# Optimal (Partial) Group Liability in Microfinance Lending*

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**Abstract**

This paper develops a model of group borrowing that incorporates partial group liability, where borrowers are penalized if their group members default but are not held responsible for the entirety of the failed loan. The model illustrates the trade-off of group liability lending: while higher levels of group liability increase within group risk-sharing, when liability becomes too high, borrowers find it optimal to strategically default. The model predicts the existence of an optimal partial liability that maximizes transfers between group members while avoiding strategic default. The model is extended to incorporate household structure and group size in order to estimate the prevalence of strategic default in the presence of correlated returns to borrowing. Using administrative data from large microfinance institution in Mexico, structural estimates suggest the presence of substantial strategic default. Exploiting variation across loan officers in *de facto* group liability, a U-shaped relationship between group liability and default rates is found empirically, as predicted by the model. Structural estimates suggest that moving to 50 percent liability could reduce the incidence of default by 14-15 percent, resulting in a 5-6 percent increase in welfare.

**Keywords:** microfinance, group lending, liability, strategic default, Mexico

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# 1 Introduction

The advent of group liability – where multiple borrowers are jointly responsible for the repayment of their loans – has been identified as an important factor in the expansion of access to credit in the developing world (Morduch, 1999; Armendariz and Morduch, 2010). Group liability helps overcome information asymmetries between borrowers and lenders and incentivizes risk sharing within borrowing groups (Ghatak and Guinnane, 1999). At the same time, however, group liability may induce borrowers to strategically default when their group members default, increasing default rates (Coate and Ravallion, 1993). While empirical evidence of the effect of group liability on repayment is scant,[1] determining the optimal group liability remains a pressing concern for lenders. For example, to avoid strategic defaults, Grameen Bank transitioned from its traditional full liability group loans to individual liability loans in 2002 (Yunus, 2002; Dowla and Barua, 2006).

This paper contends that neither full group liability nor individual liability may be optimal; instead, partial group liability – where borrowers are responsible for only a portion of their group members' loans – can retain the advantages of group liability lending (i.e. risk sharing) without incurring the disadvantages (i.e. strategic default). To make this argument, the paper comprises three parts. First, I develop a simple model of group borrowing to show that intra-group risk sharing is maximized and strategic default avoided when the group liability is equal to the present discounted value of repaying the loan and remaining eligible to borrow in the future. Second, I structurally estimate this optimal (partial) group liability using repayment data from a large microfinance organization and two separate extensions of the basic model that allow for the identification of strategic default in the presence of correlated returns to borrowing. Third, I exploit variation across loan officers in the group liability they enforced to show that loan officers who enforced partial group liabilities achieved lower default rates than both loan officers that enforced full group liability and individual liability.

[1]One notable exception is (Gine and Karlan, 2011), who experimentally remove the group liability provision in existing borrowing groups. They find no evidence of increased default rates, which is consistent with the tradeoff between individual liability and group liability presented in this paper.

Overall, I find that a move to 50 percent partial liability (i.e. a borrower is responsible for half of her group member's loan) would reduce default rates and increase welfare.

The borrowing model is based on a repeated game framework where borrowers repay their current loan in order to remain eligible to borrow in the future. Borrowers receive stochastic (and potentially correlated) returns to borrowing, which they may transfer to their group member. The lender can penalize borrowers if their group member fails to repay and can refuse to lend to defaulting borrowers in the future but cannot directly penalized defaulters. The model has two implications. First, if the penalty for having a group member default exceeds the present discounted value of repaying the loan and remaining eligible to borrow in the future ("the value of future borrowing"), then a borrower will find it optimal to strategically default. Second, the transfer a borrower is willing to make is bounded above by the cost incurred by letting her group member default, which is the lesser of the group liability penalty and the value of future borrowing. As a result, the optimal group liability is the value of future borrowing; any less liability reduces within-group risk sharing, and any greater liability induces strategic default.

Disentangling strategic default from correlated returns to borrowing is empirically difficult, as both increase the probability an individual defaults conditional on her group member defaulting. To separate the two, I extend the basic model in two ways. First, I incorporate household structure. By focusing on a subset of borrowers where fellow household members borrow in a different microfinance groups ("household-individual groups"), it is possible to disentangle correlated returns from strategic default if household members pool resources. Intuitively, in the absence of strategic default, the effect of a group member defaulting on an individual's repayment should be the same as the effect of that individual's household member's group member defaulting, as both defaults affect the household budget in the same way. With strategic default, however, an individual should be more likely to default when her group member defaults than when her household member's group member defaults. Second, I extend the model to allow for groups with more than two members. Incorporating group size separately identifies strategic default and correlated returns to borrowing by exploiting the relationship between the number of group members defaulting and the group liability penalty incurred by the remaining group members. Intuitively, as more members of a borrowing group default, the incentive to strategically default increases, as the total penalty is larger and is distributed amongst a smaller number of repaying individuals. As a result, with strategic default, one should only observe a small number of group members defaulting or all group members defaulting.

I structurally estimate both model extensions using repayment data from a large microfinance institution (MFI) in southern Mexico for the subsample of household-individual groups. In both extensions, I estimate that borrowers have high but variable returns to borrowing that are positively correlated with the returns of other group members. Both extensions estimate that 16 percent of the defaults observed in the data were strategic and that moving to partial liability could reduce default rates and increase welfare. While the estimated optimal default liability differ between the two model extensions, both predict that a move to 50 percent liability would be nearly optimal, reducing the probability of default by 14-15 percent and increasing welfare by 5-6 percent.

In qualitative interviews with the administrators of the MFI, it was emphasized that loan officers have substantial latitude in determining the group liability that he or she enforced. In the final part of the paper, I exploit this variation of group liability across loan officers to demonstrate that loan officers enforcing partial liability had lower default rates than either loan officers enforcing full group liability or individual liability. To do so, for each loan officer, I structurally estimate the liability he or she enforced using the sub-sample of household-individual groups and then examine the default rates the loan officer achieved for all other loans outside this sub-sample. I find a strong U-shaped relationship between the group liability enforced and the probability of default. This relationship is robust to controlling for the default rate the loan officer achieved within the sample as well as municipality fixed effects, suggesting that it is unlikely to be driven by unobservable differences in loan officer or borrowing group quality. Consistent with the structural results, these out-of-sample results suggest moving to 50% liability could substantially reduce default rates.

This paper contributes to the large theoretical literature examining the role of group liability in lending (e.g. Stiglitz, 1990; Banerjee, Besley, and Guinnane, 1994; Armendariz de Aghion, 1999; Ghatak, 1999; Rai and Sjöström, 2004; Chowdhury, 2005), as well as the much smaller empirical literature examining the effect of group liability on repayments (Gine and Karlan, 2011; Giné, Krishnaswamy, and Ponce, 2011). Like (Ahlin and Townsend, 2007), this paper attempts to bridge the gap between theory and empirics; in particular, it is the first attempt (that I am aware) to use a structural approach to identify model parameters from the observed combinations of repayment and default within a borrowing group. Given the modest data requirements necessary to implement the estimation procedure, the paper provides a general methodology that can be used to determine the optimal degree of group liability in other contexts.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 presents the household structure and group size model extensions used for the structural estimation. Section 4 describes the empirical context and data. Section 5 presents the structural estimation methodology and the results. Section 6 examines the relationship between default rates and group liability using variation in loan officer's enforced group liability. Section 7 concludes.

# 2 The basic model

In this section, I introduce a simple model of group liability borrowing. For now, I assume borrowing groups are comprised of two risk-neutral individuals. Borrowing is modeled as a repeated game in which every period borrowers have an incentive to default but continue to repay in order to remain eligible for future loans. Partial group liability is modeled as a penalty that a borrower incurs if her group member fails to repay. The basic intuition of the role played by partial group liability is simple: as the penalty increases, borrowers will be willing to transfer more money to their group member to allow their group member to repay; however, if the penalty becomes too high, then borrowers prefer to strategically default when their group member fails to repay rather than incur the penalty.

Each period of the model comprises three stages; the time line of the model is presented in Figure 1. In the first stage, individuals eligible to borrow choose whether or not to borrow.[2] Those that choose not to borrow (or are ineligible) pursue an outside option with normalized value 0. The stochastic returns to borrowing are then realized and become known to fellow group members but not lender. These returns may be correlated across individuals within a borrowing group. In the second stage, group members may choose to transfer some of their returns to each other. In the third stage, individuals choose whether or not to repay their loan, given the realized returns and net transfers received. If an individual is unable or unwilling to repay her loan, she keeps her returns and transfers but becomes ineligible to borrow in future periods.[3] Individuals who repay may continue to borrow in the future, even if their group member fails to repay; however, if their group member fails to repay, they will incur a group liability penalty.

Before introducing the model, some notation is required. Let $i$ refer to one borrower and $g(i)$ refer to her group member. Let $T$ refer to the transfer made from $i$ to $g(i)$ in a particular period ($T$ can be negative). Let $I$ refer to the cost of repaying the loan (principal plus interest). Let $P$ be the penalty an individual incurs if her group member defaults and she continues to borrow.[4] It is assumed that $P \in [0, I]$, where $P = 0$ indicates individual

[2]The model focuses on the extensive margin of the borrowing decision (i.e. whether to borrow or not) rather than on the intensive margin (i.e. how much to borrow) because in this particular empirical context, the amount each borrower borrows within a group is similar. Appendix A.2 extends the model to allow for different borrowers within a group to borrow different amounts.

[3]The inability of the bank to levy any punishment upon a defaulter other than refusing to lend in the future is consistent with the empirical setting that I examine. In Appendix A.1 I show how the model can be extended to incorporate a fixed cost to defaulting, such as a loss of collateral. It is straightforward to show that default rates are minimized when the bank never lends to an individual who has defaulted in the past, as any sort of policy of forgiveness reduces the present discounted value of repaying the loan.

[4]It is assumed the choice to pay the penalty $P$ and continue to borrow when a group member defaults remains feasible to an individual even if the returns from borrowing are not sufficient to cover both the repayment of the loan and the penalty, i.e. when $R_i < I + P$. This assumption is consistent with the

liability and $P = I$ indicates full group liability. Let each player discount the future by $\beta < 1$. Let $R_i$ indicate the realized return to borrowing for individual $i$. Finally, let $V$ indicate the present discounted value of being eligible to borrow.[5]

In what follows, I assume that borrowers are symmetric and $\beta E[R_i] > I$; i.e. the present discounted value of expected returns in the next period is greater than the cost of repayment. I show in Appendix B.1 that this implies $\beta V > I$, which ensures that individuals would choose to repay their loan so that they can continue to borrow if the penalty they incur is small enough.

The model is solved by backwards induction.

## 2.1 Stage 3: choosing whether or not to repay

In Stage 3, after returns to borrowing have been realized and transfers have been made, if both $i$ and $g(i)$ are able to repay their loans (i.e. $R_i - T \geq I$ and $R_{g(i)} + T \geq I$), both individuals decide simultaneously whether or not to repay. If they both repay, they receive their returns net of transfers and are able to borrow in the next period, but have to pay back the loan at cost $I$. If neither repays, they both receive their returns net of transfers and avoid paying back $I$ and incurring penalty $P$, but are unable to borrow in the future. If $i$ repays and $g(i)$ defaults, then they both receive their returns net of transfers, $g(i)$ becomes ineligible to borrow in the future but avoids paying back the loan, and $i$ pays back the loan, incurs penalty $P$, and remains eligible to borrow in the future.[6] Since regardless of the action, both individuals get to keep their returns net of transfers ($R_i - T$ and $R_{g(i)} + T$, respectively) the strategic form of the game is equivalent to:

empirical setting I consider, as the penalties normally take the form of additional fees or higher interest rates on future loans.

[5]The value of borrowing$V$ depends on $P$ as well as the rest of the model parameters, but I refrain from using the notation $V(P)$ where possible for the sake of readability.

[6]In Appendix A.3, I show how the model can be extended to allow individuals to impose social sanctions on group members. The threat of incurring social sanctions provides an additional incentive for within-group risk sharing. However, because a borrower incurs a social sanction when her group member defaults regardless if she repays or strategically defaults herself, social sanctions do not directly affect the decision of whether or not to strategically default.

| $i$ / $g(i)$ | Repay | Default |
|---|---|---|
| Repay | $\beta V - I$, $\beta V - I$ | $\beta V - I - P$, 0 |
| Default | 0, $\beta V - I - P$ | 0, 0 |

Since $\beta V > I$, one Nash equilibrium of this game is for both group members to repay. If $P > \beta V - I$, another possible Nash equilibrium is for both group members to default. Such an equilibrium is undesirable both theoretically (as its payoffs to both players are strictly lower than if they both repaid) and empirically (since the most common outcome we observe in group lending is both members repaying), so I assume that if both group members are able to repay, then the loan is repaid.

If one player can't repay because of insufficient funds, then the game is more interesting. Without loss of generality, assume that $R_i - T \geq I$ and $R_{g(i)} + T < I$. Then $g(i)$ will be forced to default, leaving $i$ with the following decision:

$$\max\{\text{Repay,Default}\} = \max\{\beta V - I, P\}$$

Clearly, $i$ will repay if $P^* \equiv \beta V - I > P$, and strategically default if $P^* < P$: if the penalty for having a group member default is sufficiently high then the value of being able to continue to borrow will not be worth paying the penalty. In what follows, I refer to the case where $P > P^*$ as the strategic default equilibrium (or SD equilibrium) and the case where $P < P^*$as the non-strategic default equilibrium (or NSD equilibrium).

## 2.2 Stage 2: sending transfers to group members

In the second stage, individuals determine how much to transfer to their group member, knowing the realized returns and foreseeing the results of stage 3. Since sending transfers is costly, an individual will only send a transfer when it is affordable and when it allows their group member to repay her loan. An individual will be willing to make transfers up to the point where she is indifferent between sending the transfer and letting her group member default. This maximum transfer depends on the equilibrium. Without loss of generality, suppose $g(i)$ cannot afford to repay her loan without a transfer from $i$, and $i$ can afford to repay her loan as well as $g(i)$'s shortfall. From stage 3, it can be seen that in the SD equilibrium, $i$ will send a transfer if and only if:

$$\underbrace{R_i - T + \beta V - I}_{\text{if } g(i) \text{ repays}} \geq \underbrace{R_i}_{\text{if } g(i) \text{ defaults}} \Leftrightarrow \beta V - I \geq T$$

Similarly, in the NSD equilibrium, $i$ will send a transfer if and only if:

$$\underbrace{R_i - T + \beta V - I}_{\text{if } g(i) \text{ repays}} \geq \underbrace{R_i + \beta V - P}_{\text{if } g(i) \text{ defaults}} \Leftrightarrow P \geq T$$

Hence, the maximum transfer that $i$ will be willing to send, $T^*$, is:

$$T^* = \min\left(P, \beta V - I\right) = \min\left(P, P^*\right) \tag{1}$$

These results are intuitive: in the SD equilibrium (i.e. $P \geq P^*$), if $i$ does not cover $g(i)$, then $g(i)$ will default, causing $i$ to default too (because it is optimal for $i$ to default). This makes $i$ ineligible for future loans (but saves her from having to repay the current loan); the net cost to $i$ from not transferring anything to $g(i)$ is $\beta V - I$. Hence, it is optimal for $i$ to transfer up to this amount in order to avoid having $g(i)$ default. Similarly, in the NSD equilibrium (i.e. $P < P^*$), $g(i)$ defaulting will cause $i$ to incur a penalty $P$ and so $i$ will be willing to transfer any amount up to $P$ to avoid this penalty.

Given the optimal maximum transfers, it is possible to determine whether or not $i$ and $g(i)$ repay based entirely on the realized returns $R_i$, $R_{g(i)}$, the maximum transfers between group members, and the equilibrium. This is depicted graphically in Figure 2. If both $R_i > I$ and $R_{g(i)} > I$ (region C), both $i$ and $g(i)$ will repay without the need of transfers. If $R_{g(i)} \in [I - T^*, I]$ and $R_i > 2I - R_{g(i)}$(region F), then $i$ will cover $g\left(i\right)$'s shortcoming and both will repay. If both $R_i < I$ and $R_{g(i)} < I$ (region D), then both $i$ and $g(i)$ will default.

Finally, if $R_i > I$ and $R_{g(i)} < \max\{I - T^*, 2I - R_i\}$ (region E), $g(i)$ will surely default and $i$ will either strategically default or incur the penalty and continue to borrow. (Regions A and B are the equivalent of regions E and F, respectively, with the roles reversed).

