and the unique equilibrium matching pattern is PAM if $N \in (1, 2)$. Hence, suppose $N_1 > 2$, and consider a price stabilization policy which reduces the convexity of variance so that $N_2 \in (1, 2)$.

We want to characterize the aggregate and distributional welfare effects of this policy, taking the total sum of certainty-equivalents to be the social welfare function.

Note that using the power function as a functional form for $V(p)$ has one small drawback. We want to analyze a policy that reduces the variance of every project (reduces the riskiness of an environment), and particularly reduces the variance of the riskiest projects, which makes "decreasing N" a natural choice for representing this policy. However, when N falls, the variance of the projects $p \in (0, 1)$ actually increases. To focus on our policy analysis, it is most straightforward to simply assume that the population of risk types in $G1$ and $G2$ is such that no possible pair ever wishes to undertake a project $p \in (0, 1)$. So, assume:

$$\frac{\overline{r}_1 + \overline{r}_2}{\overline{r}_1 \overline{r}_2} \geq \frac{N_1}{2}$$

where $\overline{r}_1$ is the most risk-averse agent in $G1$, and $\overline{r}_2$ is the most risk-averse agent in $G2$. This is simply for convenience, and places no substantive restrictions on the intuition or the policy analysis, especially as my matching results are distribution-free. Now, decreasing N unambiguously reduces the riskiness of the project environment and captures our policy experiment.

There are two effects of altering the shape of $V(p)$: first, there is a pure effect from decreasing the riskiness of an environment (which causes any matched pair to select a higher p project), and second, there is the endogenous network response resulting from it, which then further affects welfare.

We know the expression for a partnership's sum of certainty-equivalents:

$$\begin{aligned} CE(r_1, r_2) &= p^*(r_1, r_2) - \frac{1}{2}\frac{r_1 r_2}{r_1 + r_2} V(p^*(r_1, r_2)) \\ &= V'^{-1}\left(\frac{2(r_1 + r_2)}{r_1 r_2}\right) - \frac{1}{2}\frac{r_1 r_2}{r_1 + r_2} V\left(V'^{-1}\left(\frac{2(r_1 + r_2)}{r_1 r_2}\right)\right) \end{aligned}$$

Suppose first that the matching pattern (NAM) was held fixed when the riskiness of the environment was reduced. What would happen to aggregate CE?

$$CE(r_1, r_2) = e^{\frac{1}{N-1}\log\left(\frac{1}{N}\frac{2(r_1+r_2)}{r_1 r_2}\right)}\left[1 - \frac{1}{N}\right]$$

$$\frac{d}{dN} = -\frac{1}{N(N-1)}\log\left[\frac{1}{N}\frac{2(r_1 + r_2)}{r_1 r_2}\right] e^{\frac{1}{N-1}\log\left(\frac{1}{N}\frac{2(r_1+r_2)}{r_1 r_2}\right)} < 0$$

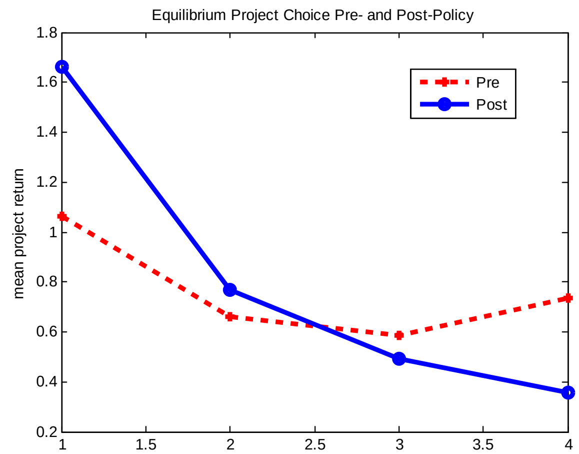
Hence, it is straightforward to see that, in the absence of an endogenous network response, decreasing N is strictly welfare-improving, both in the aggregate sense and in the individual (per pair) sense. In other words, if $N_1 > 2$ fell to $N_2 \in (1, 2)$, but the match continued to be negative-assortative, then all pairs would have higher certainty-equivalents and the sum of certainty-equivalents would increase.

However, as the main result of this paper shows, unless there are exogenous rigidities in matching, the risk-sharing network will *not* stay fixed. In this example, if $N_1 > 2$ falls to $N_2 \in (1, 2)$, the unique equilibrium matching pattern switches from negative-assortative to positive-assortative.

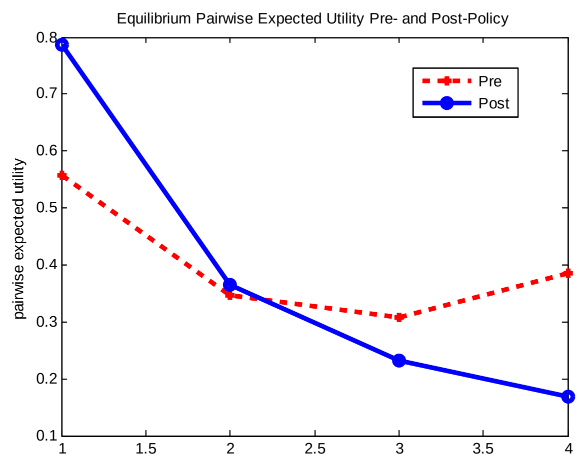
A numeric example clearly illustrates the mechanism by which this policy could have substantial adverse effects on welfare, particularly for the most risk-averse agents. For example, suppose $G1 = \{1, 2, 3, 4\}$, and $G2 = \{2, 4, 6, 8\}$. I want to focus on the welfare effect of the policy deriving from the endogenous change in equilibrium matching, and shut down the effect deriving from a change in the curvature of $V(p)$. A natural way to do this is to consider a *small but discrete* change in N, from a value of N under which NAM is the unique equilibrium, to a value of N under which PAM is the unique equilibrium.

For example, let $N_1 = 2.1$ and $N_2 = 1.9$. So, the levels of the function change very little, but the marginal variance cost is convex pre-policy and concave post-policy, which we know will shift the unique equilibrium match from NAM to the other extreme.

Equilibrium project choice then experiences the following change: the red line is before the policy, and the blue line is after the policy:



The sum of certainty-equivalents for matched pairs experiences the following change:

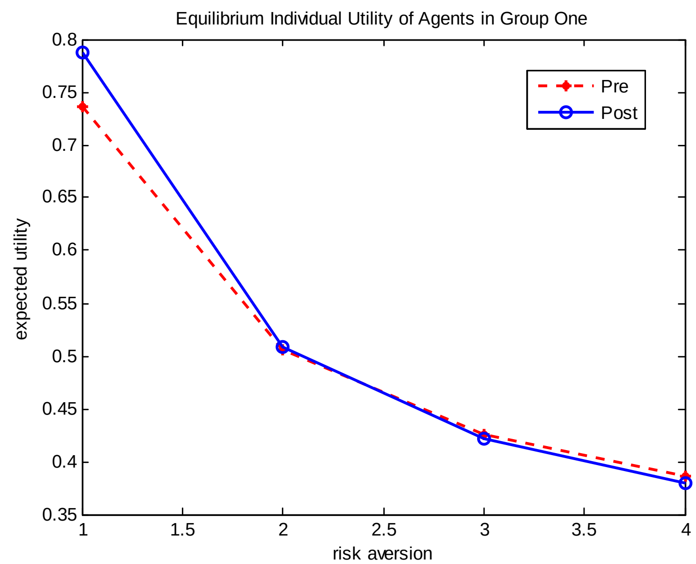


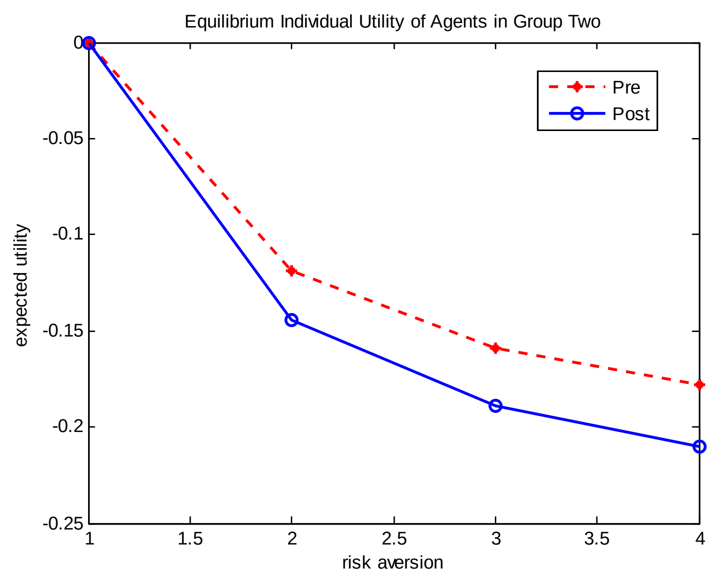
Clearly, there is more dispersion and inequality in joint expected utilities post-policy, where the most risk-averse agents partnered with each other are the worst off. However, we might also want to know what happens to individual utilities. Such analysis is challenging in a model of endogenous matching, as the vector of equilibrium individual expected utilities is not unique (please refer to the discussion in the Results section). Rather, there is a degree of freedom, which is the equilibrium expected utility received by one agent. Once this is specified (for any agent), the equilibrium expected utilities of each individual in the rest of the population are pinned down.

Because risk reduction policies are generally intended to target the most risk-averse people, let us study the vector of equilibrium expected utilities where the most risk-averse agent in group two[20] is best off–that is, the most risk-averse agent in group two has a higher utility in this vector than any other possible vector of equilibrium expected utilities.

Comparing pre- and post-policy individual utilities of group one and group two:

[20] Equivalently, group one.





We see that the more risk-averse agents in group one and group two are worse off after implementation of the policy, purely as a result of the endogenous network response: a price stabilization policy intended to reduce the risk burden of the poor, and particularly the risk of the higher mean yield crops, may actually decrease aggregate welfare[21] and increase the risk borne by the most risk-averse agents, precisely because the policy causes the least risk-averse agents to abandon their roles as informal insurers of the most risk-averse, in favor of entrepreneurial partnerships with fellow less risk-averse agents.

This intuition adds interestingly to a few existing results. Attanasio and Rios-Rull (2000) model the introduction of formal insurance as a policy which reduces the riskiness of the environment. They also find that such a policy may hurt the welfare of the most risk-averse agents. However,

[21] As measured by the total sum of certainty-equivalents.

their model of (non-endogenous) informal risk-sharing, which builds off Ligon, Thomas, Worrall (2001), centers around limited commitment–two agents interacting repeatedly can only sustain informal risk-sharing with the threat of cutting off all future ties for someone who reneges. This particular type of punishment has the important implication that the value of future ties directly trades off with equilibrium risk-sharing. So, anything that lowers the cost of autarky (remaining alone and being informally uninsured) will decrease the level of informal insurance that can be sustained. They then frame the introduction of formal insurance as a policy that would reduce the cost of being in autarky, and subsequently argue that the level of informal insurance falls, which may reduce aggregate welfare.

