Physics 41, Winter 1998
Lab 2 - The Velocity of Light

Theory

In this experiment, you will measure c, the velocity of light, by two totally different methods: a purely electrical measurement of the resonant frequency of an LC circuit and a direct measurement of the velocity of propagation of a light wave emitted by a laser diode. In the first of these methods, the inductance L and capacitance C of a simple circuit are calculated from their dimensions by standard electromagnetic theory, and $\varepsilon_o\mu_o$ calculated from the product LC, which can be found by measuring the resonant frequency of the LC circuit. In the second method, you will directly determine the velocity of propagation of a light wave, which is amplitude-modulated at a high frequency, by measuring the phase change of the modulation as a function of distance. These can be quite accurate experiments if done properly and if all possible sources of error are taken into account by appropriate corrections to your experimental results.

It was Maxwell who first noticed that the velocity with which an electromagnetic wave propagated in his theory, $c = (\varepsilon_o\mu_o)^{-1/2}$, was numerically identical to the measured velocity of light. This observation led him to a conclusion that was not obvious at the time -- that light is an electromagnetic wave. In his *Treatise on Electricity & Magnetism,* Vol. 2, Chapter XX (3rd. editions, 1891, republished by Dover Press, 1954), Maxwell argues “If it should be found that the velocity of propagation of electromagnetic disturbances is the same as the velocity of light...we shall have strong reasons for believing that light is an electromagnetic phenomenon...” Whether these two velocities are indeed the same is precisely what you will find out in this experiment.

A. Lumped circuit method.

The air-spaced capacitor consists of two coaxial metal cylinders d meters long. The outer cylinder has an *inside* diameter of b and the inner cylinder has an *outside* diameter of a. The inner cylinder is held in place by two narrow Teflon rings (width 2.5 mm, $\varepsilon = 2.1$). If end corrections and the contribution of the plastic are neglected, the capacitance is given by

$$C_c = \frac{2\pi\varepsilon_o d}{\ln\left(b/a\right)} \qquad (1)$$

The corrections to this formula are not negligible, but they can be calculated and the ratio C_c / ε_o determined (see question 2).

The inductor is a coil of length l and mean radius r with N uniformly spaced turns wound on a material with $\mu = 1$. If end corrections are neglected, the inductance is

$$L = \frac{\pi\mu_o N^2 r^2}{l} \qquad (2)$$

Again , the corrections to this formula can be calculated and the ratio L/μ_o determined (see question 3).

The resonant frequency of an LC circuit is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad (3)$$

where C is the *total* capacitance in parallel with the inductor. Some of this capacitance may come from the inductor itself (see step A.3 in the Procedure). This simple formula is also subject to a small correction for the losses in the circuit .

Hence if f_r is measured for a circuit in which C_C/ε_o and L/μ_o are known, $\varepsilon_o\mu_o$ and hence $c = (\varepsilon_o\mu_o)^{-1/2}$ can be obtained.

B. Propagation method

A diode laser is modulated at 95 MHz. Its output beam is collimated and transmitted to a retroreflector which returns it to a photodiode detector close to the laser. By moving the retroreflector along a track parallel to the light beam, you can shift the phase of the 95 MHz modulation in the detector current relative to the signal which drives the diode. By measuring this phase shift as a function of the distance travelled by the light, you can make a direct determination of the speed of light.

References

Reitz,Milford and Christy,*Foundations of electromagnetic theory*, 4th ed.
pp. 151-154, 277-278, 386-390, 394-398.

Procedure

A. Lumped Circuit Method

1. The apparatus for this part of the experiment is shown below. Check to see that the apparatus is actually setup as shown in this diagram. The circuit consists of a coaxial capacitor in parallel with a coil inductor. A sine wave voltage supplied by the HP 209A oscillator is applied to the LC circuit via a loop of wire coiled around one end of the inductor. [Since the loop of wire is not physically attached to the inductor, how does the sine wave voltage get "applied" to the LC circuit?]