## 2.3 Stage 1: choosing whether or not to borrow

In the first stage, eligible individuals choose whether or not borrow prior to their returns being realized. With the outside option normalized to 0, individuals will choose to borrow as long as $V > 0$. Let $\pi_{R,R}$ be the probability that the realized returns are such that both $i$ and $g(i)$ repay (i.e. regions B, C, and F in Figure 2) and let $\pi_{R,D}$ be the probability that the realized returns are such that $g(i)$ defaults and $i$ either strategically defaults or repays while incurring the penalty (i.e. region E in Figure 2). The expected return to borrowing can then be written as:[7]

$$V(P) = E[R_i] + \pi_{R,R}(\beta V(P) - I) + \pi_{R,D} 1\{P \leq P^*\}(P^* - P) \tag{2}$$

where transfers do not enter the expression because, from symmetry, expected net transfers are equal to zero. Since the last term of equation (2) is weakly positive, we have:

$$V(P) \geq \frac{E[R_i] - \pi_{R,R} I}{1 - \beta \pi_{R,R}} > 0 \; \forall P \in [0, I]$$

since $\beta E[R_i] > I$. Hence, all eligible individuals will choose to borrow.

## 2.4 Optimal (partial) group liability

From equation (1), the optimal maximum transfer is increasing one-for-one with the penalty $P$ until the penalty exceeds $P^*$, the threshold at which strategic default becomes optimal. Since a larger $T^*$ increases within-group risk sharing, conditional on remaining in the NSD

[7]While $\pi_{R,R}$ and $\pi_{R,D}$ depend in part on $P$ (since optimal transfers are determined in part by $P$), I suppress this dependence when possible for readability.

equilibrium, an increase in $P$ decreases the probability of default. However, if $P$ is large enough to induce strategic default, both group members will default in regions $A$ and $E$ of Figure 2, causing a discontinuous increase in default rates. Figure 3 depicts this relationship between group liability $P$, probability of default and the maximum transfers. As is evident, neither full group liability ($P = I$) nor zero group liability ($P = 0$) minimizes default rates.[8] Instead, the $P$ that minimizes defaults is an arbitrarily small amount below $P^*$. At this amount, transfers between group members are maximized and the penalty is not large enough to induce strategic default.

Since $P^* \equiv \beta V(P^*) - I$, equation (2) yields a simple fixed point expression for the optimal group liability penalty:[9]

$$P^* = \frac{\beta E[R_i] - I}{1 - \beta \pi_{R,R}(P^*)}, \tag{3}$$

where I now emphasize the fact that $\pi_{R,R}$ is dependent upon $P^*$. When the returns to borrowing are high (low), the incentive to repay and continue to borrow is large (small), so strategic default can be avoided with a high (low) group liability. All other moments of the distribution of returns affect the optimal group liability rate only through $\pi_{R,R}$: the greater the probability that both group members repay, the larger the present discounted value of continuing to borrow and hence the greater the ability to sustain higher group liability (an effect which is amplified by the fact that $\pi_{R,R}$ is an increasing function of $P^*$).

Figure 4 depicts the optimal group liability rate for a range of model parameters. Returns to borrowing of $i$ and $g(i)$ are assumed to be distributed according to a bi-variate normal distribution with a mean $\mu \equiv \frac{E(R_i)-I}{I} \in [0,1]$, a standard deviation $\sigma \in [0,1]$ and a correlation coefficient of $-0.5$ (top left) 0 (top right), 0.5 (bottom left), or 0.9 (bottom right). I assume $\beta = 0.95$. The optimal group liability rate varies substantially, indicating that different

[8] If $\beta V(P^*) - I \geq I$, then the optimal liability is full liability, i.e. $P^* = I$. Note that a lender cannot effectively enforce a liability greater than $I$, since a borrower would always prefer to repay her group member's loan rather than incurring the group liability penalty.

[9] In Appendix B.2 I prove that $P^*$ exists and is unique provided that $\beta$ is sufficiently below 1. In calculations, convergence to $P^*$ using an iterated function approach is almost instantaneous with $\beta = 0.95$.

empirical contexts may have very different optimal liabilities. Conditional on the variance and correlation, high mean returns to borrowing increases the optimal liability. Conditional on the mean and correlation, increases in the variance decrease the optimal liability, as they reduce the probability of repayment. Since within-group risk sharing is more effective the less positively correlated the returns to borrowing are between group members, the optimal liability tends to be lower the greater positive correlation in returns for a given mean and variance.

## 2.5 Welfare implications

In this section I show that the optimal group liability rate not only minimizes default rates, it also maximizes welfare. Normalize the size of the loan to 1 so that the amount to be repaid is equal to one plus the interest rate: $I = 1 + r$, and define the group liability rate to be the fraction of the group member's loan that an individual is penalized: $p \equiv \frac{P}{I}$. I assume that each MFI takes the interest rate as given. The per borrower profits of an MFI can be written as a function of the group liability rate $p$ and the given interest rate $r$:

$$\Pi\left(r,p\right) = \pi_{R,R}\left(1+r\right) + \pi_{R,D}1\{p \leq p^*\}\left(1+r+p\right) - 1 \tag{4}$$

From the previous section, $\pi_{R,R}$ is maximized when optimal transfers are maximized, which occurs when $p \geq p^*$. Furthermore, $\pi_{R,D}$ is decreasing in $p$ for $p \leq p^*$ but constant for $p > p^*$. As a result, MFI profits are maximized when $p = p^*$. This is intuitive: in addition to preferring minimal default rates, if one group member defaults, the MFI prefers to receive the maximum penalty that does not incentivize the other group member to default.

Suppose there is free entry into the MFI market, so that the interest rate adjusts to ensure there are zero profits. For any fixed group liability rate, the zero profit interest rate $r^*\left(p\right)$ is:

$$1 + r^*\left(p\right) = \frac{1 - 1\{p \leq p^*\}\pi_{R,D}p}{\pi_{R,R} + 1\{p \leq p^*\}\pi_{R,D}} \tag{5}$$

Since MFI profits are maximized at $p = p^*$, it is straightforward to show that the interest rate is minimized at $p = p^*$. With zero profits, all welfare accrues to the borrowers, so total welfare is the value to the borrower of borrowing at interest rate $r^*(p)$. From equations (2), (3), and (5) it can be shown (see Appendix B.3) that welfare $W(p)$ can be written as:

$$W(p) = \frac{E[R_i] - I}{1 - \beta\left(\pi_{R,R} + \pi_{R,D} 1\{P \leq P^*\}\right)} \tag{6}$$

Equation (6) is intuitive: when the MFI is earning zero profits, all the gains each period accrue to the borrower. The gains are the excess expected returns over the cost of the loan, $E[R_i] - I$. Since the borrower remains eligible to borrow with probability $\pi_{R,R} + \pi_{R,D} 1\{P \leq P^*\}$, the present discounted value of the per period gains borrowing is simply equation (6). Since the probability of remaining eligible to borrow is maximized at $P = P^*$, it follows immediately that welfare is maximized at $P = P^*$.

How substantial are the welfare gains associated with choosing the optimal partial group liability? Figures 5 and 6 depict the potential welfare gains of moving from individual liability ($P = 0$) and full liability ($P = I$), respectively, to the optimal liability for a range of model parameters. In both cases, the welfare gains associated with the move to optimal liability exceed 10 percent for certain combinations of model parameters. The welfare gains are the greatest moving away from individual liability when the mean and standard deviation of returns to borrowing are of approximately equal magnitudes; in these cases, the risk of default can be substantially reduced when a high group liability encourages within-group risk sharing. In contrast, the welfare gains are the greatest moving away from full liability when the returns to borrowing are low but variable; in these cases, the risk of strategic default is high, as it is likely one group member will be able to repay while the other cannot.

# 3 Extending the model

The model presented in Section 2 depicts the relationship between group liability and strategic default and generates a clear policy implication: to minimize default, group liability should be set high enough to promote within-group risk sharing, but low enough to avoid strategic default. To determine the optimal group liability, I would like to structurally estimate the model using repayment data obtained from a large microfinance institution. Unfortunately, with repayment data alone, strategic default cannot be disentangled from correlated returns to borrowing in the basic model. Intuitively, just because it is observed in the data that any time one group member defaults, all other group members default, it cannot be claimed that strategic default is present - it could very well be that group members have highly correlated shocks, so when one gets poor returns, all other group members do too. To disentangle strategic default from correlated returns, this section of the paper extends the basic model to incorporate both household structure and group size.

## 3.1 Household structure

Consider a household with two members, each of whom is in a lending group with an individual in a single-member household. Let individuals $i$ and $h(i)$ be the two individuals in the two-member household (hereafter "household members") and let $g(i)$ and $g(h(i))$ refer to their respective borrowing group members. This household-group structure, hereafter referred to as "household-individual groups," is illustrated in Figure 7.

I assume $i$ and $h(i)$ maximize household returns to borrowing. This assumption implies that the household will pool the returns both receive from borrowing and determine how to best allocate those returns to the repayment of the two household loans. Given this assumption, extending the basic model to household-individual groups enables the disentanglement of correlated returns and strategic default by providing information about how a borrower $i$ responds to actions by her household member's group member $g(h(i))$. Intuitively, in the absence of strategic default, the effect of $g(i)$ defaulting on $i's$ repayment should be the same as the effect of $g(h(i))$ defaulting, as the two defaults should equally affect the total household returns of $i$ and $h(i)$. When there is strategic default, however, the probability that $i$ defaults conditional on $g(i)$ defaulting should exceed the probability that $i$ defaults conditional on $g(h(i))$ defaulting since $i$ defaults in response to $g(i)$ defaulting.

The implications of the basic model outlined above generalize to the household-individual groups under one additional assumption: that the expected net transfers between group members in any period are equal to zero. This assumption is not innocuous: since $i$ and $h(i)$ fully share resources, they will be able to repay their loans more often than either $g(i)$ or $g(h(i))$, suggesting that they will transfer more to $g(i)$ and $g(h(i))$ than they receive. Hence, this assumption requires that $g(i)$ and $g(h(i))$ compensate $i$ and $h(i)$ for their higher repayment rates by making side payments in periods to $i$ and $h(i)$ when everyone can repay.

When expected transfers are zero, all four individuals have the same optimal group liability. This occurs because at the point that the borrowers are indifferent between repaying and strategically defaulting, the only state in the world in which they receive positive payouts is when they both are able to repay, which occurs with probability $\pi_{R,R}$. Since $\pi_{R,R}$ is the same for both group members, from equation (2), the value of borrowing for $P = P^*$ will be the same for the two group members. Since $V_i(P^*) = V_{g(i)}(P^*)$ and $P^* \equiv \beta V(P^*) - I$, it follows immediately that $P_i^* = P_{g(i)}^*$.

As in Section 2.2, the maximum transfer any individual will be willing to make is the maximum of the penalty $P$ and $P^*$. Since the group members share the same $P^*$, the group members will share the same willingness to make transfers. Hence, incorporating household structure only changes the budget constraint of borrowers within the same household; it otherwise does not affect the willingness of borrowers to repay loans or make transfers. As a result, given household returns $R_h \equiv R_i + R_{h(i)}$, household group member's returns $R_{g(i)}$ and $R_{g(h(i))}$, maximum transfers $T^*$, and the equilibrium, it is possible to determine which individuals repay and which individuals default, just like in the basic model; Appendix C.1

presents an algorithm for doing so.

### 3.2 Group size

Empirically, most borrowing groups are larger than two members. In this subsection, I show how the model can be extended to groups with $N > 2$ members. To do so, I assume that in the case of default, the group is able to to determine how the group liability penalty is split amongst the remaining group members.

As in the basic model, no borrower will be willing to transfer more than $\beta V(P) - I$ (the value of remaining eligible to borrow in the future) to another group member. Suppose that $n$ individuals are facing shortcomings and in need of transfers. With group liability $P$, the group faces a total penalty of $nP$. Since the group can allocate the penalty as it sees fit, it can maximize within-group risk sharing by force any group member to incur the entire penalty if that group member is not willing to make a transfer. As a result, any borrower would be willing to transfer at most $T_n^* = \min\{\beta V(P) - I, nP\}$.

Given the maximum transfer, should $n$ group members default, by symmetry the actual penalty incurred by each borrower is her share of the total penalty, $\frac{n}{N-n}P$. Symmetry also implies that expected net transfers between group members are zero. Let $\pi_n$ be the probability that $n$ group members default (where, for clarity, I do not explicitly note the dependence on model parameters). I show in Appendix B.4 that the methodology presented in Section 2.5 yields the following generalization of the expression for welfare given in equation (6):

$$W(P) = \frac{E[R_i] - I}{1 - \beta \sum_{n=0}^{N} \left(\frac{N-n}{N}\right) \pi_n 1\{\beta V - I \geq \frac{n}{N-n} P\}} \tag{7}$$

As in the basic model with two borrowers, welfare is maximized by maximizing the probability the individual borrower repays. Also as in the basic model, the group liability affects the probably of repayment in two ways: first, higher group liabilities incentivize greater within-group risk sharing, as $T_n^* = \min\{\beta V(P) - I, nP\}$; second, higher group liabilities incentivize strategic default, as the maximum number of group members a borrower is willing to let default before strategically defaulting herself is $k(P) = \lfloor \left( \frac{\beta V - I}{\beta V - I + P} \right) N \rfloor$. Since transfers are maximized at the fixed point $P = \beta V(P) - I$ for all $n$, it is straightforward to show that the optimal group liability is no higher than the present discounted value of continuing to borrow: $P^* \leq \beta V(P^*) - I$. Whether or not $P^*$ is strictly below $\beta V(P^*) - I$, however, depends on the relative importance of strategic default and within-group risk sharing, which will vary depending on the distribution of returns. Figure 8 depicts the relationship between the group liability and default rates; as $P$ increases, within-group transfers increase up to $P = \beta V(P) - I$, lowering the default rate. However, $k(P)$ declines, causing discontinuous increases in strategic default rates. The optimal group liability $P^*$ may be equal to $\beta V(P) - I$ (left panel) or strictly lower (right panel). Overall, the model predicts a U-shaped relationship between group liability and default rates.

In groups with more than two members, it is possible to disentangle strategic default from correlated returns to borrowing using only information on repayment rates. This identification arises from the fact that the group liability incurred by a borrower rises mechanically with the number of group members who default (as the total penalty increases and must be shared amongst a smaller number of people), causing strategic default to be optimal when the number of defaults exceeds $k(P)$. As a result, in the presence of strategic default, we should only observe groups with $n \leq k(P)$ group members defaulting or all group members defaulting. When there is no strategic default but returns amongst group members are correlated, however, we should observe groups with $n \in (k(P), N)$ group members defaulting with positive probability.

# 4 Empirical context

This section summarizes the lending environment and data that will be used in remainder of the paper.

## 4.1 The lending environment

To structurally estimate the models extensions, I use repayment data from Grameen Trust Chiapas (GTC), a large microfinance institution located in Chiapas, Mexico. At the time of study, GTC had 17 branches covering 94 municipalities located throughout the state. GTC offers both individual loans and group loans; individual loans require more collateral (typically 20% instead of 10% for the group loans) and usually are for smaller amounts.

During the period of study, the lending process was decentralized. While the central office determined the interest rates, bank branches had almost complete autonomy in determining other policies regarding loans, and substantial leeway was granted to individual loan officers. Typically, after clients chose to pursue a group loan, they formed their own group, were assigned a loan officer, and each client was given their own loan with their own repayment plan. If all the loans in the group were successful, each group member was offered a new loan with an increased amount of credit.[10] If any member of the group was unable to repay their loan at the end of the repayment period, the entire group was supposed to be dissolved and all the loans were supposed to be "restructured" into individual loans with a new payment schedule and continuing interest payments. Once restructured, an individual repay the restructured loan to be able to continue to borrow. However, as emphasized by qualitative interviews with the General Director and several branch managers, there was a substantial amount of variability in the default process. In some cases, if one group member failed to repay, the group was allowed to continue without that group member. In other cases, individuals who had been restructured were discouraged from continuing to borrow, even after repaying. There were also cases where borrowers were not allowed to continue to borrow until all group members had repaid their restructured loans in full. The specific penalty to having a group member fail to repay was determined by the loan officer. This variation in $P$ across loan officers is exploited in Section 6.