However, in this paper, commitment is taken to be perfect. Perfect commitment in Attanasio and Rios-Rull (2000) would preclude the conclusion that the introduction of formal insurance could hurt the most risk-averse agents, because lowering the cost of autarky matters only through the punishment of cutting off future ties, which is no longer relevant. I show that, *even under perfect commitment*, introducing formal insurance might still reduce the welfare of the most risk-averse agents, because the composition of the informal risk-sharing network changes in response. In my model, reducing the riskiness of the environment raises the value of autarky, but it *also* raises the value of being in a relationship, and raises it heterogeneously across partnerships of different risk compositions.

By contrast, Chiappori et. al. (2011) estimate that the *least* risk-averse individuals are the ones left worse off after the introduction of formal insurance. The informal argument is that the least risk-averse agents are displaced as informal insurers. However, this exactly illuminates the need for a model of the *equilibrium network* of relationships–I show that the least risk-averse agents do leave their roles as informal insurers, but only because they prefer to undertake entrepreneurial pursuits instead. It would be interesting to see how the estimation changes after accounting for the endogeneity of matching.

5 Testing the Theory: An Experiment

This model provides a natural setting for an experimental test of the theory. In this section, I propose a basic experimental design to test the key predictions. First, I discuss subject and setting choice. Next, I describe important elements of the setup. In particular, I describe how to elicit risk type, and how to design different sets of gamble choices. Finally, I outline the empirical strategy suggested by the main theorem and corollary from Section 4.

5.1 Subject and Setting Choice

The theoretical framework is a good fit for an agricultural context. As discussed in the introduction and background sections, farmers in a broad cross-section of regions are demonstrably risk-averse, and heterogeneous in their risk aversion. Moreover, this heterogeneity in risk aversion translates into a heterogeneity in equilibrium behavior; in particular, crop portfolios, spatial plot cohices, input and irrigation choices, and openness to innovative techniques varies by farm. Finally, there are plenty of examples of the prevalence of collectivity in farming–farmers seeking advice from each other, farmers entering risk-sharing agreements and self-help groups with each other, and most directly, landowners employing farmers, for example as sharecroppers.

This suggests several advantages in selecting subjects from villages which are primarily agricultural, and, correspondingly, recruiting subjects for whom farming is the primary occupation. This is not because the experimental setup should require subjects to possess farming expertise, but rather because the strong connection between their lives and the task of choosing amongst gambles with differing yield distributions gives us a measure of confidence about the relevance and intuitiveness of the experiment, and their subsequent comprehension of it.

Since matching pattern regressions will be run at the village level, it will be necessary to have a reasonably large sample of villages to obtain power. Because the theoretical result is distribution free, it is able to accommodate heterogeneity in risk type distributions across villages. Villages should be chosen which do not communicate with each other, to prevent information about the experiment from spreading. Characteristics of the villages should be recorded, such as geographical location, agroclimate, size, primary occupation, average income, and proximity to urban center.

Field officers should hold private interview sessions with randomly-chosen households in each village to extend an invitation to send a member to participate in the experiment. As in Attanasio et. al. (2010), this should be done only a few days previous to the experiment, to minimize social selection. Standard characteristics of each interviewed household, and of each adult member of that household, should be collected, such as ethnicity and religion, physical characteristics of the dwelling, occupations, income. If the primary occupation is agricultural, collect data on crops grown, plot size, plot scattering, irrigation, fertilizer, and total harvest.

Additionally, kinship, friendship, neighbor, trust, and employment network data must be collected from households. For example, each interviewed individual may be asked for the names of her relatives, her friends, her neighbors, the people she trusts most, and the people for whom she works/whom she employs/with whom she is business partners.

5.2 Setup

5.2.1 Round 1: Risk Elicitation

The experiment will consist of two rounds. In the first round, the "risk type elicitation round", each subject will meet privately with a field researcher, where she will be presented with a set of gambles varying in riskiness and asked to choose one. (The design of this set of gambles is discussed in detail in the subsection below, "Round 2: Joint Project Choice".) The gamble choice will be explained pictorially in a manner that accounts for potential illiteracy and unfamiliarity with probability, and comprehension will be tested. A selection of a gamble with higher mean and variance will indicate a lower degree of risk aversion than a selection of a gamble with lower mean and variance. Additionally, each subject will be asked to report what gamble they think the people they reported trusting the most during the initial household interview would have chosen. This allows us to get a sense of how accurately individuals perceive the risk aversion of those with whom they are close.

After all risk types have been elicited[22], subjects will be gathered in the same room, where each individual's gamble choice will be posted publically, again in a manner that accommodates illiteracy. (Subjects are *not* told initially that their first round gamble choices will be announced.) The purpose of this announcement is to ensure common knowledge of everyone's risk types.

5.2.2 Between Rounds: Matching

Following the elicitation of risk types and the public posting of the gamble choices, subjects shall be randomly divided into two equally-sized groups (if there is an odd number of people, then one person shall remain unmatched). A researcher will then explain that they will be allowed to pair up across groups, and that once paired, each pair will be offered the same set of gambles that they faced in their private first rounds, where they will be allowed to choose a project jointly and commit *ex ante* to a return-contingent sharing rule of the realized income by reporting their agreed-upon rule to a researcher, who will enforce it. (Arguments can be made for or against the possible dependence of the second round gamble choice on first round gamble outcomes if the set of gambles is the same in both rounds, and as Attanasio et. al. (2010) have set a precedent of reusing the first round set, I leave discussion of this question to another forum).

After this explanation of the proceedings of the rest of the game, the subjects will be given half an hour to establish partnerships (if an individual chooses to remain unmatched, she will receive 0 for sure because she will not be allowed to select a gamble. Since the supports of all gambles are positive, nobody should choose to remain unmatched as long as there are partners available).

5.2.3 Round 2: Joint Project Choice

Once partnerships have been established, researchers will visit each pair privately to take down their joint project choice and the sharing rule they have agreed upon. Then, researchers will draw a realization from each pair's project choice, and distribute funds according to the sharing rules that were reported to them. This will mark the end of the experiment.

The key to the experimental test of the theory is the design of the set of gambles from which matched pairs are asked to choose. Recall that the conditions for unique positive-assortative and negative-assortative matching emerging from the model in Section 4 centered on the rate of curvature of the mean-variance tradeoff across the spectrum of available projects. In the model, the tradeoff between the desire to match with someone with a different risk type in order to smooth a given risk in a mutually satisfying way, and the desire to match with someone with a similar risk type in order to choose a mutually agreeable risk, is captured by this rate of curvature of the

[22] The Rabin (2000) critique does apply here, but many experiments elicit risk in the literature in this way, so by revealed preference the payoff in economic insight seems to be worth it. Moreover, in this paper, if the environment is such that the marginal variance cost function is globally convex or concave, then all we need from the risk elicitation round is the *ordinal* risk attitudes.

mean-variance tradeoff. When a slightly higher mean return corresponds with a drastically higher variance of return, so that the difference in variance of a low mean return project and a high mean return project is extremely stark, negative-assortative matching is more likely to result, since all possible pairs prefer the safer, low mean return project. On the other hand, when a small increase in mean return corresponds to a moderately higher increase in variance, positive-assortative matching is more likely to result, since there is significant disagreement about preferred project: less risk-averse people prefer riskier projects, while more risk-averse people prefer safer projects.

Hence, the theory suggests a clear intuition for designing sets of gamble choices: within a given set, gambles should differ in mean and variance of return, with higher mean gambles having correspondingly higher variances, and the shape of this mean-variance tradeoff should differ across different sets of gamble choices.

For example, a reasonable approach would be to design two types of gamble sets, each containing five gambles. Villages in the sample would be randomly allocated into two groups, where villages in the first group face the first set of gamble choices, and villages in the second group face the second set of gamble choices. Indexing set by superscript i, and gamble by subscript j, the challenge is to choose values for (p_j^i, V_j^i), the mean and variance of gamble j in set i, for $i \in \{1, 2\}$ and $j \in \{1, 2, 3, 4, 5\}$.

There are three significant feasibility constraints imposed by the realism of an experiment that prevent us from directly applying the theoretical result to this design problem.

1. The return distributions of the gambles must have *positive, bounded supports.* It seems unwise to extract money from an experimental subject by declaring that she received a "negative realization", and it seems expensive to allow for the possibility of a realization of infinity.

   The relevant implication is that we cannot offer gambles with distributions that are literally distributed normally.

2. The set of gamble choices is *discrete*. The key idea in the theoretical model was not that a continuum of projects existed literally, but rather that activity diversification (e.g. intercropping) enabled a spectrum of mean returns with corresponding variances that were higher for higher means (an upper envelope).

   In the experiment, subjects are not allowed to create portfolios of projects; instead, they must select a single gamble. This can be thought of as reflecting rigidities in activity diversification (e.g. intercropping is not possible because of soil type), and limits a pair's *ex ante* risk management ability. An individual r's desire to pair up with another individual r because they share project preferences is dampened by the fact that first, their favorite project is unlikely to be available in the small set of gamble choices, and second, because there is not a wide range of choices, individuals of other risk types will cluster with r on project choice.

   The relevant implication of this is that distinct pairs are no longer guaranteed to select distinct projects. Instead, there will be "clustering" on each of the gamble choices; the degree of this "clustering" depends on the distribution of risk types in the population and on the shape of the mean-variance tradeoff.

3. *Limited liability* holds in an experimental setting. In the theory, the return-contingent sharing rule of a pair is completely unconstrained. In the experiment, this will not be the case. If for example a gamble's realized return is 0, then each partner must receive 0.

   This constraint is the most serious, because it inhibits the transferability of expected utility. For example, if 0 is in the support of a gamble's return distribution, then partners cannot agree to nonzero state-independent transfers. This restriction on the sharing rule limits the pair's *ex post* risk management ability, and, importantly, limits different pairs differently depending on risk type composition. A quite risk-averse individual r's desire to pair up with a much less risk-averse individual $r' << r$ is dampened by the fact that, because of limited liability, r' may not be able to ensure her a return-independent transfer.

   Hence, the pairwise sum of certainty-equivalents calculated in Section 2 holds only a bounded range of divisions v of the expected utility pie, where these bounds depend on risk type and therefore influence endogenous partner choice.

Because of these three limitations of the experimental environment, it is necessary to construct an "experimental model" satisfying these constraints which captures the essence of the theory: given a population of heterogeneously risk-averse agents, the risk relationship of the gambles to each other in the available choice set determines whether positive-assortative or negative-assortative matching results.