[10]The increasing size of the loan over time is not specifically incorporated into the model, although the discount factor could be interpreted to represent both impatience and the increasing value of the loan (assuming the latter is less than the former).

## 4.2 Data

The administrative data include every group loan made by GTC between 2004 and 2008. During this period, over 33,000 loans were made to over 18,000 individuals in over 5,000 groups. The data include the start and end dates of the loan, the amount of the loan (principle and interest), some basic information about the client, the borrowing group, and whether or not the loan was repaid. Summary statistics for the entire sample are presented in the upper panel of Table 1. The average loan amount was slightly more than 10,000 pesos (1 US dollar is worth approximately 10 pesos during this period). The default rate was relatively low (4.5%), which is similar to default rates in other microfinance lending programs worldwide (Morduch, 1999).

The structural estimation uses only a subset of the administrative data representing loans taken out by household members in household-individual groups. Specifically, the sample includes only loans by borrowers in which another member of the household had a loan with a different group at the same time. Households are defined to comprise of all borrowers that have the same address in the same town; addresses with more than 6 borrowers are excluded as they are unlikely to be considered households in the traditional sense. Of the 33,772 total loans, 1,782 are included in this sample. The second panel of Table 1 depicts the summary statistics for this reduced sample; the values of the variables are quite similar to those of the entire database, although default rates and loan amounts are slightly lower and the number of group members is slightly higher. Note that there is no need to constrain my focus to this particular sample for the group size model extension; I choose to do so both to retain comparability with the household structure model extension and to exploit the possibility of out-of-sample tests. In particular, the last panel of Table 1 reports the summary statistics for $25,643$ borrowers not in the household-individual sample that share a loan officer with at least one household-individual group; this "out of sample" group appears similar based on observables to both the household-individual group and the full sample.

There are at least two potential issues using the household-individual sample frame to estimate the household structure model extension. First, in the model, all four individuals make the decisions of whether or not to repay simultaneously; however, in the data, different household member's loans start and end at different times. In creating this sample, I assume that any loans that overlapped (i.e. one began before the other ended) occur at the same time. Since most loans are six months in length, this could mean that the decisions on whether or not to repay may be made multiple months apart. The second problem arises from the fact that the model only considers four individuals, whereas empirically the number of group members and often the number of household members is larger. In the estimation below, the "other" group or household members are simply amalgamated into one individual. Note that neither of these problems plague the estimation of the group size model extensions; however, the structural estimation of the group model extension does exclude all groups with more than 8 members because of computation time; these groups comprise 6 percent of the sub-sample.

## 4.3 Evidence of strategic default

In this subsection, I present suggestive evidence that strategic default is prevalent GTC's lending. Recall from Section C.1 that in the absence of strategic default, since a household pools its resources, the probability that an individual defaults conditional on her own group member defaulting should be the same as the probability she defaults conditional on her household member's group member defaulting. Table 2 presents the empirical prevalence of all possible default combinations for household-individual groups. The probability an individual defaults conditional on her household member's group member defaulting is 27.4 percent (26/95). Conditional on her own group member defaulting, however, the probability $i$ defaults is 90.6 percent (58/64); this substantial difference suggests that the presence of strategic considerations. In addition, given that the probability of default conditional on one's household member's group member defaulting exceeds the unconditional probability of $i$ defaulting of 3.6 percent (64/1782), the data also suggest that the returns to borrowing are positively correlated.

The patterns of repayment within groups also suggest the presence of strategic default. Table 3 reports the proportions of various combinations of defaults across group size for all loans. The most common pattern of default observed is for all group members to default, which occurs in 83.8 percent (326/389) of cases. When not all group members default, only a few repay: 71.4 percent (45/63) of the time a single borrower repays and 25.4 percent (16/63) of the time two group members pay. Of the 61 occurrences of default observed in groups with five or more members, I never observe that more than two group members default without the entire group defaulting. This is consistent with the claim in Section 3.2 that in the presence of strategic default, one should only observe a small number of borrowers or the entire group defaulting.

# 5 Structural estimation of the models

In this section, I structurally estimate the household structure and group size extensions of the model to determine the prevalence of strategic default and infer the optimal partial liability. I first present the methodology and then discuss the results.

## 5.1 Methodology

The goal of the structural estimation is to estimate both the distribution of the returns to borrowing and the group liability imposed on each borrowing group. I assume that all borrowers face (ex-ante) identical returns to borrowing that are drawn from a multivariate normal distribution. As a result, the distribution is fully characterized by the mean and standard deviation of returns, along with the correlation in returns across individuals. Consistent with the empirical context, I assume that each loan officer imposes a common group liability on all of her groups.

The structural estimation uses a simulated maximum likelihood procedure that works as follows. First, I choose a set of parameters that characterizes the distribution of returns to borrowing. Second, given this distribution, I simulate the realized returns for a large number of borrowing groups.[11] Third, for each possible group liability penalty rate $p$ and for every simulated group,[12] I use the household structure and group size extensions of the model to determine which individuals repay and which default. Fourth, I aggregate across the simulations to calculate the estimated probability of every possible default combination given a particular group liability penalty and the distribution of returns. Fifth, for each loan officer $l$, I find the group liability penalty $\hat{P}_l$ which maximizes the likelihood of observing the particular default combination of her groups in the data. Sixth, given $\{\hat{P}_l\}$, I aggregate across loan officers to determine the total likelihood and iterate the entire process to find the distribution of returns that maximizes this likelihood.

Mathematically, suppose there are $m \in \{1, ..., M\}$ simulations, $\delta \in \{1, ..., \Delta\}$ possible default combinations, and $i \in \{1, ..., N_l\}$ borrowing groups for loan officer $l \in \{1, ..., L\}$ in the data. Let $\theta$ be the vector of parameters governing the distribution of returns, let $R_m(\theta)$ be the vector of returns of simulation $m$, let $D_x(R_m(\theta), p, \theta)$ be the default combination predicted by model extension $x \in \{\text{household structure, group size}\}$ given $R_m(\theta)$, group liability rate $p$ and set of parameters $\theta$, and let $d_i$ be the observed default combination of group $i$. Then the estimated distribution of returns $\hat{\theta}_x$ and loan officer-specific group liability $\{\hat{P}_l^x\}$ for model extension $x$ is:

$$\hat{\theta}_x \equiv \arg\max_{\theta \in \Theta} \sum_{l=1}^{L} \max_{p \in [0,1]} \sum_{i=1}^{N_l} \sum_{\delta=1}^{\Delta} 1\{d_i = \delta\} \log\left(\frac{1}{M} \sum_{m=1}^{M} 1\{D_x(R_m(\theta), p, \theta) = \delta\}\right) \tag{8}$$

$$\hat{P}_l^x \equiv \arg\max_{p \in [0,1]} \sum_{i=1}^{N_l} \sum_{\delta=1}^{\Delta} 1\{d_i = \delta\} \log\left(\frac{1}{M} \sum_{m=1}^{M} 1\{D_x\left(R_m\left(\hat{\theta}_x\right), p, \theta\right) = \delta\}\right) \tag{9}$$

The major difficulty in calculating equations (8) and (9) is calculating the function $D_x(R_m(\theta), p, \theta)$ that determines the predicted default combinations; I present the algorithms used in Ap-

[11] In the results that follow, I simulate $10,000$ borrowing groups.

[12] Since $p \in [0, 1]$, I discretize the space in 0.01 steps.

pendix C.

## 5.2 Results

Table 4 presents the results of the structural estimation for both the household structure and group size model extensions. The estimated mean returns to borrowing (net of the cost of the loan) is 51.1% for the household structure extension and 19.5% for the group size extension. Since each loan is approximately 6 months long and the annual interest rate is about 46%, this suggests that gross monthly returns to borrowing are 9.7 percent ($\frac{\ln(1.477)}{6} + \frac{\ln(1.46)}{12}$) and 6.1 percent ($\frac{\ln(1.1951)}{6} + \frac{\ln(1.46)}{12}$), respectively, which is similar to experimental estimates of the returns to borrowing among small business owners in other developing countries (de Mel, McKenzie, and Woodruff, 2008). The standard deviation of returns to borrowing is 0.46 and 0.13 for the household structure and group size extensions, respectively, implying that a borrower will get negative returns (net of the principal plus interest) on a loan 13 percent (household structure) and 7 percent (group size) of the time. Both model extensions predict that the returns to borrowers within groups are positively correlated within groups (estimated correlation coefficients are 0.28 for household structure and 0.48 for group size), and the household structure estimates suggest the returns to borrowing are positively correlated within the household as well (correlation coefficient 0.20).

The structural estimates of the loan officer group liability penalties suggest that most enforced full group liability; the mean penalty estimated penalty was 0.96 and 0.90 with a standard deviation of 0.16 and 0.29 for household structure and group size extensions, respectively. Both model extensions yield similar estimates of which loan officers deviate from full group liability; the correlation between the two estimates across loan officers is 0.49.

How well do the models predict the observed default combinations in the data? Figure 10 compares the full distribution of household-individual default combinations in the data (reported in Table 2) with the predicted distribution given the structural estimates of the distribution of returns to borrowing and the loan officer-specific group liability penalties. Overall, the distribution of default combinations predicted by the structural model closely matches the observed distribution. The only discretion is that the model under-predicts the probability that all individuals default (1.23 percent in the data and 0.11 percent in the model) and over-predicts the probability that $i$ and $g(i)$ default while $h(i)$ and $g(h(i))$ repay (1.80 percent in the data and 2.72 percent in the model). This may be due to the fact that $h(i)$ and $g(h(i))$ are multiple borrowers in the data but are treated as individuals in the model.

Figure 11 compares the observed probabilities of default by group size with the probabilities of default predicted by the group size model extension given the structural estimates. The fit of the group size model extension is not as good as the fit of the household structure model extension, although the predicted default rates are within the 95 percent confidence interval of the observed default rates for all borrowing sizes. Since larger groups are better able to insure against low returns of any particular group member, the model predicts that the probability of default is monotonically decreasing in group size; in the data, however, there is no such relationship. This may be due to the fact that the model abstracts from the group formation process, whereas in reality group size may be correlated with other factors that affect default rates.

How do the estimated group liabilities compare to the optimal group liabilities? Both models estimate that the optimal group liability is strictly smaller than full liability; the household structure extension predicts any partial liability rate between 70 percent and 99 percent minimizes default rates, while the group size extension predicts a partial liability rate between 19 percent and 33 percent minimizes default rates.[13] Since most loan officers are enforcing full group liability, currently the MFI is setting the group liability too high. At the current group liabilities, both model extensions estimate that 16 percent of defaults

[13] In both cases, while the maximum transfer borrowers are willing to make increases in the optimal range, the probability of having to make such a large transfer is so small that the models predict the default rates would be identical throughout the optimal range (the estimation tolerance is 0.01%).

in the data are due to strategic default. Moving to optimal group liability would decrease default by 16 percent (household structure) to 20 percent (group size), increasing the welfare of borrowers by 7 percent in both cases.

Figure 9 depicts the estimated relationship between group liability and default rates for the household structure and group size model extensions. Despite the differences between the structural estimates, both model extensions predict that the default rate is nearly minimized over a large range of intermediate group liabilities. As a result, if the MFI were to enforce a partial liability of 50 percent, both model extensions estimate that default rates would be lowered (and welfare raised) by nearly as much as if they were to choose the optimal liability. In particular, at 50 percent liability, both model extensions estimate that default rates would fall by 14 percent (household structure) or 15 percent (group size), resulting in 6 percent (household structure) or 5 percent (group size) welfare gain to borrowers.

# 6 Assessing the model using variation in loan officer's enforced liability

Since many important aspects of group borrowing (e.g. adverse selection in group formation, moral hazard hazard in project choice, monitoring costs, etc.) are absent in the model, it remains to be seen how well the model reflects the reality of group borrowing. To assess the model, this section examines whether the U-shaped relationship between group liability and default predicted by the model (and its extensions) is present for loans not used in the structural estimation.

To do so, I exploit the variation across loan officers in the *de facto* group liability they enforced. The model predicts that borrowers with loan officers enforcing intermediate group liabilities should have lower default rates than those with loan officers enforcing either low or high group liabilities. To see if this is true, I can regress an indicator variable equal to one if individual $i$ with loan officer $l$ in municipality $m$ in year $t$ defaults, $D_{ilmt}$, on a quadratic function of the structurally estimated loan officer specific group liability using model extension $x \in \{\text{household structure, group size}\}$, $\hat{P}_l^x$, a vector of controls $X_{ilmt}$,[14] year fixed effects $\delta_t$, and municipality fixed effects $\delta_m$:

$$D_{ilmt} = \alpha + \gamma_1 \hat{P}_l^x + \gamma_2 \left(\hat{P}_l^x\right)^2 + \beta X_{ilmt} + \delta_t + \delta_m + \varepsilon_{ilmt} \tag{10}$$

Since $\hat{P}_l^x \in [0, 1]$, the U-shaped relationship predicted by the model predicts $\gamma_1 < 0$ and $\gamma_2 > 0$. There are three potential problems with estimating equation (10). First, since the loan officer penalties are structurally estimated from repayment data, equation (10) must be evaluated on data not used in the structural estimation to avoid any mechanical correlation between default rates and group liability. Because the structural estimation was done using only the subset of borrowers that were in household-individual groups, I can estimate equation (10) for borrowers not in household-individual groups but who shared the same loan officer (so that an estimate of $\hat{P}_l^x$ exists). Second, since $\hat{P}_l^x$ is an estimate, it is subject to measurement error, which may bias coefficients toward zero. Because I conduct two separate structural estimations using two different sources of variation, I can correct for measurement error by instrumenting for $\hat{P}_l^{hhstruct}$ with $\hat{P}_l^{grpsize}$. Third, there may unobservable omitted variables negatively correlated with default and positively correlated intermediate values of group liability that cause a spurious correlation between default rates and $\hat{P}_l^{hhstruct}$. This concern is mitigated by the inclusion of municipality fixed effects, as qualitative interviews with the MFI officials suggested that allocation of loan officers to borrowers was plausibly random within a branch (and each municipality was served by only one branch). However, there remains the concern that loan officers that enforce group liabilities nearer to the optimal group liability are also better in other ways that reduce default, which I explore further below.

Table 5 presents the results of regression (10) for the household-individual sample used for

[14]Controls include the age and sex of the borrower, the size of the loan, the length of time the individual has been borrowing with the bank, the interest rate, the length of the loan, and the size of the group.

the structural estimation. In eight of nine specifications, the default rates exhibit a U-shaped relationship with the group liability penalty enforced by the loan officer, although only half of the specifications yield statistically significant results. Table 5 presents the results of regression (10) for borrowers not in the the household-individual sample who had the same loan officer. In eight of nine specifications, default rates exhibit a statistically significant U-shaped relationship with the group liability. In these eight specifications, I can strongly reject that the optimal liability is zero, and for seven of the specifications, I can strongly reject that the optimal liability is full group liability. The point estimates of $\hat{\gamma}_1$ and $\hat{\gamma}_2$ imply an optimal group liability of between 0.35 and 0.82 depending on the specification, which is similar to the structural estimates. Moving from full liability to the optimal liability results in statistically significant reductions in default in six of nine specifications and statistically significant increases in welfare in four of nine specifications. Like the structural estimates, moving from full liability to 50% liability is nearly as beneficial as moving to the optimal liability; seven of nine specifications predict that such a move would reduce in default rates and increase welfare.

One potential concern is that the structural estimation procedure merely assigns the "better" loan officers partial liability, so that the out-of-sample U-shaped relationship between group liability and default rates is only capturing differences in loan officer quality. To see if this is the case, Table 7 presents the results of regression (10) on out-of-sample borrowers controlling for the in-sample default rates of the loan officers. This procedure compares the default rates of loan officers who achieved the same default rates in-sample but, because of the particular combination of defaults within their borrowing groups, were estimated to have enforced different group liabilities. As is evident, even conditional on their in-sample default rates, a U-shaped relationship between group liability and default rates remains, suggesting that loan officer quality is not driving the results. Hence, while the model abstracts from many important components of group borrowing, its central prediction that partial group liability can achieve lower default rates than either full group liability or individual liability is supported empirically.