The idea is the following. Recall that the sum of $N > 2$ independent draws from a $uni[a, b]$ distribution approximates a normal distribution on the support $[Na, Nb]$. For example, if $a = 0$ and $b = 1$, the convolution of the N densities is (figure taken from Grimstead and Snell, Chapter 7, Intro to Probability 1997):

This allows us to offer gambles whose returns are approximately normally-distributed on bounded, positive supports. This ensures feasibility of experimental payments and allows us to apply our understanding of the role played by the mean-variance tradeoff from the theoretical results of Section 3.

Therefore, experimental subjects should be asked to choose from one of the following five gambles:

1. Receive the sum of N independent draws from a distribution which yields x for sure. (This is the safe choice.)

2. Receive the sum of N independent draws from a $uni[a_2, b_2]$ distribution, where:

$$\begin{aligned} a_2 &< x < b_2 \\ \frac{a_2 + b_2}{2} &> x \end{aligned}$$

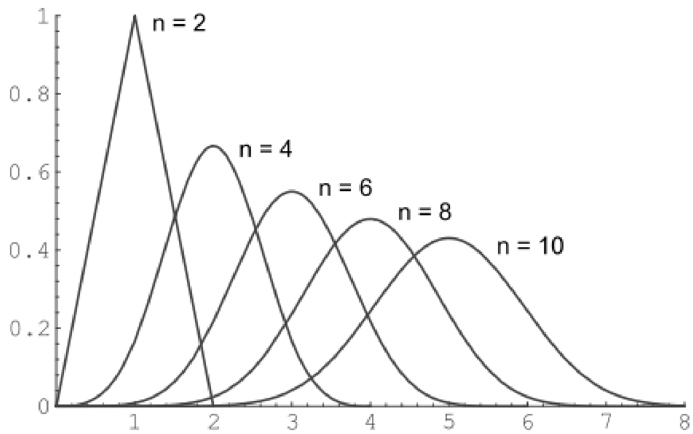


Figure 7.6: Convolution of n uniform densities.

so that this gamble has a higher mean but also a higher variance than the safe choice.

3. Receive the sum of N independent draws from a $uni[a_3, b_3]$ distribution, where:

$$\begin{aligned} a_3 &< a_2 \\ b_3 &> b_2 \\ \frac{a_3 + b_3}{2} &> \frac{a_2 + b_2}{2} \end{aligned}$$

4. Receive the sum of N independent draws from a $uni[a_4, b_4]$ distribution, where:

$$\begin{aligned} a_4 &< a_3 \\ b_4 &> b_3 \\ \frac{a_4 + b_4}{2} &> \frac{a_3 + b_3}{2} \end{aligned}$$

5. Receive the sum of N independent draws from a $uni[a_5, b_5]$ distribution, where:

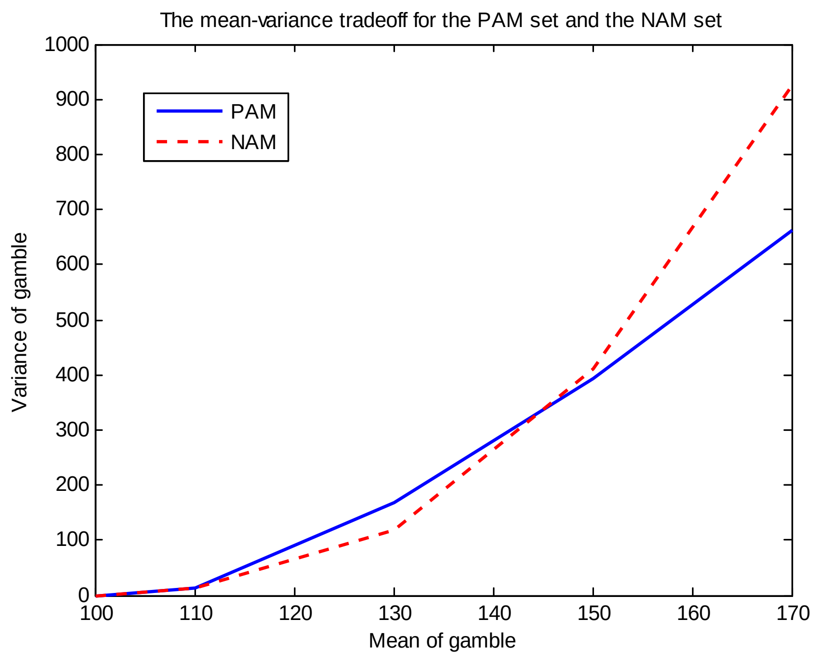
$$\begin{aligned} a_5 &< a_4 \\ b_5 &> b_4 \\ \frac{a_5 + b_5}{2} &> \frac{a_4 + b_4}{2} \end{aligned}$$

Each gamble has an approximately normal distribution on the support $[Na_k, Nb_k]$, and gambles with higher means also have higher variances. The values of a_2, a_3, a_4, a_5 and b_2, b_3, b_4, b_5 must be chosen to shape the curvature of the mean-variance tradeoff across gambles so that positive-assortative matching is supported in one set, and negative-assortative matching is supported in the other set.

It is useful to consider a concrete example. Draw $N = 10$ times independently from whichever distribution is chosen; the pair receives the realized sum of the draws. The two sets of gamble choices are:

	PAM	NAM
x :	10	10
$[a_2, b_2]$:	$[9, 13]$	$[9, 13]$
$[a_3, b_3]$:	$[6, 20]$	$[7, 19]$
$[a_4, b_4]$:	$[4, 26]$	$[4, 26]$
$[a_5, b_5]$:	$[3, 31]$	$[0, 34]$

The following figure shows the mean-variance tradeoff across gambles for the PAM set and for the NAM set:



It is worth noting the novelty of the design necessitated by the theory of endogenous risk-sharing pair formation under *both* income-smoothing and consumption-smoothing. In Attanasio et. al. (2010), for example, all experimental subjects faced the same set of gambles, which differed in mean and variance. However, the risk-sharing rule was imposed to be an equal division. Fixing the sharing rule at equal division has a variety of implications: first, positive-assortative matching will result, because the consumption-smoothing/gains-from-trade channel is shut down. (Additionally, an equal sharing rule automatically satisfies limited liability.) We do not need the normal distribution to clarify the insight–this result can be proven for a general return distribution–and so we do not need the trick of summing independent draws from uniform distributions. Consequently, the specific shape of the mean-variance tradeoff across the available gambles is not important, as long as higher mean projects also have higher variance.

Allowing for consumption-smoothing by endogenizing a risk-sharing group's sharing rule, in addition to endogenous choice of income stream, entails the precise design of different sets of gamble choices. Since the effect of the tradeoff between consumption-smoothing and income-smoothing on equilibrium matching is best distilled with normally-distributed returns, one idea is to replicate such a distribution in a feasible way by summing N independent draws from uniform distributions, which differ in mean and variance.

5.3 Empirical Strategy

Recall from Proposition 4 that support for the theory in the data would be observation of at least one of the following: (a) people match positive-assortatively in risk attitudes, and the observed means of equilibrium risky projects are convex in the representative rates of risk aversion of matched pairs; (b) people match negative-assortatively in risk attitudes, and the observed means of equilibrium risky projects are concave in the representative rates of risk aversion of matched pairs.

The basic empirical strategy consists of 3 steps. First, given a dataset containing Z villages, index a village by k. Then:

1. For each village k, characterize the equilibrium matching pattern (the correlation between the probability that individual i is matched with individual j and their corresponding risk types r_i and r_j). Is the equilibrium matching pattern the "correct" one, that is, the pattern which was meant to be induced by the set of gamble choices offered to that village?

2. Next, examine the behavior of equilibrium project choices and the risk composition of the matched pairs.

3. Then, characterize the relationship between the equilibrium matching pattern and the equilibrium behavior of project choice in risk types across all villages. If we observe convexity of project choice in the representative rates of risk aversion of matched pairs corresponding with positive-assortative matching, and/or concavity of project choice in the representative rates of risk aversion of matched pairs corresponding with negative-assortative matching, then this is support for the theory.

To be a little more specific, first focus on data from a given village k. Suppose that village k was offered the "positive-assortative" set of gamble choices, so we should expect positive-assortative matching.

To glean the matching pattern from the data, set the observation level to be a dyad (i, j) of an individual i in group one matched with an individual j in group two (recall that the experiment participants were randomly divided into two equally-sized groups). If each group has M people in it, then there should be $\frac{M(M-1)}{2}$ total observations.

Define a variable that describes the distance between the risk attitudes of i and j:

$$\Delta_{ij} = |r_i - r_j|$$

Note that this is symmetric: $\Delta_{ij} = \Delta_{ji}$.

Now, define a variable m_{ij}:

$$m_{ij} = \begin{cases} 1, & i \text{ matched with } j \\ 0, & else \end{cases}$$

So, m_{ij} is also symmetric.

Then, for each village k, regress the linkage variable m_{ij} on the distance between risk attitudes, Δ_{ij}, and the pre-existing networks X_{ij}^k (e.g. kinship):

$$m_{ij}^k = \alpha^k + \mu^k \Delta_{ij}^k + X_{ij}^k + \varepsilon_{ij}^k$$

If $\mu^k > 0$, the matching pattern is closer to being negative-assortative, since large differences in risk attitudes increase the probability that two individuals are linked, while if $\mu^k < 0$, the matching pattern is closer to being positive-assortative, since small differences in risk attitudes increase the probability that two individuals are linked.

Confirm that the matching pattern of village k is the pattern that was meant to be induced by the set of gamble choices offered to village k.

Now, look at matched dyads in village k only, and denote the mean of the equilibrium project choice of a matched (i, j) by p_{ij}.

For each matched dyad (i, j), calculate the representative rate of risk aversion:

$$\tilde{H}_{ij} = \frac{1}{r_i} + \frac{1}{r_j}$$

For the matched dyads in each village k, regress:

$$p_{ij}^k = c^k + \theta^k r_i^k + \pi^k r_j^k + \gamma^k \left(\tilde{H}_{ij}^k\right)^2 + V^k + \eta_{ij}^k$$

where V^k is village fixed effects.

It should be the case that $\theta^k, \pi^k < 0$ for all k, since more risk-averse individuals should choose safer (lower mean) gambles. If $\gamma^k > 0$, then equilibrium project mean is convex in the rate of risk aversion, and if $\gamma^k < 0$, then equilibrium project mean is concave in the rate of risk aversion.

Finally, regress:

$$\mu^k = \kappa + \delta \gamma^k + \nu$$

to ascertain the relationship in village k between the equilibrium matching pattern and the behavior of equilibrium projects in risk types.

Support for the model is $\delta > 0$: this indicates that convexity of mean income in the representative rates of risk aversion of a linked pair corresponds with positive-assortative matching, while concavity of mean income in the representative rates of risk aversion of a linked pair corresponds with negative-assortative matching.