# 7 Conclusion

This paper developed a model of group lending that contrasts the costs and benefits of group liability. The model implied that greater group liability encourages greater intra-group transfers, but if group liability is too large, it induces borrowers to strategically default. While simple, the model incorporated several realistic characteristics of group lending, including correlated returns to borrowing, limited liability, and forward-looking behavior. To disentangle strategic default from correlated returns to borrowing using observations on repayments alone, the model was extended into include household structure and group size.

Both model extensions were structurally estimated using repayment data from a large microfinance institution in southern Mexico. Despite difference sources of identification, structural estimates of both extensions suggested correlated returns across borrowers and substantial incidence of strategic default. Exploiting the fact that individual loan officers were largely able to choose their own policies regarding default, each loan officer's *de facto* group liability was estimated; these estimates were similar for the two model extensions. Using this variation, a strong U-shaped relationship between group liability and default rates was demonstrated using out-of-sample repayment data, providing support for the conclusion of the model that partial group liability results in lower default rates than either full group liability or full individual liability. Estimates of the optimal partial group liability ranged substantially, although 50% group liability was shown to reduce default rates nearly as much as the optimal group liability.

Hence, this paper suggests a easily implementable policy proscription that has the potential to reduce default rates in group lending programs. However, it should be emphasized that these empirical findings rely on the experience of one particular lender. Since the optimal liability varies substantially depending on the model fundamentals, it is important that the group liability is tailored to each empirical setting. One advantage of the estimation strategy developed in this paper is its modest data requirements, which hopefully can facilitate its replication in other contexts.

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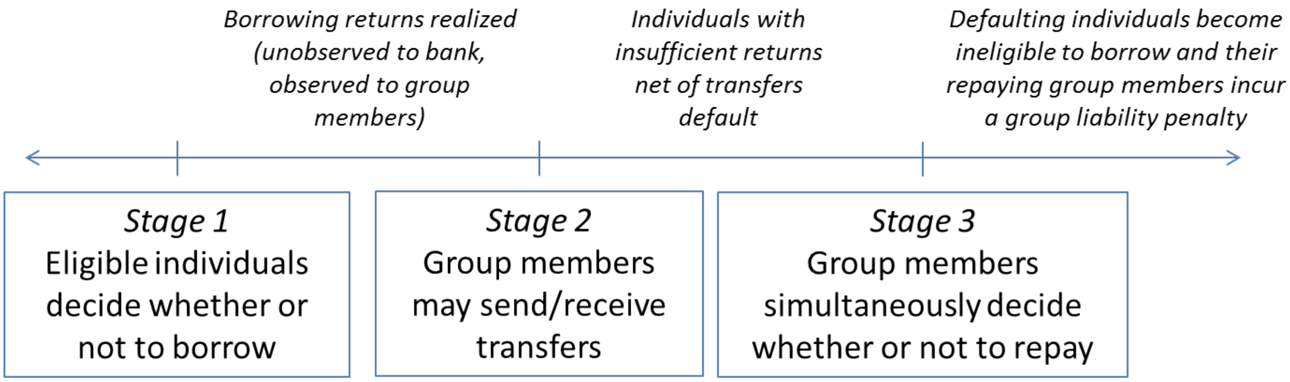
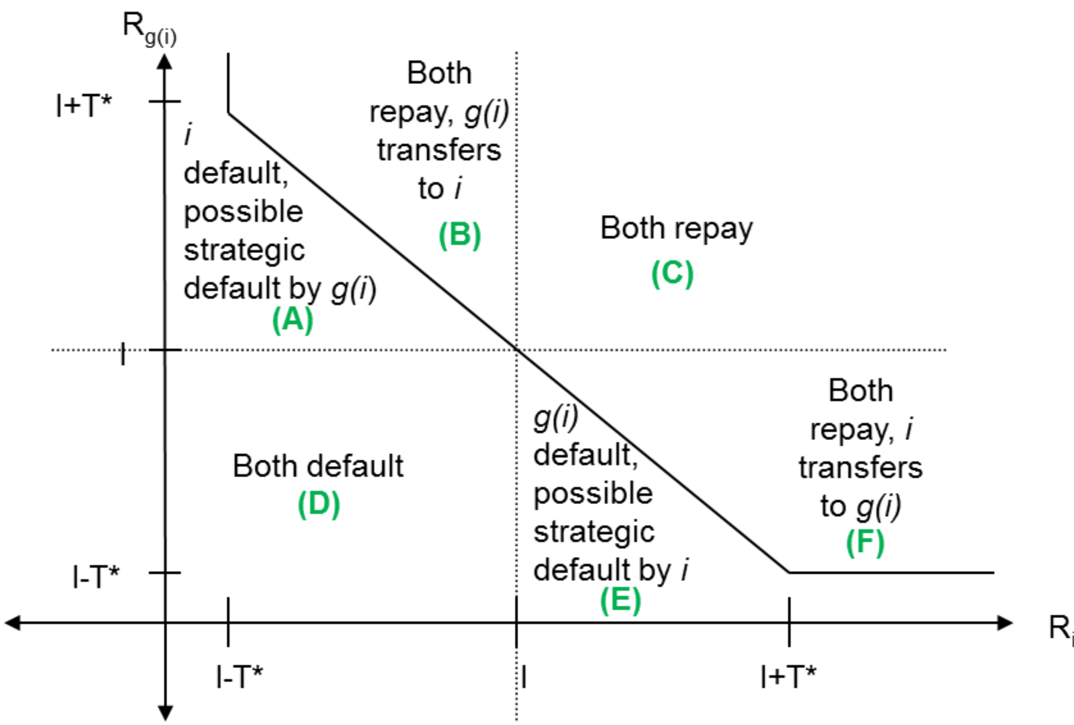
# Figures and tables

Figure 1: Model time line



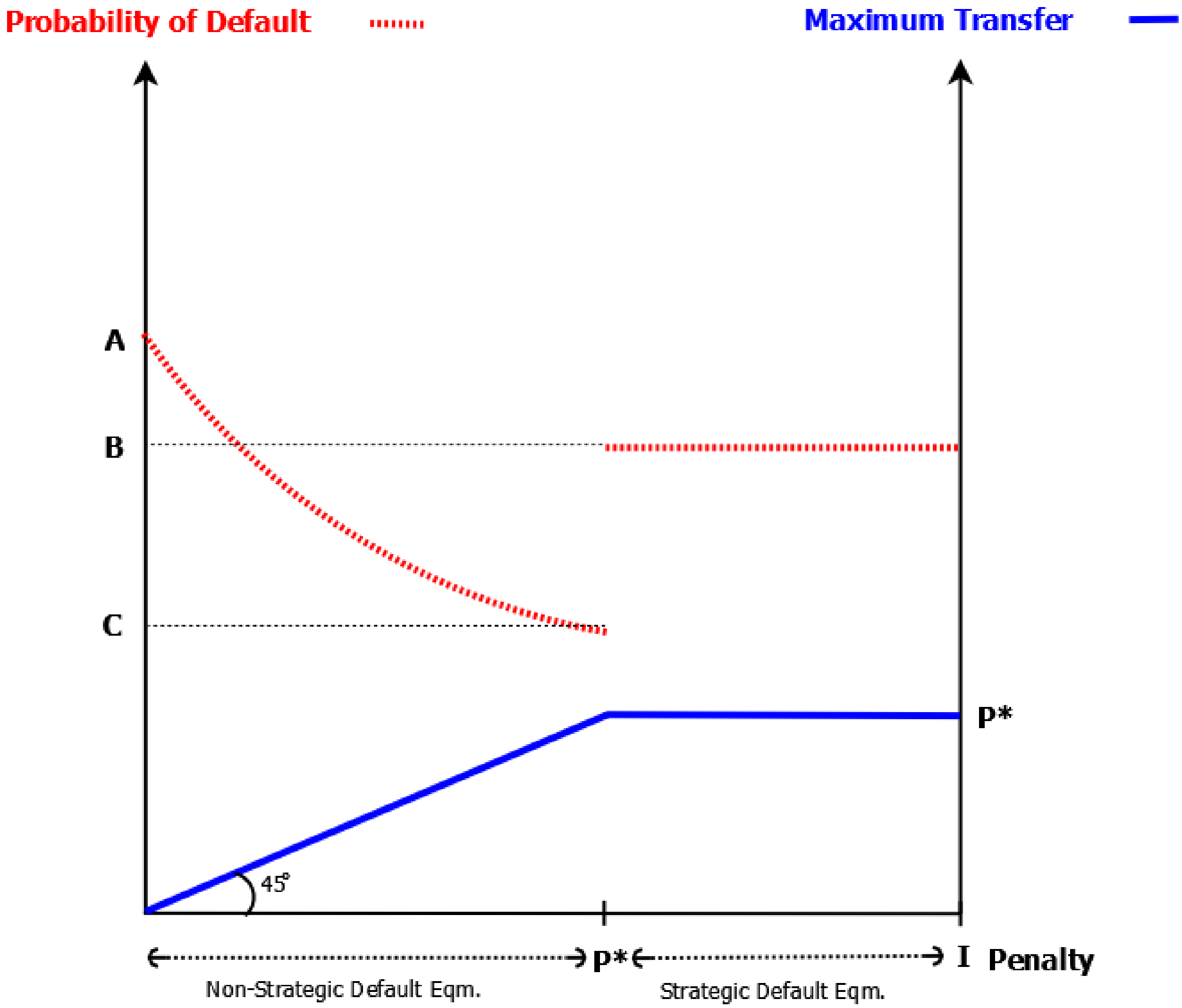

*Notes:* This figure shows the time line of a single period of the borrowing game.

Figure 2: Returns to borrowing and default in the basic model



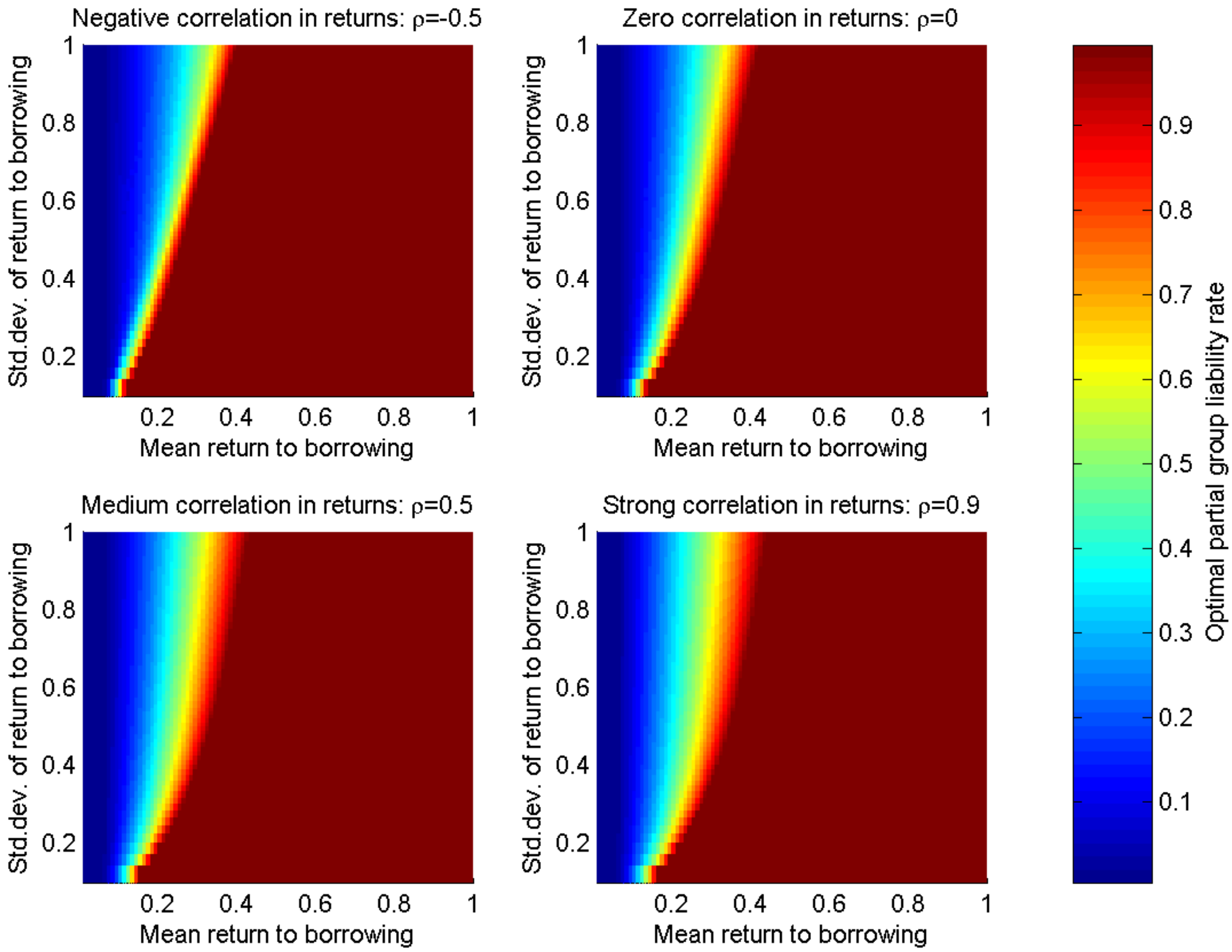

*Notes:* This figure shows the joint repayment decisions of group members $i$ and $g(i)$ as a function of their returns to borrowing.

Figure 3: Default rates and transfers as a function of group liability



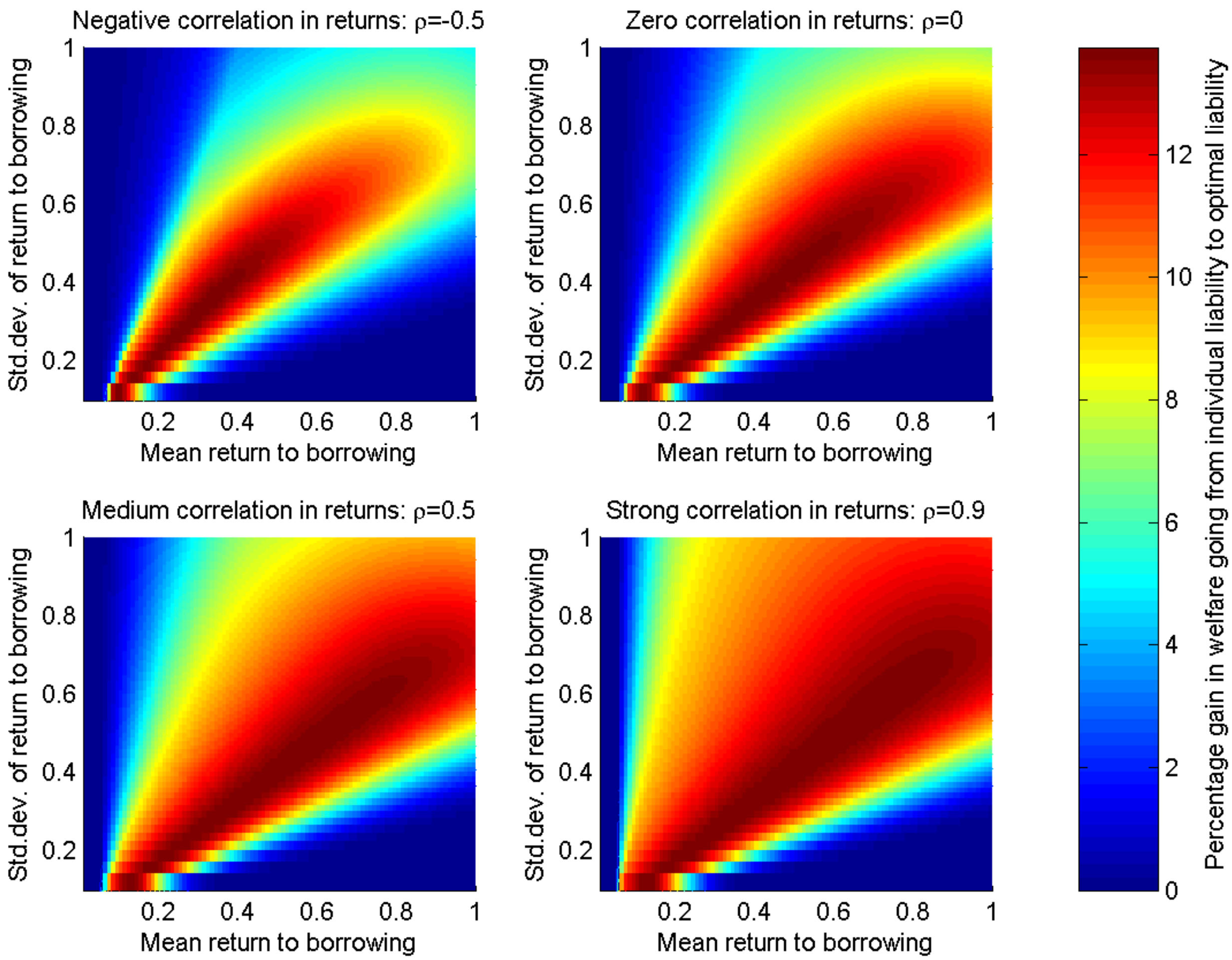

*Notes:* This figure shows the how transfers and default rates depend on the penalty incurred when a group member defaults. In the non-strategic default equilibrium, increases in the penalty increase within-group risk sharing, reducing default rates. If the penalty becomes too large, however, strategic default becomes optimal, causing a discontinuous jump in default rates.

Figure 4: Optimal partial liability as a function of returns to borrowing



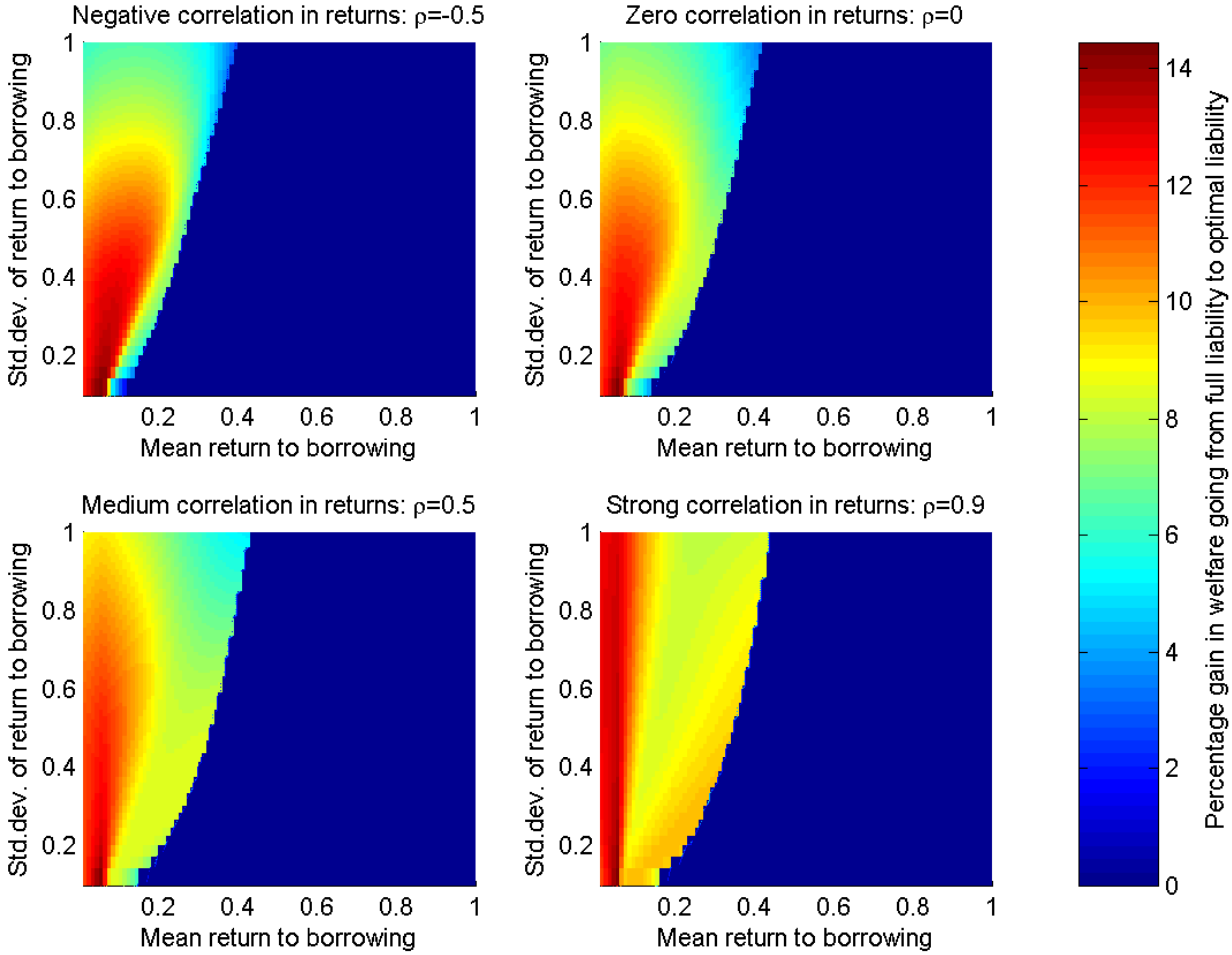

*Notes:* This figure shows the optimal partial group liability rate as a function of the mean and standard deviation of the returns to borrowing. Returns are assumed to be bivariate normally distributed with a correlation between group members of $-0.5$ (upper left), 0 (upper right), 0.5 (lower left), or 0.9 (lower right).

Figure 5: Welfare gains moving from individual liability to optimal group liability




*Notes:* This figure shows the welfare gains (in percent) of moving from individual liability to optimal group liability as a function of the mean and standard deviation of the returns to borrowing. Returns are assumed to be bivariate normally distributed with a correlation between group members of $-0.5$ (upper left), 0 (upper right), 0.5 (lower left), or 0.9 (lower right).

Figure 6: Welfare gains moving from full liability to optimal group liability




*Notes:* This figure shows the welfare gains (in percent) of moving from full group liability to optimal (partial) group liability as a function of the mean and standard deviation of the returns to borrowing. Returns are assumed to be bivariate normally distributed with a correlation between group members of $-0.5$ (upper left), 0 (upper right), 0.5 (lower left), or 0.9 (lower right). The discontinuity is due to the fact that if the optimal group liability is even marginally below full liability, then strategic default is optimal with full group liability.

Figure 7: Structure of "household-individual" groups

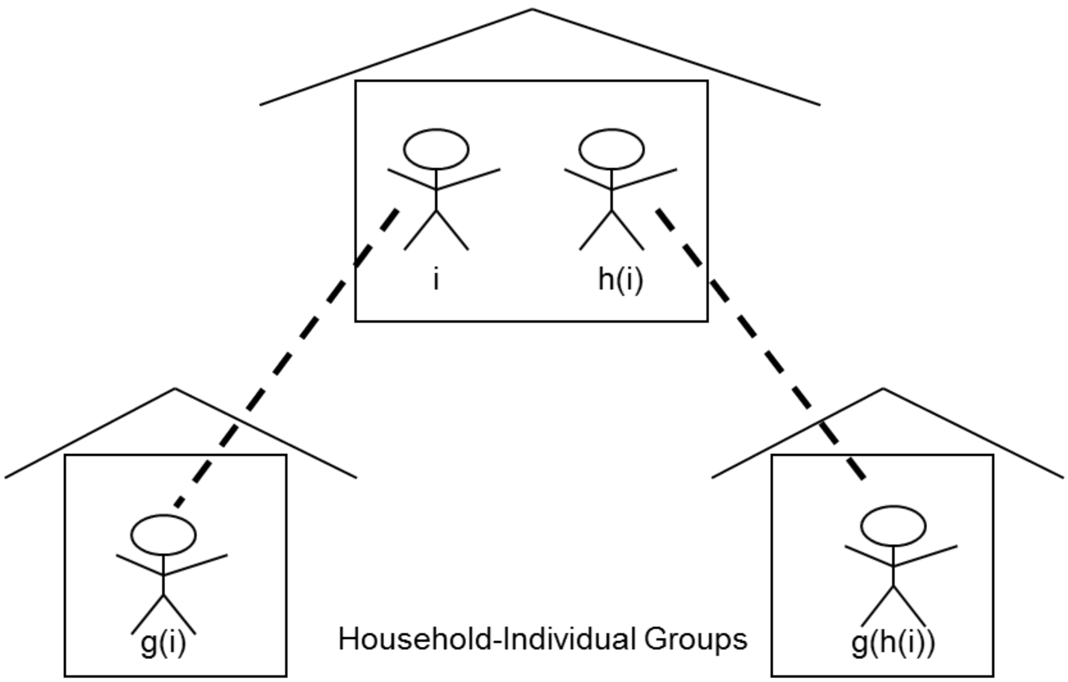
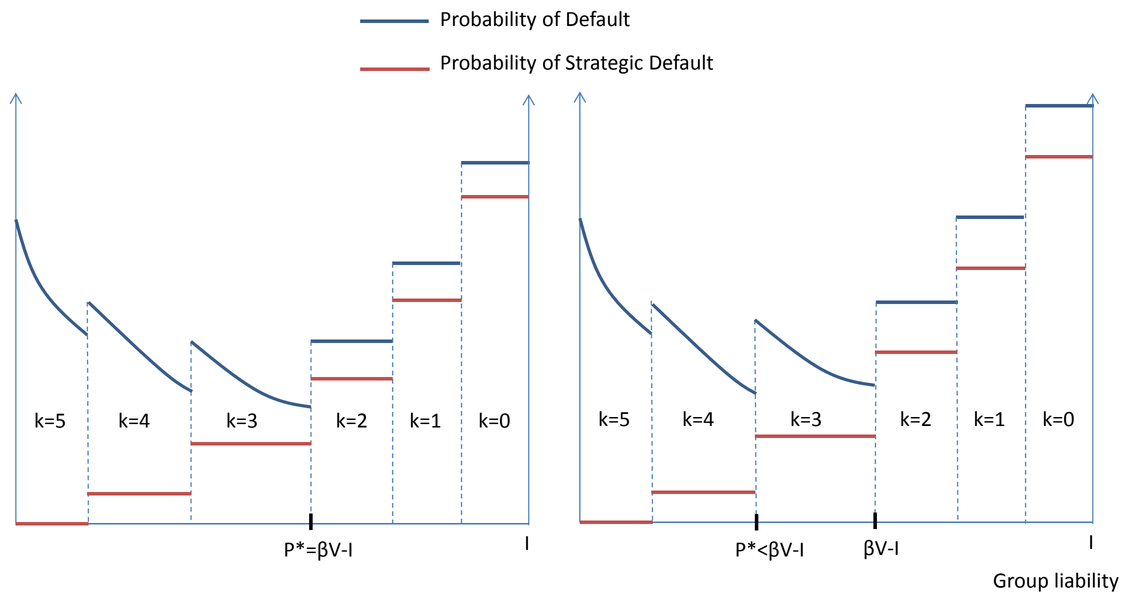
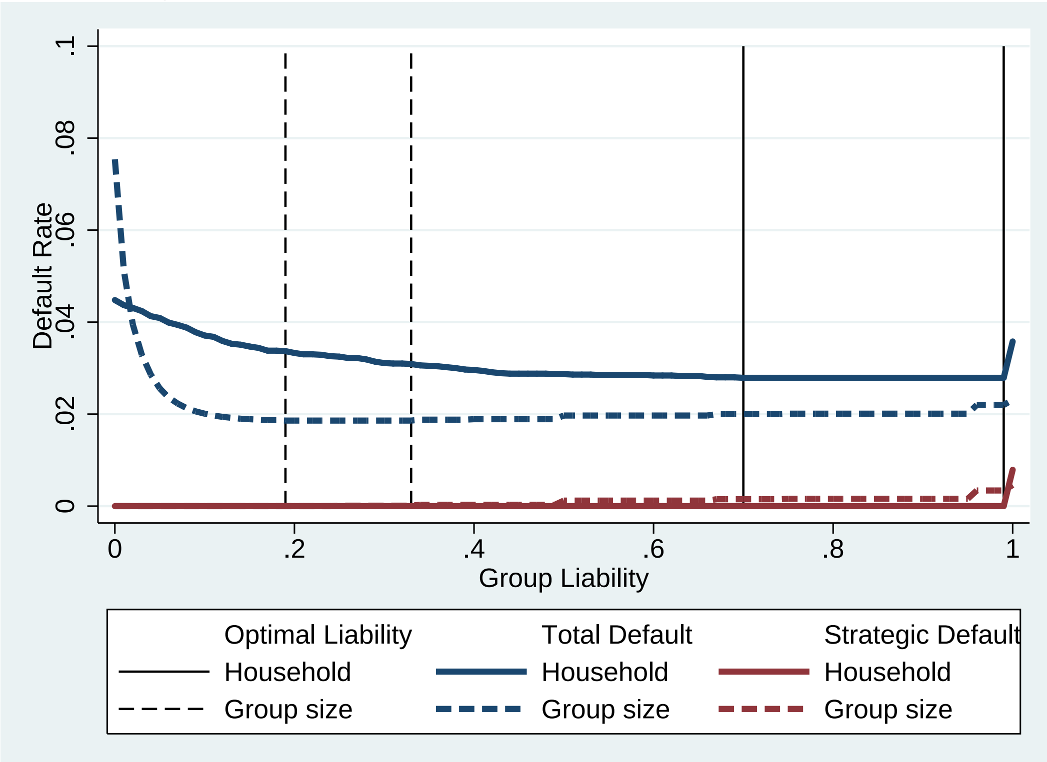


Figure 8: Default rates in groups with more than two members



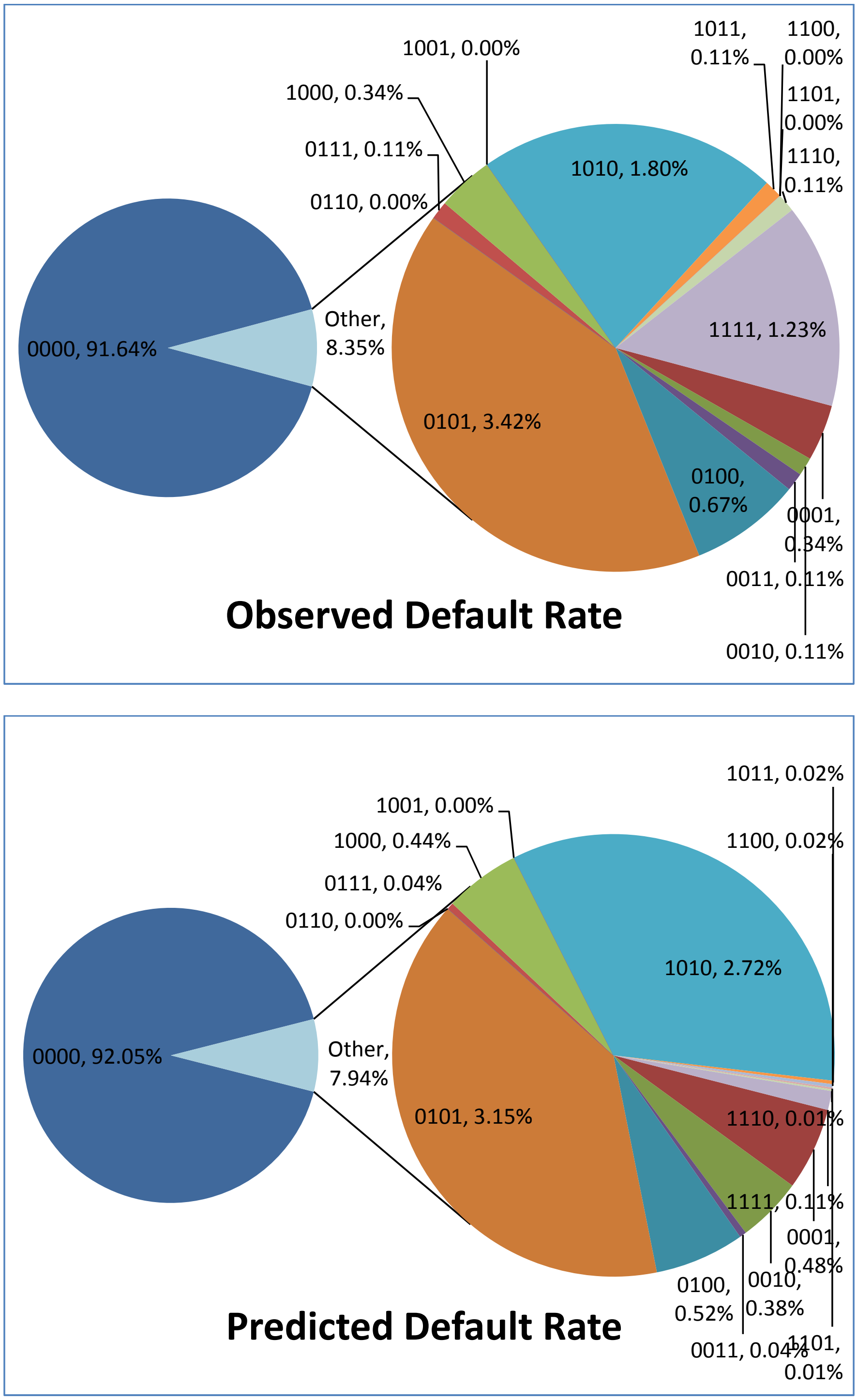

*Notes:* This figure shows the how default rates depend on the group liability penalty in a borrowing group with six members. As the penalty increases: (1) the within-group risk sharing increases up to $P = \beta V - I$ and stays level thereafter; and (2) the number of other group members that an individual is willing to allow to default before strategically defaulting herself, $k$, decreases, increasing the probability of strategic default. The optimal penalty $P^*$ that minimizes default rates is no greater than $\beta V - I$ (left panel), but may be strictly below $\beta V - I$ (right panel).