6 Conclusion

This paper proposes a novel framework for thinking about endogenous matching between heterogeneous agents as resulting from the tradeoff of two general forces: one, a "gains from trade" force, which favors matching between unlike types, and the other, a "similarity of perspective" force, which favors matching between like types. Specifically, conditional on facing a given risk, a less risk-averse agent and a more risk-averse agent benefit from being matched with each other, because the less risk-averse agent is willing to sell insurance which the more risk-averse agent is willing to buy. But conditional on being able to choose what risk to face, agents prefer partners with similar risk attitudes. In other words, consumption-smoothing forces push the match to be negative-assortative, while income-smoothing forces push the match to be positive-assortative.

I show that the equilibrium composition of risk-sharing pairs depends critically on the interaction between consumption-smoothing and income-smoothing as risk management tools, and in particular the differential evaulation of this interaction across agents of different risk attitudes. I show that this interaction is cleanly captured by the convexity of the marginal variance cost function in expected returns of projects, and provide testable conditions that tie the theory to data. In particular, I describe observable differences in the environments in which each type of assortative matching pattern emerges, and use these differences to construct a testable falsifiability condition based on the relationship of the mean returns of equilibrium projects to the risk composition of matched pairs. I find preliminary support for the theory in existing literature.

But why should we care about understanding these matching forces? This theory has a variety of meaningful implications for policy. For example, even under perfect commitment in the contracting environment, a risk-reduction policy which reduces the risk of higher mean, higher variance projects in particular, may decrease aggregate welfare and leave the most risk-averse agents strictly worse off, purely because of the endogenous network response: the least risk-averse agents abandon their roles as "informal insurers" of the most risk-averse, preferring entrepreneurial partnerships with fellow less risk-averse agents instead.

Accounting for endogeneity would also affect the analysis of a variety of land reform policies discussed in the literature. For example, Banerjee (2000) points out that the effects of land redistribution cannot be estimated without understanding the reasons behind the status quo distribution of land in the first place. If the distribution of land served a risk-sharing purpose, or alternatively, if agents optimized their risk-sharing activities given the land distribution (for example, large landowners provide informal insurance to the landless), then land redistribution may either have no effect, as people implicitly ignore it, or may actually hurt informal insurance.

Improving the security of tenant farmers is another common land reform. For example, Banerjee et. al. (2002) discuss Operation Barga, a 1970s West Bengali land reform which diminished the ability of landlords to expel their tenants. An unexplored effect of such a reform is the added matching friction: relationships will be unable to respond optimally to changes in the environment. For example, if a bad rainfall shock unexpectedly hits a farming village, increasing the steepness of the mean-variance tradeoff across crop choices, it would be efficient for less risk-averse individuals to embrace their role as informal insurers and provide insurance to more risk-averse individuals on safer crops. If Barga had locked less risk-averse individuals in their relationships with other less risk-averse individuals before the shock, they will not be able to adjust towards efficiency. On the other hand, if Barga had locked the less risk-averse into relationships with the more risk-averse, and then the government began subsidizing innovations (reducing the variance of high mean projects), the less risk-averse individuals would like to switch to less risk-averse partners and take up these innovations, but will be legally prevented from doing so.

Finally, accounting for endogeneity may substantially alter our inferences from empirical observations. For example, an observation that the strength of dependence of a sharecropper's rent contract on realized harvest is *positively* correlated with the riskiness of the crop grown may tempt us to conclude that risk is not a problem for farmers, since risk considerations do not appear to influence contract design (Allen and Lueck (1992)). However, if more risk-averse farmers work for less risk-averse landowners, cultivating safer crops under low-powered contracts, while less risk-averse farmers work for more risk-averse landowners, cultivating riskier crops under high-powered contracts, the same empirical observations would emerge, but risk concerns would be playing a significant role in contract design, through the unaccounted-for channel of contracting partner choice.

All sorts of challenges remain. This model can clearly be enriched in many ways–by generalizing preferences and the project space, by introducing limited commitment or incomplete information, by allowing for more complex forms of group formation, or by adding dynamics to study the feedback between riskiness of project choice and investment capability, to characterize a potential innovation trap. The main contribution of this paper is to propose a tractable model of the endogenous formation of risk-sharing relationships, upon which further dimensions can be layered, and to show that even this simple model can lead to substantial changes in our understanding of numerous policy debates. The unique environments of developing economies make design problems particularly challenging, but it is the magnitude of the stakes which makes them particularly important.

7 References

1. Ackerberg, D. and M. Botticini. "Endogenous Matching and the Empirical Determinants of Contract Form." Journal of Political Economy (110) 2002 564-590.

2. Akanda, M., H. Isoda, and S. Ito. "Problem of Sharecrop Tenancy System in Rice Farming in Bangladesh: A Case Study on Alinapara Village in Sherpur District." Journal of Int. Farm Management (4) 2008 23-30.

3. Alderman, H. and C. Paxson. "Do the Poor Insure? A Synthesis of the Literature on Risk and Consumption in Developing Countries." Economics in a Changing World, vol. 4 1992 49-78.

4. Allen, D. and D. Lueck. "Contract Choice in Modern Agriculture: Cash Rent versus Cropshare." Journal of Law and Economics (35) 1992 397-426.

5. Antle, J. "Econometric Estimation of Producers' Risk Attitudes." Amer. Journal of Ag. Econ. (69) 1987 509-522.

6. Attanasio, O., A. Barr, J. Cardenas, G. Genicot, and C. Meghir. "Risk Pooling, Risk Preferences, and Social Networks." Amer. Econ. Journal: Applied Economics (4) 2012 132-167.

7. Attanasio, O. and J. Rios-Rull. "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare?" European Economic Review (44) 2000 1225-1258.

8. Banerjee, A. "Prospects and Strategies for Land Reforms." *Annual World Bank Conference on Development Economics 1999*, eds. B. Pleskovic and J. Stiglitz. 253-284.

9. Banerjee, A., P. Gertler, and M. Ghatak. "Empowerment and Efficiency: Tenancy Reform in West Bengal." Journal of Political Economy (110) 2002 239-280.

10. Bardhan, P. "Interlocking Factor Markets and Agrarian Development: A Review of Issues." Oxford Econ. Papers (32) 1980 82-98.

11. Basu, K. "Limited Liability and the Existence of Share Tenancy." J. Dev. Econ. (38) 1992 203-220.

12. Becker, G.S. "A Theory of Marriage: Part I." Journal of Political Economy (81) 1973 813-846.

13. Bellemare, M. "Sharecropping, Insecure Land Rights and Land Titling Policies: A Case Study of Lac Alaotra, Madagascar." Development Policy Review (27) 2009 87-106.

14. Besley, T. "Savings, Credit, and Insurance." Handbook of Development Economics (3) 1995 2123-2207.

15. Binswanger, H.P. "Attitudes towards Risk: Experimental Measurement in Rural India." Am. J. Agr. Econ. (62) 1980 395-407.

16. Bloch, F. et. al. "Informal Insurance in Social Networks." J. Econ. Theory (14) 2008 36-58.

17. Bold, T. "Implications of Endogenous Group Formation for Efficient Risk-Sharing." The Economic Journal (119) 2009 562-591.

18. Bramoulle, Y. and R. Kranton. "Risk Sharing Networks." J. Econ. Behavior and Organization (64) 2007 275-294.

19. Chandrasekhar, A. et. al. "Testing Models of Social Learning on Networks: Evidence from a Framed Field Experiment." Working Paper 2012.

20. Chiappori, P-A., K. Samphantharak, S. Schulhofer-Wohl, and R. Townsend. "Heterogeneity and Risk-Sharing in Village Economies." Working Paper 2011.

21. Chiappori, P-A. and P. Reny. "Matching to share Risk." Working Paper 2006.

22. *Insurance Against Poverty.* ed. Stefan Dercon. 2004.

23. Dalla Marta, A., M. Mancini, R. Ferrise, M. Bindi, and S. Orlandini. "Energy crops for biofuel production: Analysis of the potential in Tuscany." Biomass and Bioenergy (34) 2010 1041-1052.

24. Dawe, D. "How far down the path to free trade? The importance of rice price stabilization in developing Asia." Food Policy (26) 2001 163-175.

25. Dercon, S. "Risk, Crop Choice, and Savings: Evidence from Tanzania." Econ. Dev. and Cultural Change (44) 1996 483-513.

26. Dercon, S. and P. Krishnan. "In Sickness and in Health: Risk Sharing within Households in Rural Ethiopia." Journal of Political Economy (108) 2000 688-727.

27. Fafchamps, M. "Risk-sharing Between Households." Handbook of Social Economics 2008.

28. Fafchamps, M. and A. Lund. "Risk-sharing Networks in Rural Phillippines." J. Dev. Econ. (71) 2003 261-287.

29. Fafchamps and Gubert. "The Formation of Risk-Sharing Networks." J. Dev. Econ. (83) 2007 326-350.

30. Genicot, G. and D. Ray. "Group Formation in Risk-Sharing Arrangements." Rev. Econ. Stud (70) 2003 87-113.

31. Knudsen, O. and J. Nash. "Domestic Price Stabilization Schemes in Developing Countries." Economic Development and Cultural Change (38) 1990 539-558.

32. Kocherlakota, N. "Implications of Efficient Risk-Sharing Without Commitment." Review of Economic Studies (63) 1996 596-609.

33. Kurosaki, T. "Risk-Sharing Arrangements and the Structure of Risk and Time Preferences: Theory and Evidence from Village India." Working Paper 1999.

34. Legros, P. and A. Newman. "Beauty is a Beast, Frog is a Prince: Assortative Matching with Nontrasferabilities (75) 2007 1073-1102.

35. Ligon, E., J. Thomas, and T. Worrall. "Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies." Review of Economic Studies (69) 2002 209-244.

36. Minot, N. "Food Price Stabilization: Lessons for Eastern and Southern Africa." Risk Management in African Agriculture Seminar Presentation 2010.

37. Morduch, J. "Income Smoothing and Consumption Smoothing." J. Econ. Perspectives (9) 1995 103-114.

38. Rabin, M. "Risk aversion and expected utility theory: A calibration theorem." Econometrica (68) 2000 1281–1292.

39. Rathore, V.S., J.P. Singh, and R.K. Beniwal. "Rainfed Farming Systems of Hyper Arid Northwestern Rajasthan: an Analysis." Annals of the Arid Zone (48) 2009 125-131.

40. Rosenzweig, M. "Risk, Implicit Contracts, and the Family in Rural Areas of Low-Income Countries." Economic Journal (98) 1988 1148-1170.

41. Rosenzweig, M. and H.P. Binswanger. "Wealth, Weather Risk, and the Composition and Profitability of Agricultural Investments." The Economic Journal (103) 1993 56-78.