Figure 9: Group liability and default: Structural estimates



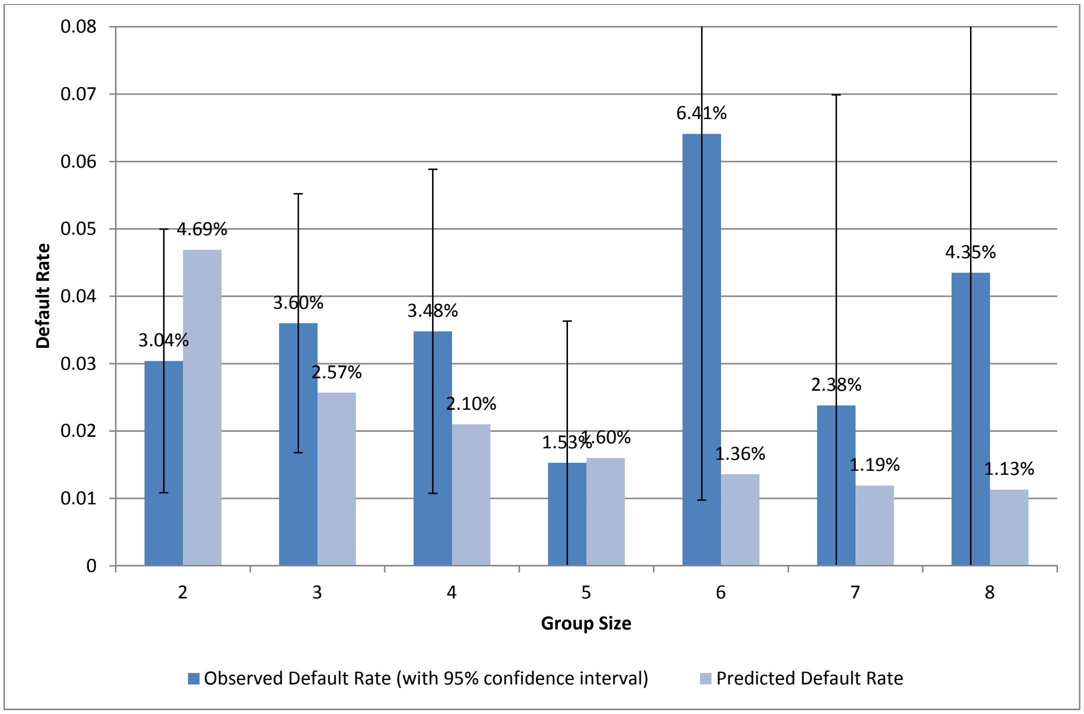

*Notes:* This figure shows the predicted relationship between group liability and default using the structural estimates from the household structure and group size model extensions. Increases in group liability at low levels reduce default rates by incentivizing within-group risk sharing. Increases in group liability at high levels, however, increase default rates by increasing the prevalence of strategic default. Default rates are minimizes at a group liability of 0.24 for the group size model extension and in the interval of $[0.7, 0.99]$ for the household structure model extension.

Figure 10: Model fit: Household structure




*Notes:* This figure compares the observed and predicted household-individual probabilities of different combinations of repayment. The four digit binary codes in the pie graph refer to the vector $\left[D_i, D_{h(i)}, D_{g(i)}, D_{g(h(i))}\right]$, where $D_x$ is an indicator variable equal to one if individual $x$ defaulted. Note that the top panel replicates the results reported in Table 2.

Figure 11: Model fit: Group size




*Notes:* This figure compares the observed probabilities of default and the predicted probabilities of default using the group size model extension by group size.

Table 1: Summary statistics

| | Mean | Std. Dev. | Min. | Max. |
|---|---|---|---|---|
| *All borrowers* | | | | |
| Default | 0.045 | (0.207) | 0 | 1 |
| Age | 38.612 | (11.44) | 13.933 | 87.441 |
| Sex (1=male) | 0.181 | (0.385) | 0 | 1 |
| Loan Amount (pesos) | 10036.727 | (12145.261) | 384.61 | 260000 |
| Tenure with Bank (weeks) | 51.208 | (33.35) | 4.429 | 311.143 |
| Interest Rate (annual) | 0.461 | (0.052) | 0 | 1.725 |
| Loan Length (weeks) | 28.417 | (6.295) | 3.714 | 107.571 |
| Group Size | 4.472 | (3.437) | 2 | 31 |
| Observations | 33,772 | | | |
| *Household-individual groups* | | | | |
| Default | 0.036 | (0.186) | 0 | 1 |
| Age | 38.691 | (11.918) | 18.209 | 82.188 |
| Sex (1=male) | 0.183 | (0.387) | 0 | 1 |
| Loan Amount | 9666.183 | (8903.115) | 763.070 | 101680 |
| Tenure with Bank (weeks) | 48.287 | (29.625) | 13.857 | 172.286 |
| Interest Rate | 0.457 | (0.046) | 0 | 0.565 |
| Loan Length (weeks) | 27.733 | (3.836) | 13.857 | 54.143 |
| Group Size | 5.036 | (3.993) | 2 | 27 |
| Observations | 1,782 | | | |
| *Borrowers sharing loan officers with household-individual groups* | | | | |
| Default | 0.033 | (0.178) | 0 | 1 |
| Age | 38.525 | (11.359) | 15.02 | 87.441 |
| Sex (1=male) | 0.176 | (0.381) | 0 | 1 |
| Loan Amount | 9889.773 | (12004.795) | 384.61 | 260000 |
| Tenure with Bank (days) | 51.408 | (33.344) | 4.429 | 248 |
| Interest Rate | 0.462 | (0.05) | 0 | 1.186 |
| Loan Length (days) | 28.312 | (6.115) | 3.714 | 107.571 |
| Group Size | 4.266 | (3.069) | 2 | 31 |
| Observations | 25,643 | | | |

*Notes:* The household-individual groups sample includes only loans by borrowers in which another member of the household had an outstanding loan with a different group at some point during the borrowing cycle. The borrowers sharing loan officer with household-individuals groups are any borrowers not in household-individual groups whose loan officer oversaw at least one loan for a household-individual group.

Table 2: Observed default combinations for household-individual groups

| Default combinations: | | | | Observed | Number of |
|---|---|---|---|---|---|
| $D_i$ | $D_{h(i)}$ | $D_{g(i)}$ | $D_{g(h(i))}$ | Probability | occurences |
| 0 | 0 | 0 | 0 | 0.9164 | 1,633 |
| 0 | 0 | 0 | 1 | 0.0034 | 6 |
| 0 | 0 | 1 | 0 | 0.0011 | 2 |
| 0 | 0 | 1 | 1 | 0.0011 | 2 |
| 0 | 1 | 0 | 0 | 0.0067 | 12 |
| 0 | 1 | 0 | 1 | 0.0342 | 61 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0.0011 | 2 |
| 1 | 0 | 0 | 0 | 0.0034 | 6 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0.0180 | 32 |
| 1 | 0 | 1 | 1 | 0.0011 | 2 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0.0011 | 2 |
| 1 | 1 | 1 | 1 | 0.0123 | 22 |
| Total: | | | | 1.0000 | 1,782 |

*Notes:* The sample includes only loans by borrowers in which another member of the household had an outstanding loan with a different group at some point during the borrowing cycle. $D_{g(i)}$ and $D_{g(h(i))}$ are equal to one if any group member defaulted.

Table 3: Observed composition of defaults by Group Size

| Defaulting members | Number of Members in Borrowing Group 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 1 | 14.08% (20) | 13.93% (17) | 7.81% (5) | 6.90% (2) | 0% (0) | 14.29% (1) | 0% (0) |
| 2 | 85.92% (122) | 7.38% (9) | 6.25% (4) | 6.90% (2) | 5.26% (1) | 0% (0) | 0% (0) |
| 3 | | 78.69% (96) | 3.13% (2) | 0% (0) | 0% (0) | 0% (0) | 0% (0) |
| 4 | | | 82.81% (53) | 0% (0) | 0% (0) | 0% (0) | 0% (0) |
| 5 | | | | 86.21% (25) | 0% (0) | 0% (0) | 0% (0) |
| 6 | | | | | 94.74% (18) | 0% (0) | 0% (0) |
| 7 | | | | | | 85.71% (6) | 0% (0) |
| 8 | | | | | | | 100.00% (6) |

*Notes:* Each column is a different group size. Each row indicates the number of group members that defaulted, conditional on there being one default. Number in parentheses indicates number of borrowing groups in cell. Full sample.

Table 4: Structural results

| **Structural Estimates** | Household structure | Group size |
|---|---|---|
| Mean returns to borrowing | 1.5105 | 1.1951 |
| Standard deviation of returns to borrowing | 0.4587 | 0.1345 |
| Correlation between group members | 0.3148 | 0.4811 |
| Correlation between household members | 0.1994 | N/A |
| Correlation between $i$ and $g(h(i))$ | 0.0001 | N/A |
| Correlation between $g(i)$ and $g(h(i))$ | 0.0000 | N/A |
| Mean loan officer penalty | 0.9614 | 0.9017 |
| Standard deviation of loan officer penalty | 0.1606 | 0.2898 |
| Fraction of loan officers enforcing full liability ($\hat{P}_l = 1$) | 0.7931 | 0.8793 |
| Correlation of $\hat{P}_l^{hhstruct}$ and $\hat{P}_l^{grpsize}$ | 0.4935 | |
| **Default rates** | | |
| Observed default rate | 3.59% | 3.43% |
| Predicted default rate | 3.34% | 2.32% |
| Predicted strategic default rate | 0.53% | 0.37% |
| Predicted fraction of strategic defaults | 15.87% | 15.95% |
| **Optimal liability** | | |
| Optimal liability | [70%,99%] | [19%,33%] |
| Predicted default rate at optimal liability | 2.79% | 1.86% |
| Predicted strategic default rate at optimal liability | 0.00% | 0.00% |
| Welfare gain from moving to optimal liability | 6.82% | 6.53% |
| **50% liability** | | |
| Predicted default rate at 50% liability | 2.87% | 1.97% |
| Predicted strategic default rate at 50% liability | 0.00% | 0.12% |
| Welfare gain from moving to 50% liability | 5.83% | 4.97% |

*Notes:* The first column reports structural estimates using the household structure model extension developed in Section 3.1; the second column uses the group size model extension developed in Section 3.2. The slight difference in default rates between the two models is because the household structure column reports the average default rate for household members, whereas the group size column reports the average default rate for all borrowers.

Table 5: Effect of loan officer penalties on default rates (in sample)

| | (1) HH | (2) HH | (3) HH | (4) Group | (5) Group | (6) Group | (7) IV | (8) IV | (9) IV |
|---|---|---|---|---|---|---|---|---|---|
| Loan officer penalty | -0.108 | -0.110 | -0.108 | -1.506*** | -1.384*** | -0.392 | -1.702** | -1.343** | 0.767 |
| | (0.158) | (0.150) | (0.142) | (0.463) | (0.437) | (0.311) | (0.682) | (0.565) | (0.513) |
| Loan officer penalty | 0.080 | 0.074 | 0.066 | 1.353*** | 1.235*** | 0.305 | 1.128*** | 0.852** | -0.901* |
| (squared) | (0.125) | (0.118) | (0.103) | (0.411) | (0.388) | (0.274) | (0.437) | (0.358) | (0.508) |
| Constant | 0.064 | 0.100 | 0.176* | 0.189*** | 0.198*** | 0.194** | 0.606** | 0.542*** | 0.212* |
| | (0.044) | (0.080) | (0.091) | (0.055) | (0.069) | (0.089) | (0.252) | (0.201) | (0.116) |
| Controls | No | Yes | Yes | No | Yes | Yes | No | Yes | Yes |
| Municipality Fixed Effects | No | No | Yes | No | No | Yes | No | No | Yes |
| Year Fixed Effects | No | Yes | Yes | No | Yes | Yes | No | Yes | Yes |
| Observations | 1782 | 1782 | 1782 | 1782 | 1782 | 1782 | 1782 | 1782 | 1782 |
| R-squared | 0.000 | 0.021 | 0.314 | 0.025 | 0.042 | 0.318 | -0.088 | -0.040 | 0.291 |
| U-shaped relationship? | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | No |
| *Optimal group liability* | | | | | | | | | |
| Optimal group liability | 67.4%*** | 74.9%*** | 81.9%*** | 55.7%*** | 56.0%*** | 64.2%*** | 75.4%*** | 78.8%*** | |
| | (22.6) | (29.7) | (27.5) | (0.6) | (0.6) | (7.3) | (3.1) | (3.8) | |
| Reduction in default rates | 23.6% | 7.3% | 1.6% | 741.0%*** | 486.2% | 36.6% | 208.4%*** | 75.5% | |
| | (56.6) | (26.7) | (7.1) | (215.6) | (756.1) | (51.8) | (69.4) | (115.1) | |
| Welfare gain | 10.7% | 4.2% | 1.2% | $\infty$ | -174.2% | 32.4% | 399.5% | 59.0% | |
| | (28.7) | (15.8) | (5.2) | | (108.9) | (57.2) | (718.4) | (85.0) | |
| *50% liability* | | | | | | | | | |
| Reduction in default rates | 17.0% | 0.2% | -3.4% | 729.0%*** | 477.2% | 30.8% | -13.6% | -63.4% | |
| | (64.0) | (34.6) | (10.2) | (210.7) | (741.5) | (50.8) | (109.5) | (127.5) | |
| Welfare gain | 7.4% | 0.1% | -2.4% | $\infty$ | -176.7% | 26.0% | -5.0% | -23.8% | |
| | (30.4) | (19.0) | (6.9) | | (114.1) | (51.6) | (37.7) | (26.2) | |

Borrowing-group clustered standard errors reported in parentheses. Controls include age, sex, loan amount, tenure with bank, interest rate, loan length, and group size. HH columns use the loan officer penalties structurally estimated by incorporating household structure. Group columns use the loan officer penalties structurally estimated by incorporating group size. IV columns instrument for the penalties estimated using household structure with the penalties estimated using group size. Infinite welfare gains are a result of the linear probability predictions not being constained to be between zero and one: when the estimated probability of default is less than $\frac{\beta-1}{\beta}$, the geometric sum needed to calculate the present discounted value does not converge. Stars indicate statistical significance: * $p<.10$ ** $p<.05$ *** $p<.01$