42. Rosenzweig, M. and O. Stark. "Consumption Smoothing, Migration, and Marriage: Evidence from Rural India." J. Polit. Econ. (97) 1989 905-926.

43. Schulhofer-Wohl, S. "Negative-assortative Matching of Risk-Averse Agents with Transferable Expected Utility." Econ. Letters (92) 2006 383-388.

44. Shapley, L. and M. Shubik. "The assignment game I: The Core." Intl. J. Game Theory (1) 1971 111-130.

45. Sharpe, William F. "The Sharpe Ratio". The Journal of Portfolio Management (21) 1994 49–58.

46. Singh, H., G. Singh, J. Samuel, and R. Pathak. "Precision Farming in Horticulture." Proceedings of the National Seminar Workshop on Hi-Tech Horticulture and Precision Farming, 2003.

47. Stiglitz 1974. "Incentives and Risk Sharing in Sharecropping." Review of Economic Studies (41) 1974 219-255.

48. Townsend, R. "Optimal Contracts and Competitive Markets with Costly State Verification." Journal of Economic Theory (21) 1979 265-293.

49. Townsend, R. "Risk and Insurance in Village India." Econometrica (62) 1994 539-591.

50. Townsend, R. and R. Mueller. "Mechanism Design and Village Economies: From Credit to Tenancy to Cropping Groups." Review of Economic Dyanmics (1) 1998 119-172.

51. Wright, B. "Storage and Price Stabilization." Handbook of Ag. Econ. (1) 2001 818-861.

8 Appendices

8.1 A1: The Model with General Distribution of Returns

Suppose all elements of the environment are the same, except now project p has returns distributed $R_p \sim f(R_p|p)$, $\Omega(R_p) \perp p$, for some well-defined pdf $f(R_p|p)$ whose moments exist.

Given a matched pair (r_1, r_2), agent r_1 solves:

$$\max_{s(R_p)} \int_{-\infty}^{\infty} -e^{-r_1[R_p - s(R_p)]} f(R_p|p) dR_p \ s.t.$$

$$\int_{-\infty}^{\infty} -e^{-r_2 s(R_p)} f(R_p|p) dR_p \geq -e^{-v}$$

Then the optimal sharing rule is

$$s^*(R_p) = \frac{r_1}{r_1 + r_2} R_p + \frac{1}{r_2} \log \int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} R_p} f(R_p|p) dR_p + \frac{v}{r_2}$$

where the optimal project solves:

$$\max_p -e^{\frac{r_1}{r_2} v} \left[\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} R_p} f(R_p|p) dR_p \right]^{1 + \frac{r_1}{r_2}}$$

Then the sum of certainty-equivalents of the pair is:

$$CE(r_1, r_2) = -\left(\frac{1}{r_1} + \frac{1}{r_2} \right) \log \int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} R_p} f(R_p|p^*(r_1, r_2)) dR_p$$

Denote the harmonic mean of (r_1, r_2) by H, and one-half the harmonic mean by:

$$\hat{H}(r_1, r_2) = \frac{r_1 r_2}{r_1 + r_2}$$

We know that a sufficient condition for the unique equilibrium matching pattern to be PAM is $\frac{dCE(r_1, r_2)}{dr_1 dr_2} > 0$, while we know that $\frac{dCE(r_1, r_2)}{dr_1 dr_2} < 0$ is sufficient for NAM to be the unique equilibrium matching pattern. What conditions are needed for these two inequalities to hold?

Recognize that the sum of certainty-equivalents is actually just:

$$\begin{aligned} CE(r_1, r_2) &= -\frac{1}{\hat{H}(r_1, r_2)} C_{R_p}(t = -\hat{H}(r_1, r_2)) \\ &= -\hat{H}(r_1, r_2) p + \sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n}}{(2n)!} k_{2n}(p) \end{aligned}$$

where $C_{R_p}(t)$ is the cumulant-generating function of the random variable R_p (where the cumulant-generating function is log of the moment-generating function).

Impose a symmetric distribution, since we know that all higher-order odd cumulants are zero.

Then it is clear that a given risk-averse agent r evaluates a project p by looking at its expected return and the sum of all its higher order cumulants. Specifically, the agent likes expected return, but expected return comes at a cost, which is a larger sum of higher order cumulants. This can be thought of as a "cost function" of projects, and can be thought of as sort of a "generalized variance".

The first-order condition characterizing optimal project choice is given by:

$$FOC_p : \sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n}}{(2n)!} k'_{2n}(p) = \hat{H}(r_1, r_2)$$

while we need the following condition for convexity:

$$SOC_p : \sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n}}{(2n)!} k''_{2n}(p) \geq 0$$

Then differentiate the first-order condition implicitly with respect to $\hat{H}$:

$$\frac{dp}{d\hat{H}} = \frac{1 - \sum_{n=1}^{\infty} \frac{\hat{H}(r_1,r_2)^{2n-1}}{(2n-1)!} k'_{2n}(p)}{\sum_{n=1}^{\infty} \frac{\hat{H}(r_1,r_2)^{2n}}{(2n)!} k''_{2n}(p)}$$

But the FOC tells us:

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n}}{(2n)!} k'_{2n}(p) &= \hat{H}(r_1, r_2) \Rightarrow \\ \sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n-1}}{(2n)!} k'_{2n}(p) &= 1 \end{aligned}$$

and

$$\sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n-1}}{(2n-1)!} k'_{2n}(p) > \sum_{n=1}^{\infty} \frac{\hat{H}(r_1, r_2)^{2n-1}}{(2n)!} k'_{2n}(p) = 1$$

So:

$$\begin{aligned} \frac{dp}{d\hat{H}} &< 0 \Rightarrow \\ \frac{dp}{dr_1} &= \frac{dp}{d\hat{H}} \frac{d\hat{H}}{dr_1} < 0 \\ \frac{dp}{dr_2} &= \frac{dp}{d\hat{H}} \frac{d\hat{H}}{dr_2} < 0 \end{aligned}$$

where $p^*(r_1, r_2)$ symmetric in r_1, r_2 since $\hat{H}(r_1, r_2)$ symmetric in r_1, r_2.

8.2 A2: Equivalence of model to alternative with individual project choice, pooled income, and IR

Suppose that a spectrum of projects $p \geq 0$ is available, where $R_p \sim N(p, V(p))$, where $V(p) > 0$, $V'(p) > 0$, and $V''(p) > 0$. Each agent r in $G1$ and $G2$ chooses a project p_r, and a matched pair pools their realized incomes and splits it according to a pre-decided pooled-return-contingent sharing rule, $s(R_{p_{r_1}} + R_{p_{r_2}})$.

Assume that the income realizations of r_1 and r_2 are independent (the return of project p_{r_1} is independent of the return of project p_{r_2}). Then the distribution of the pooled income stream is simply $f(R_p|p_{r_1}) * f(R_p|p_{r_2})$ the convolution, with mean $p_{r_1} + p_{r_2}$ and variance $V(p_{r_1}) + V(p_{r_2})$. (Actually, since I've imposed normal form, $R_{p_{r_1}} + R_{p_{r_2}} \sim N(p_{r_1} + p_{r_2}, V(p_{r_1}) + V(p_{r_2}))$.) Note that IR is clearly satisfied, because the autarkic sharing rule (where each person keeps her realized income) is always an option for the pair, but isn't chosen in equilibrium.

But note that $N(p_{r_1} + p_{r_2}, V(p_{r_1}) + V(p_{r_2}))$ for $p_{r_1}, p_{r_2} > 0$: $p_{r_1} = 1, p_{r_2} = 3$, then $N(4, V(1) + V(3))$, versus $p_{r_1} = 2, p_{r_2} = 2$, then $N(4, V(2) + V(2))$ since $V()$ convex, this means that any pair (r_1, r_2) would always choose $2, 2$ instead of $1, 3/3, 1$. Suppose $p_{r_1} + p_{r_2} = \hat{p}_{r_1} + \hat{p}_{r_2}$. then because $V()$ convex, the lower number is the one with smaller $|p_{r_1} - p_{r_2}|$ (e.g. if the sum is X, then the "best" sum of projects is $\frac{X}{2}, \frac{X}{2}$). We can partition all possible pairs (p_{r_1}, p_{r_2}) into equivalence classes by their sum–e.g. the class 4 is represented by the line $x + y = 4$. Then, call the variance of the project with mean 4 (class 4), $V(2) + V(2)$. In fact, for class X (which is the mean of the pooled return), let the variance of this be $2 * V(\frac{X}{2})$ (this is the smallest variance of the pooled return for all pairs of projects having the same expected pooled return). (We do this because we know that any risk-averse agent deciding between two projects with normally-distributed returns having the same mean would always choose the one with lower variance.)

This implies that we can think of this problem in the following way: a matched pair jointly selects a project X with return distributed $N(X, 2 * V(\frac{X}{2}))$, where we can define $\widetilde{V}(X) = 2 * V(\frac{X}{2})$, a convex function since $V()$ is convex. And, assume IR is satisfied, that is, that agents would rather match with any partner and start a project than being alone and not undertaking a project.

But this is the environment I study.

8.3 A3: Proof that NTU problem has TU representation in expected utility

We know from the appendix discussing the generalized model that the indirect utility of an agent r_1 who ensures his partner, r_2, a level of expected utility $-e^{-v}$ is:

$$\phi(r_1, r_2, v) = -e^{\frac{r_1}{r_2} v} \left[\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1 + r_2} R_p} f(R_p|p) dR_p \right]^{1 + \frac{r_1}{r_2}}$$

Partner r_2 of course receives:

$$EU_{r_2}(v) = -e^{-r_2 \frac{v}{r_2}}$$

Then the certainty-equivalent of each member is:

$$\begin{aligned} CE_{r_1}(v) &= -\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\log\left[\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p\right] - \frac{v}{r_2} \\ CE_{r_2}(v) &= \frac{v}{r_2} \end{aligned}$$

There is clearly a one-to-one tradeoff between the certainty-equivalent of r_1 and the certainty-equivalent of r_2. In other words, the slope of the Pareto frontier of expected utility is -1. Hence, expected utility is transferable.

It is also interesting to see that the generalized increasing/decreasing differences technique from Legros and Newman (2007) corresponds to the supermodularity and submodularity of $CE(r_1, r_2)$.

Recall the Legros-Newman generalized differences condition for matching under nontransferable utility: the following is a sufficient condition for positive-assortative matching to be the unique equilibrium matching pattern:

$$\begin{aligned} \phi(r_1, r_2, p^*(r_1, r_2), v) &= \phi(r_1, r_2', p^*(r_1, r_2'), u) \Rightarrow \\ \phi(r_1', r_2, p^*(r_1', r_2), v) &< \phi(r_1', r_2', p^*(r_1', r_2'), u)\ \forall r_1, r_2 \end{aligned}$$

We want to show that this is equivalent to a transferable utility approach, that is, showing that $\frac{dCE(r_1,r_2)}{dr_1 dr_2} > 0$, where $CE(r_1, r_2)$ is the sum of certainty-equivalents in a matched pair (r_1, r_2) (see previous appendix section).

We know from the proof of the optimal sharing rule and sum of certainty-equivalents in the general case that the indirect utility of agent r_1 when matched with agent r_2 and ensuring him expected utility $-e^{-v}$ is:

$$\phi(r_1, r_2, p^*(r_1, r_2), v) = -e^{\frac{r_1}{r_2}v}\left[\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p\right]^{1+\frac{r_1}{r_2}}$$

So

$$\phi(r_1, r_2, p^*(r_1, r_2), v) = \phi(r_1, r_2', p^*(r_1, r_2'), u) \Leftrightarrow$$

$$-e^{\frac{r_1}{r_2}v}\left[\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p\right]^{1+\frac{r_1}{r_2}} = -e^{\frac{r_1}{r_2'}u}\left[\int e^{-\frac{r_1 r_2'}{r_1+r_2'}R_p} f(R_p|p^*(r_1,r_2'))dR_p\right]^{1+\frac{r_1}{r_2'}} \Leftrightarrow$$

$$\begin{aligned} \frac{r_1}{r_2}v + \left(1+\frac{r_1}{r_2}\right)\log\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p &= \\ \frac{r_1}{r_2'}u + \left(1+\frac{r_1}{r_2'}\right)\log\int e^{-\frac{r_1 r_2'}{r_1+r_2'}R_p} f(R_p|p^*(r_1,r_2'))dR_p &\Leftrightarrow \end{aligned}$$

$$
\begin{aligned}
\frac{1}{r_2'}u - \frac{1}{r_2}v &= \left(\frac{1}{r_1}+\frac{1}{r_2}\right)\log\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p - \\
&\left(\frac{1}{r_1}+\frac{1}{r_2'}\right)\log\int e^{-\frac{r_1 r_2'}{r_1+r_2'}R_p} f(R_p|p^*(r_1,r_2'))dR_p
\end{aligned}
$$

Then the condition $\phi(r_1', r_2, p^*(r_1', r_2), v) < \phi(r_1', r_2', p^*(r_1', r_2'), u)$ is equivalent to:

$$
\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\log\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p -
$$

$$
\left(\frac{1}{r_1}+\frac{1}{r_2'}\right)\log\int e^{-\frac{r_1 r_2'}{r_1+r_2'}R_p} f(R_p|p^*(r_1,r_2'))dR_p >
$$

$$
> \left(\frac{1}{r_1'}+\frac{1}{r_2}\right)\log\int e^{-\frac{r_1' r_2}{r_1'+r_2}R_p} f(R_p|p^*(r_1',r_2))dR_p -
$$

$$
\left(\frac{1}{r_1'}+\frac{1}{r_2'}\right)\log\int e^{-\frac{r_1' r_2'}{r_1'+r_2'}R_p} f(R_p|p^*(r_1',r_2'))dR_p
$$

That is, the condition is equivalent to the following expression having increasing differences in (r_1, r_2):

$$
-\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\log\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p
$$

Now, let's find the certainty-equivalent of r_1 when she is paired with r_2 and ensuring him $-e^{-v}$:

$$
\begin{aligned}
-e^{-r_1 CE_{r_1}} &= \phi(r_1, r_2, p^*(r_1, r_2), v) \Rightarrow \\
CE_{r_1} &= -\frac{1}{r_1}\log\left[-\phi(r_1, r_2, p^*(r_1, r_2), v)\right]
\end{aligned}
$$

And the certainty-equivalent of r_2 is $\frac{v}{r_2}$. Then the sum $CE(r_1, r_2)$ is:

$$
\begin{aligned}
CE(r_1, r_2) &= -\frac{1}{r_1}\log\left[-\phi(r_1, r_2, p^*(r_1, r_2), v)\right] + \frac{v}{r_2} \\
&= -\frac{1}{r_1}\left[\frac{r_1}{r_2}v + (1+\frac{r_1}{r_2})\log\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p\right] + \frac{v}{r_2} \\
&= -\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\log\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p} f(R_p|p^*(r_1,r_2))dR_p
\end{aligned}
$$

But this is exactly the same condition we got from the generalized differences condition from Legros-Newman.

The argument is analogous for showing that Legros-Newman generalized decreasing differences condition for negative-assortative matching to be the unique equilibrium matching pattern is equivalent to showing that $\frac{dCE(r_1,r_2)}{dr_1 dr_2} < 0$.

8.4 A4: Proof of main matching result

The cumulant-generating function for a normal distribution $N(p, V(p))$ has a very nice form:

$$C_{R_p}(t = -\widehat{H}|p) = -\widehat{H}p + \frac{\widehat{H}^2}{2}V(p)$$

Hence:

$$CE(\widehat{H}) = p - \frac{1}{2}\widehat{H}V(p)$$

Recall that:

$$\widehat{H} = \frac{r_1 r_2}{r_1 + r_2}$$

Then:

$$\frac{dCE(r_1,r_2)}{dr_1 r_2} = -\frac{r_1 r_2}{(r_1+r_2)^3}V\left(V'^{-1}\left(\frac{2(r_1+r_2)}{r_1 r_2}\right)\right) + \frac{2}{r_1 r_2 (r_1+r_2)}V'^{-1\prime}\left(\frac{2(r_1+r_2)}{r_1 r_2}\right)$$

where the project chosen optimally by a pair (r_1, r_2) is:

$$p^*(r_1, r_2) = V'^{-1}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right]$$

Therefore, $\frac{dCE(r_1,r_2)}{dr_1 dr_2} > 0$ iff:

$$\frac{2(r_1+r_2)^2}{r_1^2 r_2^2}V'^{-1\prime}\left(\frac{2(r_1+r_2)}{r_1 r_2}\right) > V\left(V'^{-1}\left(\frac{2(r_1+r_2)}{r_1 r_2}\right)\right)$$

Then, define:

$$g(x) = \frac{x^2}{2}V'^{-1\prime}(x) - V(V'^{-1}(x))$$

Finding a sufficient condition for PAM is equivalent to finding conditions on $V(\cdot)$ such that $g(x) > 0$ for $x > 0$.

Assume $g(0) = 0$, that is, $V(0) = 0$ and $V'(0) = 0$. Then:

$$g'(x) = \frac{x^2}{2}V'^{-1\prime\prime}(x)$$

So, $g'(x) > 0$ if $V'^{-1\prime\prime}(x) > 0 \Leftrightarrow V'''(x) < 0$ for $x > 0$, and $g'(x) < 0$ if $V'^{-1\prime\prime}(x) < 0 \Leftrightarrow V'''(x) > 0$, which proves the result.

On the Corollaries: recall that optimal project choice of (r_1, r_2) is given by:

$$p^*(r_1, r_2) = V'^{-1}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right]$$

The sufficient condition for PAM to be the unique equilibrium is $V'^{-1\prime\prime}(p) > 0$ ($V'''(p) < 0$). It is clear that this implies project choice will be convex in the "aggregated type" of the pair, $\frac{2(r_1+r_2)}{r_1 r_2}$, that is, optimal project will vary a lot across different aggregated types. Similarly, if $V'^{-1\prime\prime}(p) < 0$, so that NAM is the unique equilibrium, it is clear that project choice will be concvae in the "aggregated type" of the pair, that is, optimal project will not vary much across different aggregated types.

How does project choice vary in individual type?

$$\begin{aligned}
\frac{dp}{dr_1} &= -V'^{-1\prime}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right]\left(\frac{2}{r_1^2}\right) \\
\frac{dp}{dr_2 dr_1} &= V'^{-1\prime\prime}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right]\frac{4}{r_1^2 r_2^2}
\end{aligned}$$

Clearly, $\frac{dp}{dr_2 dr_1} > 0$ iff $V'^{-1\prime\prime}(p) > 0$, and $\frac{dp}{dr_2 dr_1} < 0$ iff $V'^{-1\prime\prime}(p) < 0$.

Now, let's look more closely at the Sharpe ratio of a project p when $V(p) = p^N$, $N > 1$:

$$\begin{aligned}
SR(p) &= \frac{p}{V(p)^{\frac{1}{2}}} \\
&= p^{1-\frac{N}{2}}
\end{aligned}$$

Then:

$$\frac{dSR(p)}{dp} = \left(1 - \frac{N}{2}\right)p^{-\frac{N}{2}}$$

Then

$$\begin{aligned}
\frac{dSR(p)}{dp} &> 0 \; if \; N \in (1,2) \\
&< 0 \; if \; N > 2
\end{aligned}$$

But $N \in (1,2)$ is precisely the condition for PAM to be the unique matching equilibrium, and $N > 2$ is precisely the condition for NAM to be the unique matching equilibrium.

8.5 A5: NAM is the unique equilibrium when income-smoothing is shut down

Suppose that all pairs must undertake the same project, p. For instance, the government mandates that all farmers must grow rice. This effectively shuts down the income-smoothing channel.

Differentiate $CE(r_1, r_2)$ with respect to r_1 and r_2 when there is no project choice, so that all pairs (r_1, r_2) face the same risky income stream $f(R_p|p)$: the cross-partial $\frac{dCE(r_1,r_2)}{dr_1 dr_2}$ is:

$$-\frac{r_1 r_2}{(r_1+r_2)^3}\frac{\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p \int R_p^2 e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p - \left[\int R_p e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p\right]^2}{\left[\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p\right]^2}$$

But we know that:

$$\int e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p \int R_p^2 e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p > \left[\int R_p e^{-\frac{r_1 r_2}{r_1+r_2}R_p}f(R_p|p)dR_p\right]^2$$

since we know variance is always positive, thus:

$$\int f(R_p|p)dR_p \int R_p^2 f(R_p|p)dR_p > \left[\int R_p f(R_p|p)dR_p\right]^2$$

and $g(R_p) = e^{-\frac{r_1 r_2}{r_1+r_2}R_p}$ is a convex function.
Therefore

$$\frac{dCE(r_1, r_2)}{dr_1 dr_2} < 0$$

and negative-assortative matching results as the unique equilibrium.
This corresponds with the result from Chiappori and Reny (2006) and Schulhofer-Wohl (2006).

8.6 A6: PAM is the unique matching when consumption-smoothing is shut down

We know from the main matching result that the optimal sharing rule is linear. Suppose the slope of the sharing rule $s(R_p) = a + bR_p$ is fixed at b for all possible pairs of risk types. (For example, the government mandates an equal split of the output, but partners are free to make fixed transfers to each other.)

This removes consumption-smoothing as an effective risk management tool, leaving only income-smoothing (project choice). What happens to equilibrium risk-sharing relationships?