Table 6: Effect of loan officer penalties on default rates (out of sample)

| | (1) HH | (2) HH | (3) HH | (4) Group | (5) Group | (6) Group | (7) IV | (8) IV | (9) IV |
|---|---|---|---|---|---|---|---|---|---|
| Loan officer penalty | -0.066** | -0.040 | 0.002 | -0.797*** | -0.707*** | -0.430*** | -1.333*** | -1.091*** | -0.421** |
| | (0.029) | (0.029) | (0.044) | (0.114) | (0.113) | (0.120) | (0.265) | (0.235) | (0.166) |
| Loan officer penalty | 0.082*** | 0.056** | -0.000 | 0.736*** | 0.651*** | 0.384*** | 0.926*** | 0.750*** | 0.258* |
| (squared) | (0.025) | (0.025) | (0.042) | (0.101) | (0.101) | (0.107) | (0.164) | (0.147) | (0.138) |
| Constant | 0.019*** | 0.085* | 0.086 | 0.095*** | 0.165*** | 0.126* | 0.441*** | 0.439*** | 0.249*** |
| | (0.007) | (0.044) | (0.065) | (0.013) | (0.045) | (0.067) | (0.103) | (0.100) | (0.092) |
| Controls | No | Yes | Yes | No | Yes | Yes | No | Yes | Yes |
| Municipality Fixed Effects | No | No | Yes | No | No | Yes | No | No | Yes |
| Year Fixed Effects | No | Yes | Yes | No | Yes | Yes | No | Yes | Yes |
| Observations | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 |
| R-squared | 0.001 | 0.029 | 0.151 | 0.009 | 0.035 | 0.154 | -0.055 | -0.011 | 0.144 |
| U-shaped relationship? | Yes | Yes | No | Yes | Yes | Yes | Yes | Yes | Yes |
| *Optimal group liability* | | | | | | | | | |
| Optimal group liability | 40.1%*** | 35.4%*** | | 54.2%*** | 54.3%*** | 56.1%*** | 72.0%*** | 72.8%*** | 81.6%*** |
| | (6.0) | (10.3) | | (0.4) | (0.4) | (0.8) | (1.9) | (1.9) | (17.1) |
| Reduction in default rates | 83.5%*** | 23.1%** | | 454.0%*** | 125.2%*** | 93.6% | 215.0%*** | 56.7%** | 10.2% |
| | (14.4) | (10.8) | | (58.3) | (49.5) | (75.5) | (16.7) | (24.4) | (23.8) |
| Welfare gain | 50.3%*** | 17.9%*** | | $\infty$ | 539.2% | 128.2% | 526.2%* | 58.5%** | 6.8% |
| | (14.0) | (7.4) | | | (965.8) | (151.7) | (286.2) | (26.5) | (16.5) |
| *50% liability* | | | | | | | | | |
| Reduction in default rates | 81.2%*** | 21.9%** | | 450.2%*** | 124.1%*** | 91.8% | 82.4%** | 17.1% | -19.8% |
| | (14.7) | (10.5) | | (57.2) | (49.0) | (74.1) | (37.9) | (13.4) | (47.5) |
| Welfare gain | 48.3%*** | 16.9%*** | | $\infty$ | 510.1% | 122.8% | 47.5% | 12.5% | -10.9% |
| | (13.8) | (7.2) | | | (869.7) | (141.9) | (33.3) | (10.4) | (22.4) |

Borrowing-group clustered standard errors reported in parentheses. Controls include age, sex, loan amount, tenure with bank, interest rate, loan length, and group size. HH columns use the loan officer penalties structurally estimated by incorporating household structure. Group columns use the loan officer penalties structurally estimated by incorporating group size. IV columns instrument for the penalties estimated using household structure with the penalties estimated using group size. Infinite welfare gains are a result of the linear probability predictions not being constained to be between zero and one: when the estimated probability of default is less than $\frac{\beta-1}{\beta}$, the geometric sum needed to calculate the present discounted value does not converge. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01

Table 7: Effect of loan officer penalties on default rates, controlling for loan officer quality

| | (1) HH | (2) HH | (3) HH | (4) Group | (5) Group | (6) Group | (7) IV | (8) IV | (9) IV |
|---|---|---|---|---|---|---|---|---|---|
| Loan officer in-sample | 0.507*** | 0.512*** | 0.435*** | 0.498*** | 0.506*** | 0.430*** | 0.503*** | 0.509*** | 0.428*** |
| default rate | (0.046) | (0.044) | (0.053) | (0.047) | (0.045) | (0.053) | (0.046) | (0.044) | (0.053) |
| Loan officer penalty | -0.007 | 0.024 | 0.083** | -0.337*** | -0.205* | -0.281** | -0.598*** | -0.331** | -0.241 |
| | (0.027) | (0.028) | (0.038) | (0.111) | (0.109) | (0.116) | (0.182) | (0.158) | (0.155) |
| Loan officer penalty | 0.037 | 0.011 | -0.059* | 0.318*** | 0.192** | 0.248** | 0.435*** | 0.237** | 0.126 |
| (squared) | (0.024) | (0.025) | (0.035) | (0.098) | (0.097) | (0.104) | (0.113) | (0.100) | (0.131) |
| Constant | -0.015** | 0.012 | 0.026 | 0.035*** | 0.061 | 0.077 | 0.179** | 0.139* | 0.163* |
| | (0.007) | (0.044) | (0.067) | (0.013) | (0.047) | (0.069) | (0.070) | (0.079) | (0.092) |
| Controls | No | Yes | Yes | No | Yes | Yes | No | Yes | Yes |
| Municipality Fixed Effects | No | No | Yes | No | No | Yes | No | No | Yes |
| Year Fixed Effects | No | Yes | Yes | No | Yes | Yes | No | Yes | Yes |
| Observations | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 | 25643 |
| R-squared | 0.126 | 0.152 | 0.202 | 0.126 | 0.152 | 0.203 | 0.114 | 0.147 | 0.197 |
| U-shaped relationship? | Yes | No | No | Yes | Yes | Yes | Yes | Yes | Yes |
| *Optimal group liability* | | | | | | | | | |
| Optimal group liability | 8.8% | | | 53.1%*** | 53.4%*** | 56.6%*** | 68.7%*** | 69.8%*** | 95.3%** |
| | (30.7) | | | (1.2) | (1.7) | (1.3) | (3.5) | (5.3) | (45.5) |
| Reduction in default rates | 193.6%*** | | | 443.9%*** | 86.7% | 104.8% | 270.4%*** | 47.8% | 0.6% |
| | (37.1) | | | (117.4) | (78.8) | (151.8) | (31.9) | (44.1) | (11.6) |
| Welfare gain | 82.3%*** | | | $\infty$ | 70.6% | 92.7% | 165.1%*** | 28.3%* | 0.3% |
| | (26.0) | | | | (65.1) | (130.1) | (55.4) | (16.9) | (5.7) |
| *50% liability* | | | | | | | | | |
| Reduction in default rates | 154.0%*** | | | 442.0%*** | 86.2% | 102.4% | 173.6%*** | 27.2% | -52.5% |
| | (33.8) | | | (115.6) | (78.2) | (148.3) | (58.5) | (31.9) | (106.8) |
| Welfare gain | 56.1%*** | | | $\infty$ | 70.0% | 88.7% | 66.6%* | 14.3% | -20.2% |
| | (19.4) | | | | (63.9) | (122.1) | (39.9) | (14.3) | (25.6) |

Borrowing-group clustered standard errors reported in parentheses. Controls include age, sex, loan amount, tenure with bank, interest rate, loan length, and group size. HH columns use the loan officer penalties structurally estimated by incorporating household structure. Group columns use the loan officer penalties structurally estimated by incorporating group size. IV columns instrument for the penalties estimated using household structure with the penalties estimated using group size. Infinite welfare gains are a result of the linear probability predictions not being constained to be between zero and one: when the estimated probability of default is less than $\frac{\beta-1}{\beta}$, the geometric sum needed to calculate the present discounted value does not converge. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01

# Appendix

# A Model extensions

This section extends the model to incorporate several additional characteristics of microfinance borrowing, namely collateral, within-group heterogeneity in loan size and social sanctions. The implication that a partial liability which maximizes group transfers without inducing strategic default minimizes default rates remains true in all extensions.

## A.1 Incorporating collateral

Suppose that borrowers are required to put down collateral $C \leq I$ in order to borrow which the bank may keep if the borrower fails to repay. Without loss of generality, assume that $i$ is considering whether or not to make a transfer $T$ to $g(i)$ to allow $g(i)$ to repay her loan. Individual $i$ will make the transfer if the value of doing so exceeds the maximum of letting $g(i)$ default and either repaying or defaulting herself:

$$R_i + \beta V - I - T \geq \max\{R_i + \beta V - I - P, R_i - C\}$$

As a result, the maximum transfer that $i$ is willing to make is the minimum of the penalty she incurs when $g(i)$ defaults and the present discounted value of continuing to borrow less plus the value of the collateral she would have forfeited if she were to have defaulted:

$$T^* = \min\{P, \beta V - I + C\}$$

Since individual $i$ will choose to strategically default if and only if $P > \beta V - I + C$, the optimal liability is $P^* = \beta V - I + C$. Hence, the greater the amount of collateral that the bank requires, the higher the group liability (and hence within-group risk sharing) that can be maintained without inducing strategic default, as the cost of strategically defaulting increases in the amount of collateral.

## A.2 Within-group heterogeneity in loan size

Suppose that borrower $i$ has a loan of size $I$ and her group member $g(i)$ has a loan of size $\alpha I$. Let the group liability rate be the same for both borrowers (i.e. each borrower is penalized a fraction $p$ of their group member's loan size if their group member fails to repay). Suppose that $g(i)$ is unable to repay and $i$ is considering whether to make a transfer to $g(i)$. Borrower $i$ will make the transfer if and only if her value of making the transfer exceeds the maximum of repaying and allowing her group member to default or defaulting herself:

$$R_i - T + \beta V_i - I \geq \max\{R_i - p\alpha I + \beta V - I, R_i\}$$

As a result, the maximum transfer $i$ is willing to make to $g(i)$ is:

$$T_i^* = \min\{p\alpha I, \beta V_i - I\}$$

and, as in the basic model, strategic default becomes optimal when $\beta V_i - I < p\alpha I$. Hence, the optimal group liability that maximizes $i's$ transfers without inducing her to strategically default is $p_i^* = \beta\frac{V_i}{\alpha I} - 1$. By a similar logic, if $i$ is unable to repay, $g(i)$ will be willing to transfer up to $T_{g(i)}^* = \min\{pI, \beta V_{g(i)} - \alpha I\}$, and will strategically default when $\beta V_{g(i)} - \alpha I < pI$, so that the optimal group liability that maximizes $g(i)$'s transfers without incentivizing strategic default is $p_{g(i)}^* = \beta\frac{V_{g(i)}}{I} - \alpha$. Hence, as in the basic model, any liability below $\min\{p_i^*, p_{g(i)}^*\}$ or above $\max\{p_i^*, p_{g(i)}^*\}$ is suboptimal.

Calculating the value functions for $i$ and $g(i)$ and substituting this into the optimal group liabilities yields expressions for each group members optimal liabilities (where I retain the assumption of zero expected net transfers):

$$p_i^* = \frac{\beta}{\alpha}\frac{\frac{E[R_i]}{I} - \pi_{R,R}}{1 - \beta\pi_{R,R}} - 1$$

$$p_{g(i)}^* = \beta\frac{\frac{E\left[R_{g(i)}\right]}{I} - \pi_{R,R}\alpha}{1 - \beta\pi_{R,R}} - \alpha$$

If the expected returns to borrowing are a constant fraction of the amount (i.e. $\frac{E\left[R_{g(i)}\right]}{I} = \alpha\frac{E[R_i]}{I}$), then:

$$p_{g(i)}^* = \alpha^2 p_i^* + (\alpha - 1)$$

If $\alpha > 1$ (i.e. $g(i)$ has a larger loan than $i$), then $g(i)$ can incur a higher group liability rate than $i$ for two reasons: first, her value of borrowing is greater since expected returns are proportional to the loan amount; second, the cost of her group member defaulting is lower since she is penalized a fraction of her group member's loan, which is smaller than her own.

## A.3 Social Sanctions

Suppose there is an additional stage (Stage 4) of the basic game where borrowers can choose to inflict a utility cost (a "social sanction") $s \in [0, S]$ on their group member at zero cost to themselves. Borrowers will find it optimal to inflict the maximum social sanction $S$ on their group members in the event that the group member does not make a transfer that she is willing to afford. While social sanctions will increase the amount of within-group risk sharing, because an individual can inflict a social sanction regardless of whether their group member repays, social sanctions will not directly affect the optimal group liability.

To see this, suppose that $i$ is considering whether or not to make a transfer $T$ to $g(i)$. Individual $i$ will make the transfer if and only if the value of making the transfer exceeds the maximum of the value of not making the transfer, incurring the social sanction, and either repaying or strategically defaulting:

$$R_i + \beta V - I - T \geq \max\{R_i + \beta V - I - P - S, R_i - S\}$$

As a result, the maximum transfer that $i$ is willing to make is:

$$T^* = S + \min\{\beta V - I, P\},$$

so that the maximum transfer is increasing monotonically in the maximum social sanction. Individual $i$ will choose to strategically default if and only if $P > \beta V - I$. Hence, as in the basic model, the optimal group liability is $P^* = \beta V - I$, which depends on the social sanction only to the extent that the value of borrowing $V$ increases in $S$ due to the increased within-group risk sharing.

# B Additional Derivations

## B.1 $\beta E[R_i] > I$ implies $\beta V > I$

Multiplying both sides of equation (2) by $\beta$ and subtracting $I$ yields:

$$\beta V - I = \beta \left( E[R_i] + \pi_{R,R} (\beta V - I) + \pi_{R,D} \max\{0, \beta V - I - P\} \right) - I$$
$$\beta V - I \geq \frac{\beta E[R_i] - I}{1 - \pi_{R,R}} > 0$$

Since $\pi_{R,D} \max\{0, \beta V - I - P\}$ is weakly positive, we have:

$$\beta V - I \geq \frac{\beta E[R_i] - I}{1 - \pi_{R,R}}$$

Hence, $\beta E[R_i] > I$ implies $\beta V - I > 0$.

## B.2 $P^*$ exists and is unique

In this subsection, I show that $P^*$ exists and is unique as long as $\beta$ is sufficiently below 1. Recall equation (3):

$$P^* = \frac{\beta E[R_i] - I}{1 - \beta \pi_{R,R}(P^*)}$$

where $\pi_{R,R}(P)$ is the probability that both group members repay when the group liability is $P$. From figure 2, we have:

$$\pi_{R,R}(P) \equiv \int_{I-\min\{P,P^*\}}^{\infty} \int_{I-\min\{P,P^*\}}^{\infty} f(x,y)\, dxdy - \int_{I-\min\{P,P^*\}}^{I+\min\{P,P^*\}} \int_{I-\min\{P,P^*\}}^{2I-R_g(i)} f(x,y)\, dxdy,$$

where $f(x,y)$ is probability density function of the returns of $R_i = x$ and $R_{g(i)} = y$. Define the function $G(P) \equiv \frac{\beta E[R_i]-I}{1-\beta\tilde{\pi}_{R,R}(P)} - P$, where $\tilde{\pi}_{R,R}(P) \equiv \int_{I-P}^{\infty} \int_{I-P}^{\infty} f(x,y)\, dxdy - \int_{I-P}^{I+P} \int_{I-P}^{2I-y} f(x,y)\, dxdy$. Note that: a) $G(P^*) = 0$; b) since $\beta E[R_i] > I$ and $\pi_{R,R}(P) \in [0,1]$, $G(0) > 0$; c) $G(\bar{P}) < 0$ where $\bar{P} \equiv \frac{\beta E[R_i]-I}{1-\beta} > 0$; and d) $G(P)$ is continuous. By (b), (c), (d) and the intermediate value theorem, there exists a $\tilde{P} \in [0, \bar{P}]$ such that $G(\tilde{P}) = 0$; by (a), $\tilde{P} = P^*$; hence $P^*$ exists.

To prove uniqueness, I use the contraction mapping theorem. Define the metric $d(x,y) \equiv \max\{\ln\left(\frac{x}{y}\right), \ln\left(\frac{x}{y}\right)\}$ on the space $[0, \bar{P}]$. Define $H(P) = \frac{\beta E[R_i]-I}{1-\beta\tilde{\pi}_{R,R}(P)}$. Note that $H(P^*) =$

$P^*$. By the contraction mapping theorem, $P^*$ exists and is unique if for all $P_1, P_2 \in \left[0, \bar{P}\right]$, $d\left(H\left(P_1\right), H\left(P_2\right)\right) < d\left(P_1, P_2\right)$. Without loss of generality, let $P_2 > P_1$, so that $d\left(P_1, P_2\right) = \ln \frac{P_2}{P_1}$. Then

$$d\left(H\left(P_1\right), H\left(P_2\right)\right) = \ln\left(\frac{\frac{\beta E[R_i]-I}{1-\beta\tilde{\pi}_{R,R}(P_2)}}{\frac{\beta E[R_i]-I}{1-\beta\tilde{\pi}_{R,R}(P_1)}}\right) = \ln\left(\frac{1-\beta\tilde{\pi}_{R,R}\left(P_1\right)}{1-\beta\tilde{\pi}_{R,R}\left(P_2\right)}\right)$$

Hence, for all $P_2 > P_1$, there exists a contraction mapping if:

$$\ln\left(\frac{1-\beta\tilde{\pi}_{R,R}\left(P_1\right)}{1-\beta\tilde{\pi}_{R,R}\left(P_2\right)}\right) < \ln\frac{P_2}{P_1} \iff P_1\left(1-\beta\tilde{\pi}_{R,R}\left(P_1\right)\right) < P_2\left(1-\beta\tilde{\pi}_{R,R}\left(P_2\right)\right)$$

This occurs if and only if $\frac{\partial}{\partial P}P\left(1-\beta\pi_{R,R}\left(P\right)\right) > 0$ for all $P \in \left[0, \bar{P}\right]$, which is equivalent to:

$$\frac{1}{\beta} > \tilde{\pi}_{R,R}\left(P\right) + P\frac{\partial}{\partial P}\tilde{\pi}_{R,R}\left(P\right)$$

Since $\tilde{\pi}_{R,R}\left(P\right) \in [0,1]$ and $P \in \left[0, \bar{P} \equiv \frac{\beta E[R_i]-I}{1-\beta}\right]$, it is sufficient to show that:

$$\frac{\partial}{\partial P}\tilde{\pi}_{R,R}\left(P\right) < \left(\frac{1-\beta}{\beta}\right)^2\left(E\left[R_i\right]-I\right)$$

Using Leibniz's rule, the definition of $\tilde{\pi}_{R,R}\left(P\right)$, and the fact that $f\left(x,y\right) = f\left(y,x\right)$ by symmetry, we have:

$$\frac{\partial}{\partial P}\tilde{\pi}_{R,R}\left(P\right) = 2\int_{I+P}^{\infty} f\left(x, I-P\right)dx$$

Hence, for a $\beta$ sufficiently small so that $\int_{I+P}^{\infty} f\left(x, I-P\right)dx < \frac{1}{2}\left(\frac{1-\beta}{\beta}\right)^2\left(E\left[R_i\right]-I\right)$, there will exist a unique value of $P^*$. (For Figures 4 and the structural estimation, I calculated $P^*$ using an iterated function with $P_0 = 0.5$ $P_n = H\left(P_{n-1}\right)$. With $\beta = 0.95$, convergence was almost instantaneous.)

## B.3 Welfare calculations

From equation (2), welfare is:

$$W\left(p\right) = E\left[R_i\right] + \pi_{R,R}p^*\left(p\right) + \pi_{R,D}1\{p \leq p^*\left(p\right)\}\left(p^*\left(p\right)-p\right),$$

which is equivalent to:

$$W\left(p\right) = E\left[R_i\right] + p^*\left(p\right)\left(\pi_{R,R} + \pi_{R,D}1\{p \leq p^*\left(p\right)\}\right) - \pi_{R,D}1\{p \leq p^*\left(p\right)\}p \tag{11}$$

From equation (3), we can write:

$$p^*\left(p\right) = \beta W\left(p\right) - \left(1 + r^*\left(p\right)\right)$$

Substituting in equation (11) yields:

$$p^*(p) = \frac{\beta\left(E\left[R_i\right] - \pi_{R,D}1\{p \leq p^*\}p\right) - \left(1 + r^*(p)\right)}{1 - \beta\left(\pi_{R,R} + \pi_{R,D}1\{p \leq p^*\}\right)} \tag{12}$$

Substituting equation (12) into (11) and rearranging yields:

$$W(p) = \frac{E\left[R_i\right] - \pi_{R,D}1\{p \leq p^*\}p - \left(\pi_{R,R} + \pi_{R,D}1\{p \leq p^*\}\right)\left(1 + r^*(p)\right)}{1 - \beta\left(\pi_{R,R} + \pi_{R,D}1\{p \leq p^*\}\right)} \tag{13}$$

Finally, substituting equation (5) into equation (13) yields:

$$W(p) = \frac{E\left[R_i\right] - 1}{1 - \beta\left(\pi_{R,R} + \pi_{R,D}1\{p \leq p^*\}\right)},$$

as claimed.

## B.4 Welfare calculations with $N > 2$ group members

Normalize the loan amount to one so that $I = 1 + r$. Then the per borrower profits to the MFI are:

$$\Pi(p, r) = \frac{1}{N}\sum_{n=0}^{N}\pi_n\left(1\{\beta V - (1+r) \geq \frac{n}{N-n}p\}(N-n)\left(1 + r + \frac{n}{N-n}p\right) - N\right)$$

With free entry, given a group liability $p$, the interest rate ensures that each MFI earns zero profits:

$$1 + r^*(p) = \frac{1 - \sum_{n=0}^{N}\pi_n 1\{\beta V - (1+r) \geq \frac{n}{N-n}p\}\frac{n}{N}p}{\sum_{n=0}^{N}\pi_n 1\{\beta V - (1+r) \geq \frac{n}{N-n}p\}\left(\frac{N-n}{N}\right)} \tag{14}$$

Using the fact that if $n$ group members default with probability $\pi_n$, the probability any particular group member repays is $\frac{N-n}{N}$, the borrowers value function is:

$$V = E\left[\frac{R_i}{I}\right] + \sum_{n=0}^{N}\pi_n\left(\frac{N-n}{N}\right)\max\{\beta V - (1+r) - \frac{n}{N-n}p, 0\} \tag{15}$$

which is equivalent to:

$$V = \frac{E\left[\frac{R_i}{I}\right] - (1+r)\sum_{n=0}^{N-1}\pi_n\left(\frac{N-n}{N}\right)1\{\beta V - (1+r) \geq \frac{n}{N-n}p\} - \sum_{n=0}^{N-1}\pi_n 1\{\beta V - (1+r) \geq \frac{n}{N-n}p\}\frac{n}{N}p}{1 - \beta\sum_{n=0}^{N-1}\pi_n\left(\frac{N-n}{N}\right)1\{\beta V - (1+r) \geq \frac{n}{N-n}p\}} \tag{16}$$

Substituting in the zero profit interest rate given in equation (14) into the borrower value function given in equation (16) yields an expression for welfare:

$$W(p) = \frac{E\left[R_i\right] - I}{1 - \beta\sum_{n=0}^{N-1}\pi_n\left(\frac{N-n}{N}\right)1\{\beta V - I \geq \frac{n}{N-n}P\}},$$

as claimed.

# C Algorithms determining repayment combinations

In this section, I describe the algorithms I used to determine who defaults and who repays within a group given a vector of returns to borrowing $R$. By simulating a large number of $R$, I am able to estimate the probability of various default combinations given a vector of model parameters, which allows me to estimate the likelihood function of the observed data.

## C.1 Household structure

The household-individual group is comprised of four individuals: $i$, $h(i)$, $g(i)$, and $g(h(i))$. Let $R \equiv \left[R_i, R_{h(i)}, R_{g(i)}, R_{g(h(i))}\right]$ be the $4 \times 1$ vector of returns and let $D \equiv \left[D_i, D_{h(i)}, D_{g(i)}, D_{g(h(i))}\right]$ be the $4 \times 1$ vector of indicator variables equal to one if that individual defaults. The goal is to write the algorithm for a function that yields $D$ as a function of $R$, the group liability rate $p$, and the model parameters $\theta$: $D_{hh}(R, p, \theta) = D$. Let $\tilde{D}_{hh}(R, p, p^*, \theta)$ be the function that yields $D$ as a function of $R$, the group liability rate $p$, and the optimal group liability. The algorithm then works as follows:

1. First I calculate $\tilde{D}_{hh}\left(R, p, \beta\frac{V(p)}{I} - 1, \theta\right)$. Let $T^* \equiv \max\{pI, p^*I\}$, which is the maximum transfer any group member is willing to send. Note that strategic default is optimal if and only if $p > p^*$. Note too that it will never be the case that the household pays back the loan with the group member with the lower return and does not pay back the loan with the group member with the higher return. For what follows, assume that $R_{g(i)} \geq R_{g(h(i))}$ (the opposite case is of course symmetric). I first determine the repayment of $i$ and $g(i)$ and then determine the repayment of $h(i)$ and $g(h(i))$.

    (a) Define $E_{hh} \equiv 1\{p > \beta\frac{V(p)}{I} - 1\}\min\{\max\{R_i + R_{g(i)} - I, 0\}, T^*\} + 1\{p \leq p^*\}\min\{\max\{R_i + R_{g(i)} - 2I, 0\}T^*\}$ to be the maximum transfers (if anything) that the household can and is willing to transfer to $g(i)$ and define $E_{g(i)} \equiv \min\{\max\{R_{g(i)} - I, 0\}, T^*\}$ to be the maximum transfers that $h(i)$ can and is willing to transfer to $i$. Define $S_{hh} \equiv \max\{I - R_i - R_{h(i)}, 0\}$ and $S_{g(i)} \equiv \max\{I - R_{g(i)}, 0\}$ to be the shortfalls (if any) that the household and $g(i)$, respectively, face. If $E_{hh} + E_{g(i)} > S_{hh} + S_{g(i)}$, then both $i$ and $g(i)$ will repay. Furthermore, if $p \leq p^*$ and $R_i + R_{h(i)} > I$, then $i$ will repay. Similarly, if $p \leq p^*$ and $R_{g(i)} > I$, then $g(i)$ will repay. In all other cases, $i$ and $g(i)$ will default.

    (b) Define $\tilde{R}_{hh} \equiv R_i + R_{g(i)} - I - 1\{D_i = 0\} \times 1\{D_{g(i)} = 1\}(S_{hh} - S_{g(i)})$ to be the amount of returns that the household has left after paying for $i$'s loan. Let $\tilde{E}_{hh} \equiv \min\{\max\{\tilde{R}_{hh} - I, 0\}, T^*\}$ and $E_{g(h(i))} \equiv \min\{\max\{R_{g(h(i))} - I, 0\}, T^*\}$ be the maximum amount $h(i)$ and $g(h(i))$, respectively, are willing and able to transfer to each other. Define $\tilde{S}_{hh} \equiv \max\{I - \tilde{R}_{hh}, 0\}$ and $S_{g(h(i))} \equiv \max\{I - R_{g(h(i))}, 0\}$ to be the shortfalls (if any) that $h(i)$ and $g(h(i))$ face in paying back their respective loans. If $\tilde{E}_{hh} + E_{g(h(i))} > \tilde{S}_{hh} + S_{g(h(i))}$, then both $h(i)$ and $g(h(i))$ will repay. Furthermore, if $p \leq p^*$ and $\tilde{R}_{hh} > I$, then $h(i)$ will repay. Similarly, if $p \leq p^*$

and $R_{g(h(i))} > I$, then $g(h(i))$ will repay. In all other cases, $h(i)$ and $g(h(i))$ will default.

2. The previous step determined which individuals in the household-individual group repaid as a function of the realized returns to borrowing, the optimal group liability $p^*$ and the imposed group liability rate $p$. From equation (3), $p^* = \frac{\beta\mu-1}{1-\beta\pi_{R,R}(p)}$, where $\mu$ is the expected returns of borrowing. Since $\beta$ and $\mu$ are known model parameters, all that is necessary to calculate $p^*$ is $\pi_{R,R}(p)$. Note that $\pi_{R,R}(p) = E_R\left[\tilde{D}_{hh}\left(R,p,\beta\frac{V(p)}{I}-1,\theta\right)_1 \times \tilde{D}_{hh}\left(R,p,\beta\frac{V(p)}{I}-1,\theta\right)_3\right]$ where the subscripts $\tilde{D}_{hh}\left(R,p,\beta\frac{V(p)}{I}-1,\theta\right)$ denotes the $n^{th}$ element of $\tilde{D}_{hh}\left(R,p,\beta\frac{V(p)}{I}-1,\theta\right)$ and the expectation is taken over the distribution of returns. As a result, the following iterative procedure can be used to find $D_{hh}(R,p,\theta)$:

   (a) Let $n = 0$ and define $s_0 = 0.5$.

   (b) For a large number of $M$ of realized returns, calculate $\tilde{D}_{hh}(R,p,s_n,\theta)$.

   (c) Let $\tilde{\pi}_{R,R}(p) \equiv \frac{1}{M}\sum_m \tilde{D}_{hh}(R,p,s_n,\theta)_1 \times \tilde{D}_{hh}(R,p,s_n,\theta)_3$. Define $s_{n+1} = \frac{1}{2}\frac{\beta\mu-1}{1-\beta\tilde{\pi}_{R,R}(p)} + \frac{1}{2}s_n$.

   (d) Repeat the previous two steps until $|s_N - s_{N-1}| < \varepsilon$, where $\varepsilon$ is small. Define $D_{hh}(R,p,\theta) \equiv \tilde{D}_{hh}(R,p,s_N,\theta)$.

## C.2 Group size

Consider a group of $G$ individuals with a $G \times 1$ vector of realized returns $R$. The goal of the algorithm is to generate a function that takes the realized returns $R$, group liability rate $p$, and model parameters $\theta$ and returns the number of group members that default $d$: $d = D_{grp}(R,p,\theta)$. To do this, I first characterize the function $\tilde{D}_{grp}(R,p,\beta V(p)-I,\theta)$ which takes as an additional input the present discounted value of remaining eligible to borrow. I then show how it is possible to recover $D_{grp}(R,p,\theta)$ from $\tilde{D}_{grp}(R,p,\beta V(p)-I,\theta)$ using an iterative procedure.

1. Define $T^*(n) = \min\{npI, \beta V(p) - I\}$ to be the maximum transfer a borrower is willing to make as a function of the number of borrowers that default. Let $n = 0$ and $\tilde{R} \equiv R$. The following iterative process defines $\tilde{D}_{grp}(R,p,\beta V(p)-I,\theta)$:

   (a) Define $E(n) \equiv \sum_{g=1}^{G}\min\{\max\{\tilde{R}_g - I, 0\}, T(n+1)\}$ to be the total amount of transfers that group members are willing and able to make if $n$ group members default, where $\tilde{R}_g$ is the $g^{th}$ element of $\tilde{R}$. Define $S(n) \equiv \sum_{g=1}^{G}\max\{I - R_g, 0\}$ to be the total shortfall of group members if $n$ group members default.

   (b) If $E(n) \geq S(n)$, then set $d = n$ and continue to step (c). Otherwise, let $n = n+1$, define $\tilde{R}$ to be the previous $\tilde{R}$ with its lowest element removed and return to step (a).

   (c) If $\frac{d}{G-d}pI > \beta V(p) - I$ (so that strategic default is optimal) set $d = G$. Otherwise, keep $d$ unchanged.

2. Given step 1 defines $\tilde{D}_{grp}(R, p, \beta V(p) - I, \theta)$, it is possible to recover $D_{grp}(R, p, \theta)$ using the following iterative procedure. Note from equation (15) that $\beta V(p) - I = \frac{\beta\left(E[R_i] - \sum_{n=0}^{N} \pi_n 1\{\beta V - I > \frac{n}{N-n} pI\} \frac{n}{N} pI\right) - I}{1 - \beta \sum_{n=0}^{N} \pi_n \left(\frac{N-n}{N}\right) 1\{\beta V - I > \frac{n}{N-n} pI\}}$, where $\pi_n \equiv E_R\left[1\{\tilde{D}_{grp}(R, p, \beta V(p) - I, \theta) = n\}\right]$.

   (a) Let $n = 0$ and define $s_0 = 0.5$.

   (b) Let $R_m$ be the $m^{th}$ simulated return. Define $\tilde{\pi}(d) \equiv \frac{1}{M} \sum_{m=1}^{M} 1\{\tilde{D}_{grp}(R, p, s_n, \theta) = d\}$, where $M$ is large. Calculate $s_{n+1} \equiv \frac{1}{2} \frac{\beta\left(\mu I - \sum_{n=0}^{N} \tilde{\pi}_n 1\{\beta V - I > \frac{n}{N-n} pI\} \frac{n}{N} pI\right) - I}{1 - \beta \sum_{n=0}^{N} \tilde{\pi}_n \left(\frac{N-n}{N}\right) 1\{\beta V - I > \frac{n}{N-n} pI\}} + \frac{1}{2} s_n$.

   (c) Iterate step (b) until $|s_N - s_{N-1}| < \varepsilon$, where $\varepsilon$ is small. Define $D_{grp}(R, p, \theta) = \tilde{D}_{grp}(R, p, s_N, \theta)$.