A matched pair (r_1, r_2) chooses the relationship-specific transfer a and project p:

$$\max_{a,p} E[-e^{-r_1[R_p - a - bR_p]}|p] \text{ s.t.}$$

$$E[-e^{-r_2[a+bR_p]}|p] \geq -e^{-v}$$

The fixed transfer just serves to satisfy the constraint, since the division of output is fixed at b

by the government. Therefore:

$$a = \frac{v}{r_2} + \frac{r_2}{2} b^2 V(p) - bp$$

Then, the project chosen is:

$$\max_p -e^{-r_1\left[p - \frac{v}{r_2} - \left(\frac{r_1}{2}(1-b)^2 + \frac{r_2}{2}b^2\right)V(p)\right]}$$

$$p^*(r_1, r_2) = V'^{-1}\left(\frac{2}{r_1(1-b)^2 + r_2 b^2}\right)$$

Then the sum of certainty-equivalents of a given pair (r_1, r_2) is:

$$CE(r_1, r_2) = V'^{-1}\left(\frac{2}{r_1(1-b)^2 + r_2 b^2}\right) - \left(\frac{r_1}{2}(1-b)^2 + \frac{r_2}{2}b^2\right) V\left(V'^{-1}\left(\frac{2}{r_1(1-b)^2 + r_2 b^2}\right)\right)$$

And:

$$\frac{dCE}{dr_1} = -\frac{(1-b)^2}{2} V\left(V'^{-1}\left(\frac{2}{r_1(1-b)^2 + r_2 b^2}\right)\right)$$

$$\frac{dCE}{dr_2 dr_1} = V'^{-1\prime}\left(\frac{2}{r_1(1-b)^2 + r_2 b^2}\right) \frac{2b^2(1-b)^2}{\left(r_1(1-b)^2 + r_2 b^2\right)^3} > 0$$

Hence, positive-assortative matching arises as the unique equilibrium.

So, if the government regulates wages by fixing the slope of the sharing rule at some $b \in [0, 1]$, where pairs can still make within-pair state-independent transfers, the unique equilibrium matching pattern is always positive-assortative, verifying our intuition that, because consumption-smoothing is held fixed, the "similarity of decisionmaking framework" dominates and people match with people who are like them because they will agree about project choice. This can be thought of as the counterpoint to holding income-smoothing fixed (Appendix 5), as in Chiappori and Reny (2006) and Schulhofer-Wohl (2006)–a policy example suppressing choice of income stream might be, say, the government specifying that only wheat can be grown.

What are some implications of this understanding? The government may be motivated by equality concerns to specify an equal division of output in every relationship, but this may actually generate even more inequality by weakening the informal risk-management toolkit available to individuals, which then triggers endogenous change in risk-sharing networks. Specifically, if agents had been matched negative-assortatively in the status quo (because the "cost function" of project mean is quite convex, say), then this imposition of wage equality leads to positive-assortative matching, which may actually exacerbate inequality: there is a bigger spread in projects, with less risk-averse agents on projects with much higher expected returns while more risk-averse agents are on projects with much smaller expected returns, and less risk-averse agents abandon their roles as informal insurers, and more risk-averse agents wind up bearing more risk.

8.7 A7: Differently-sized groups

We know by now (see, e.g., Appendix 4) that the sum of certainty-equivalents for a given pair (r_1, r_2) in the benchmark case is given by:

$$CE(r_1, r_2) = V'^{-1}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right] - \frac{r_1 r_2}{2(r_1+r_2)} V\left[V'^{-1}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right]\right]$$

Then:

$$\begin{aligned} \frac{dCE}{dr_2} &= -\frac{r_1^2}{2(r_1+r_2)^2} V\left[V'^{-1}\left[\frac{2(r_1+r_2)}{r_1 r_2}\right]\right] \\ &< 0 \end{aligned}$$

Hence, $CE(r_1, r_2)$ is decreasing in risk aversion.

Thus, if $|G1| < |G2|$, for example, it is the most risk-averse excess agents of $G2$ who will remain unmatched in equilibrium. The main matching results apply to the rest of the agents.

8.8 A8: Background and Empirical Context

The purpose of this section is to provide further empirical context for the model. First, I will discuss the substantial role played by informal insurance motivations in building relationships in risky environments with missing formal insurance and credit markets. Additionally, I will show that risk attitudes are a significant determinant of risk-sharing partner choice.

Next, I will provide evidence that there is a great deal of heterogeneity in risk aversion across individuals in a wide range of settings.

Finally, I will provide evidence of heterogeneity in the riskiness of activities available to individuals, as well as heterogeneity in the relative riskiness of these activities across different environments.

To fix ideas, it may be helpful to envision an agricultural setting, which captures nicely the key elements of the model. Much of the literature discussed in this section is drawn from an agricultural context, where landowners and farmers are heterogeneous in the extent of their risk aversion, and landowners must decide which farmers to work with. Different crops have different yield and profit distributions: some crops are very robust to drought but correspondingly tend to produce low yields on average ("safe" crops), while other crops have the potential for very high yields, but are extremely sensitive to rainfall and other inputs, and blight easily ("risky" crops). In addition to crop portfolio and plot locations, fertilizer and other inputs, irrigation, planting times, and general farming methods and technologies must also be chosen.

Furthermore, the yield and profit distribution of each crop varies across agroclimactic region. Different parts of the world experience different levels of rainfall, soil quality, irrigation, elevation, heat, and other such ecological characteristics, and this influences the stochastic yield and profit of each crop. It is no surprise, then, that equilibrium cropping methods, crop mixes, and contracting institutions vary so widely across region. A goal of this paper is to advance the understanding of these differences.

8.8.1 Risk Attitude and Informal Insurance Relationships

An abundance of work discusses the considerable role of informal insurance concerns in network formation. People rely on each other to smooth consumption risk and income risk in a wide variety of ways (Alderman and Paxson (1992), Morduch (1995)). A very prevalent consumption-smoothing technique between people is transfers and remittances, and much work has been done to study the nature of the transfers that can be sustained given a risk-sharing group, the shapes of equilibrium networks holding fixed some transfer rule, and who is empirically observed to make transfers with whom (Townsend (1994), Fafchamps and Lund (2003), Genicot and Ray (2003), Bramoulle and Kranton (2007), to name a few). A general message these papers convey is that the need to manage risk in the absence of formal insurance institutions has huge effects on interpersonal relationships among the poor.

In fact, risk management can affect relationship formation in very specific ways. Rosenzweig and Stark (1989) show that daughters are often strategically married to villages located in environmentally dissimilar regions with minimally correlated farming incomes, for the purposes of consumption-smoothing; households exposed to more income risk are more likely to invest in longer-distance marriage arrangements. Ligon et. al. [cite], Fafchamps [1999], and Kocherlakota [1996], among many others, analyze a pure risk-sharing relationship between two heterogeneously risk-averse households who perfectly observe each other's income. Ackerberg and Botticini [cite] study agricultural contracting in medieval Tuscany, and find evidence that heterogeneously risk-averse tenant farmers and landlords strategically formed sharecropping relationships based on differing risk attitudes, motivated by risk management concerns. Hence, informal insurance motivations play a substantial role in the formation of relationships.

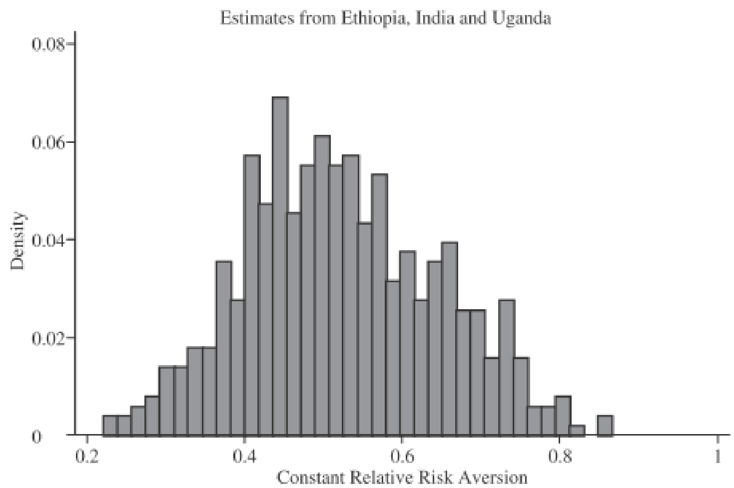
But how much do individuals care about the risk attitudes of potential partners when forming risk-sharing groups? Naturally, there are many other reasons people might match with each other, but the point of the model is to focus on one important determinant of risk-sharing relationship formation, and to study how equilibrium matching patterns shift along that dimension. Furthermore, there is a great deal of evidence that the risk attitudes of partners are indeed a primary determinant of risk-sharing partner choice. Ackerberg and Botticini (2002) provide empirical evidence supporting the presence of endogeneity of matching along risk attitude of landowners and sharecroppers in medieval Tuscany. In their data, they find that share contracts were associated with the safer crop of cereal, while fixed rent (residual claimancy) contracts were associated with the riskier crop of vines. They argue that this is the outcome of endogenous matching–risk-neutral tenants may have been assigned to the riskier crops, resulting in fixed rent contracts for vines, while risk-averse tenants may have been assigned to the safer crops, resulting in share contracts on cereals.

Additional evidence for the importance of risk attitudes as a determinant of risk-sharing relationships is found in Gine et. al. (2010) and Attanasio et. al. (2012). Gine et. al. (2010) run an experiment on small-scale entrepreneurs in urban Peru and allow joint liability groups to form endogenously in a microfinance setting. They find strong evidence of assortative matching along risk attitude. Attanasio et. al. (2012) run a unique experiment with 70 Colombian communities. They gather data about risk attitudes and pre-existing kinship/friendship networks, and then allow individuals to form risk-sharing groups of any size. Attanasio et. al. find that, when members know each other's risk types, and trust each other (family and friends are in the same group), conditioning on all other potential reasons for matching which they are able to account for (gender, age, geography), there is strong evidence of positive assortative matching along risk aversion.

To further emphasize the significant role of risk attitude in determining risk-sharing relationship formation, I use the dataset from Attanasio et. al. (2010) to calculate the proportion of formed links that involved at least one family or one friendship tie, for each municipality. The mean of these proportions is 0.005, or 0.5%. Since it's possible that there were very few family and friend ties reported in the entire dataset to begin with, I also calculate the proportion of all possible links that could have involved at least one family or friendship tie, for each municipality. The mean of this number is 0.05. Hence, this back-of-the-envelope calculation suggests that, in this setting, only about 10% of all possible risk-sharing relationships which could have involved a family or friendship tie, actually did involve such a tie. Hence, while one might expect kinship and friendship to be major influences in partner choice, there is strong evidence that risk attitude is the more prominent consideration when the partner is being chosen specifically for the purposes of informal insurance. In particular, the family and friendship tie is likely to influence the pool of potential partners (because individuals are less likely to know the risk attitudes of strangers, or to trust them), but the choice of partner from this pool for the purposes of insurance is primarily determined by risk attitudes.

8.8.2 Heterogeneity in Risk Aversion

The second key piece of the model is heterogeneity in risk attitudes across individuals. There is plenty of evidence that people are risk-averse and that they are heterogeneous in their risk-aversion. Harrison et. al. (2010) asked 531 experimental subjects drawn from India, Ethiopia, and Uganda to choose a gamble from a set of gambles varying in riskiness (a riskier gamble has higher mean but correspondingly higher variance), in a similar spirit as the seminal study by Binswanger (1980), and estimated the density of CRRA risk attitudes:



It's clear that there is a substantial amount of variation, and almost every point in the $[0, 1]$ range is represented.

Chiappori et. al. (2010) use two distinct methods to measure heterogeneity in risk preferences in Thai villages, where these villages are spread across several regions in Thailand. The first method is based off the co-movement of individual consumption with aggregate consumption, and the second is based off of optimal portfolio choice theory. Using both methods, they find substantial heterogeneity in risk attitudes in each village. Moreover, this heterogeneity varies across villages and regions. The following table reports the means of risk tolerance for each of 16 villages, and the test statistic for heterogeneity:

		risk aversion γ_i			risk tolerance $1/\gamma_i$		
village	households	mean	χ^2	p-value	mean	χ^2	p-value
			Chachoengsao				
2	13	2.00	277.29	0.0000	1.56	3543.60	0.0000
4	21	0.79	78.44	0.0000	2.47	1646.42	0.0000
7	6	0.98	6.69	0.3509	1.28	32.21	0.0000
8	14	0.61	31.11	0.0053	5.11	7986.64	0.0000
			Buriram				
2	18	0.62	12.54	0.8184	2.97	368.59	0.0000
10	8	0.34	5.87	0.6618	4.02	147.64	0.0000
13	10	0.41	14.27	0.1610	7.61	2255.00	0.0000
14	15	0.84	73.55	0.0000	3.55	4209.49	0.0000
			Lop Buri				
1	19	1.20	96.08	0.0000	1.36	1011.17	0.0000
3	8	2.12	348.07	0.0000	1.33	3981.73	0.0000
4	27	1.40	173.59	0.0000	1.29	2061.54	0.0000
6	24	1.82	485.27	0.0000	1.29	3074.97	0.0000
			Sisaket				
1	22	0.43	21.94	0.4633	3.78	457.10	0.0000
6	34	0.78	117.07	0.0000	1.85	2010.67	0.0000
9	22	0.76	33.96	0.0495	3.24	2141.48	0.0000
10	13	0.47	9.68	0.7199	2.90	36.03	0.0006
			pooled				
-	274	0.98	1358.43	0.0000	2.64	77568.89	0.0000

Again, it is clear that there is widespread variation in the degree of risk aversion across households.

8.8.3 Heterogeneity in Risky Activities and Settings

Finally, agents in a given setting have a wide variety of investment options and household decisions to make, which vary in riskiness. For example, a farmer must choose a spatial distribution of his plots, what lumpy purchases to make (e.g. bullocks), and when and how to plant his crop. A microentrepreneur must decide what kind of business he wants to start. Parents must decide how to invest household resources, and whom their children will marry. Individuals face a diversity of choices, and how much diversity, as well as the relative riskiness of one decision compared to another, varies across settings.

For example, Rosenzweig and Binswanger (1993) consider the equilibrium crop portfolio choices of heterogeneously risk-averse farmers living in six ICRISAT villages located across three distinct agroclimactic regions in India. The first region is characterized by low levels of erratically distributed rainfall and soils with limited water storage capacity (this is the riskiest environment), the second region by similarly erratic rainfall and irrigation but better soil storage capacity, and the third region by low levels of more reliable rainfall with reasonable soil storage capacity (this is the safest environment). The principal crops grown are sorghum, pigeon peas, pearl millet, chickpeas, and groundnuts, and their yield distributions vary across environment. They show that differences in risk aversion do translate into differences in choice of risky investments. Individuals are influenced by risk-reduction when choosing income streams, particularly in response to limitations on *ex post* consumption-smoothing, and the degree to which they are influenced depends on their risk aversion.

Dercon (1996) also studies the variation in riskiness of agricultural investment decisions by heterogeneously risk-averse rural households. His data is drawn from Tanzania, a country with very underdeveloped credit markets (in 1989, only 5% of commercial bank lending went to the private sector, and less than 10% of this lending went to individual farmers). The UN Food and Agriculture Organization provides an interesting look at the vast heterogeneity in crop yield distributions and equilibrium crop mix across regions in Tanzania in 1998. The following table shows the area, yield, and production of each of five crops across ten agroclimactically heterogeneous regions in Tanzania[23]:

Table 2: Tanzania - 1998 Vuli Crop Production in Cereal Equivalent by Region

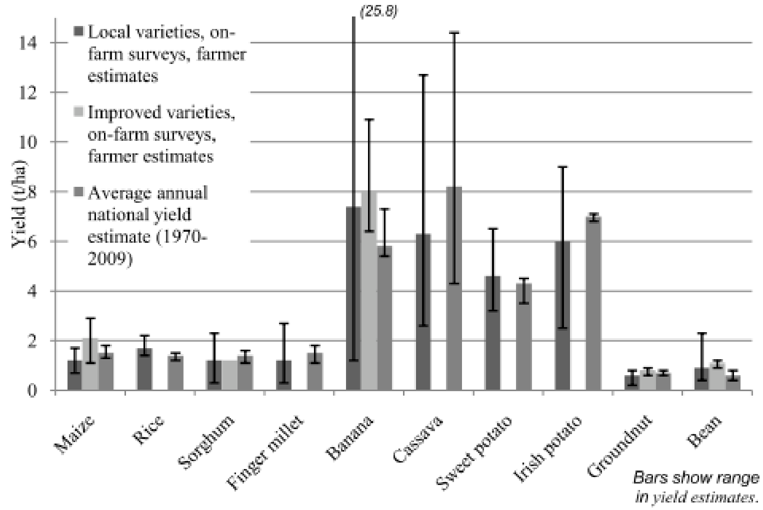
	Maize			Sorghum			Paddy			Millet			Wheat		
Region	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**	**Area ('000 ha)**	**Yield (kg/ha)**	**Production (tonnes)**
Mara	13.0	1 700	22.1	20.0	1 300	26.0	1.0	1 600	1.6	0.5	1 100	0.6	0.0	0	0.0
Arusha	50.0	1 600	80.0	10.0	1 200	12.0	2.6	2 300	5.98	3.4	1 000	3.4	14.0	1 700	23.8
Kilimanjaro	30.0	1 500	45.0	1.5	1 300	2.0	3.2	2 500	8	-	-	-	1.2	1 400	1.7
Tanga	20.0	1 500	30.0	-	-	-	2.1	1 900	3.99	-	-	-	0.7	1 000	0.7
Morogoro	40.0	1 400	56.0	8.0	1 200	9.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mbeya	20.0	1 600	32.0	0.0	0.0	0.0	1.0	1 800	1.8	0.0	0.0	0.0	0.0	0.0	0.0
Coast/DSM	11.0	1 100	12.1	-	-	-	-	-	-	-	-	-	-	-	-
Kagera	51.0	1 500	76.5	2.7	1 300	3.5	0.1	2 000	0.2	-	-	-	-	-	-
Kigoma	40.0	1 800	72.0	3.0	1 500	4.5	3.0	2 000	6	0.1	1 000	0.1	-	-	-
Mwanza	100.0	1 200	120.0	110.0	1 100	121.0	10.0	1 800	18	1.0	1 000	1.0			
Total	**375**		**545.7**	**155.2**		**178.6**	**23.0**		**45.6**	**5.0**		**5.1**	**15.9**		**26.2**

Unfortunately, this table excludes estimates of the *variance* of yield of each of these crops across regions. Dercon (1996) provides a discussion of this in his paper. He describes a multiplicity of soils and irrigation systems in Tanzania, which support different crops. Paddy, a crop which can yield a high return, is restricted only to specific soils and areas close to a river, and is the least drought and locust resistant. Despite the potential for high returns, only 11% of the total cultivation sample grew paddy. On the other hand, sorghum yields only a low-moderate return, but all soils can sustain it, and it is more resistant to drought and pests. Even though it had a lower mean return, it was grown by all but two households in the sample.

Uganda and Ethiopia are similar to Tanzania in the set of crops grown, though the actual crop mix grown differs due to differences in environmental conditions. An IFPRI report from 2011 estimating crop yields in Uganda provides a useful illustration of how the variance of crop yields differs across crops, and the typical relationship of the variance with the mean:

[23] Of course, in addition to levels and fluctuations of crop yields, farmers care about the levels and fluctuations of crop prices, as they care ultimately about the distribution of profits.

Figure A.1—Average yield and yield range of selected unfertilized crops observed in on-farm surveys (farmer recall) in Uganda, by variety, type, and average of annual national yield estimates (1970–2000)



Source: Compiled by authors based on database of crop yields in Uganda.

There is a clear positive relationship between mean yield and variance of yield. Groundnuts have low mean yields and correspondingly low fluctuation of yields, making it a "safer" crop, while banana has much higher mean yields but correspondingly higher fluctuation of yields, making it a "riskier" crop.

Abebe et. al. (2010) provide a similar graphic for Ethiopia:

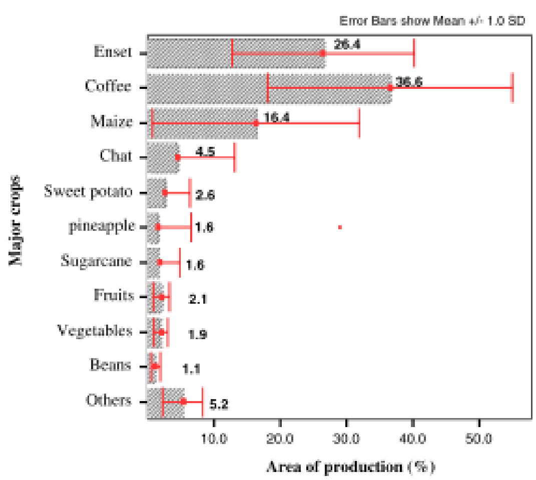


Fig. 3 Mean area share (in percentage of the farm area) of the major crops in farms. All 144 farms are used here. *Error bars* indicate one standard deviation

(Enset is a type of banana.)

Again, we get a general sense that higher mean yield crops have a higher variance of yield, while lower mean yield crops have a smaller variance of yield. Comparing across Uganda and Ethiopia, we see that maize is a safer crop relative to sweet potato in Uganda, but the opposite is true in Ethiopia. Thus, the same set of crops have very different yield distributions in different settings, and furthermore, each crop's relative riskiness with the other crops also varies across setting.