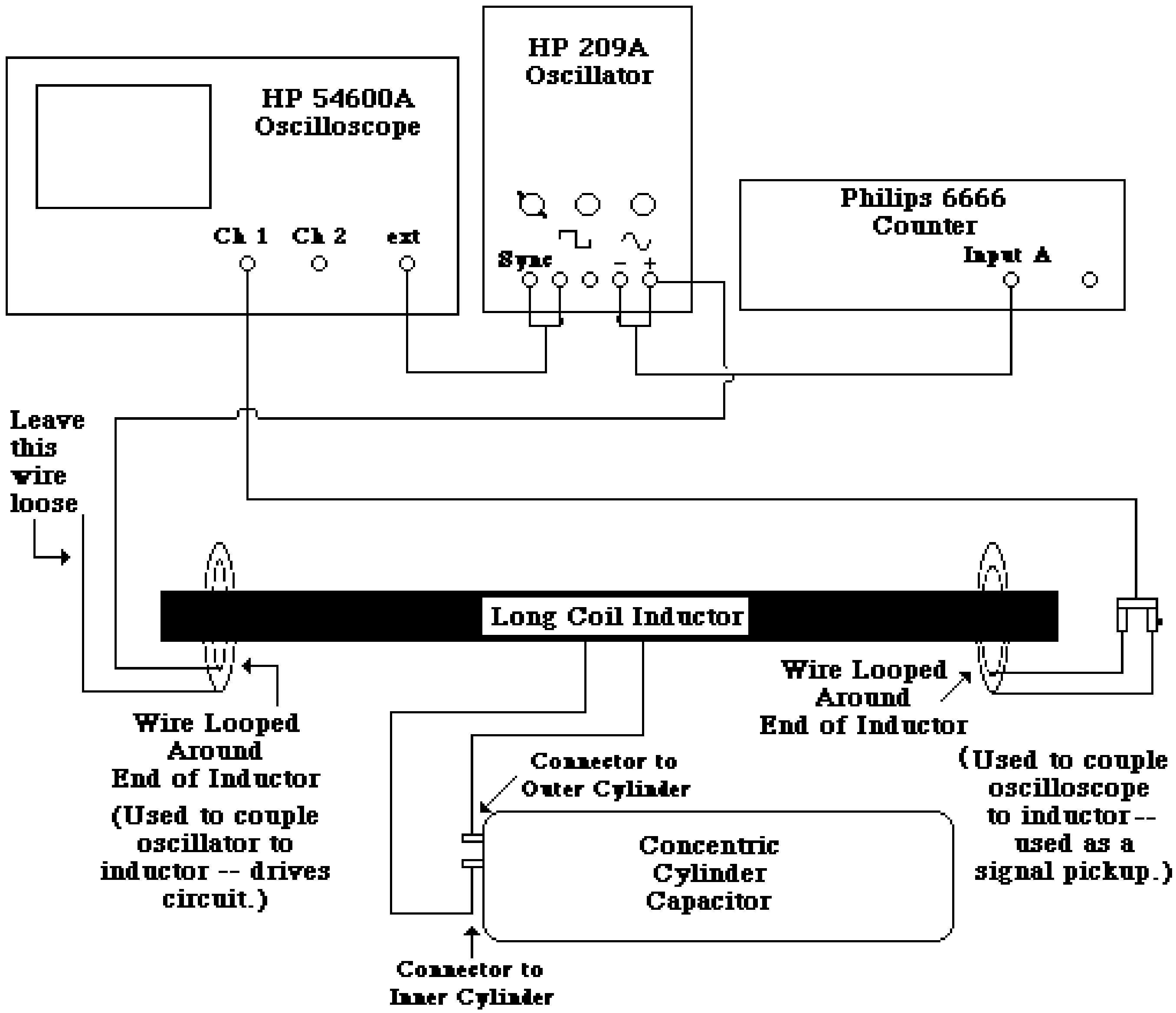

Figure 1

The frequency counter is used to measure the frequency of the sine wave voltage. It allows a much more accurate frequency measurement than reading the frequency from the dial on the oscillator. The response of the LC circuit to the applied voltage is picked up by a loop of wire around the other end of the inductor and displayed on the oscilloscope.

The relevant dimensions of the inductor and capacitor are:

Coil		Capacitor	
Length:	94.8 cm	Diameter of inner cylinder:	47.7 mm
# of turns:	877	Average gap separation:	3.26 mm
Diameter:	4.24 cm	Length:	50.1 cm

Take a few minutes to note any imperfections in the construction of the coil inductor and the coaxial conductor and what effect these imperfections might have on your experimental results.

Some questions to consider include: What is the effective diameter of the coil? Is the number of turns of wire per unit length uniform in the coil? In other words, is the number of turns per unit length constant along the whole inductor? Is the gap in the capacitor uniform, both around one end and from end to end? [Note: the value of the capacitance is much more sensitive to the gap (b-a) than to the individual radii a and b . Why?]

Estimate the errors due to these non-uniformities and, if necessary, figure out a way to correct for them.

2. If you have never used this type of oscilloscope, you may find it more confusing to configure than a conventional analog oscilloscope. All settings are made via menus shown at the bottom of the display screen. For example, press the SETUP button on the control panel of the oscilloscope. The setup menu will appear at the bottom of the display screen. One of the choices (on the far right) is "Default Setup". To choose this option, press the button below the words "Default Setup". This will put the oscilloscope in its default setup, thereby eliminating any setup options chosen by a previous user which might not be appropriate for this experiment. The only other thing you need to do to put the oscilloscope into the configuration you need for this part of the experiment is to select external triggering. To do this, press the SOURCE button in the trigger area of the control panel. Select external triggering by pressing the button under the letters "Ext" on the display. A users manual for this oscilloscope is available should you need it.

3. Use equations (1) and (2) to compute the capacitance C of the capacitor and the inductance L inductor. Compute an approximate resonant frequency for this LC circuit by substituting these values of C and L into equation (3). Set the frequency on the HP 209A oscillator to this resonant frequency. Press the AUTO-SCALE button on the oscilloscope. You should be close enough to the actual resonant frequency to see a signal on the oscilloscope display. [Note: AUTO-SCALE is a very useful function of this oscilloscope. Anytime you push AUTO-SCALE, the oscilloscope automatically adjusts the vertical and time-base gain settings to give a good display of the signal on the screen. This is very helpful when you are having a hard time making the proper adjustments manually due to unfamiliarity with the oscilloscope.]

   You may see a lot of noise on the displayed signal. This is pick-up from various sources of interference around the lab such as the fluorescent lights. Your coil is a very efficient antenna. Try moving the wires to reduce the pick up and if necessary turn off the lights.

   Another method of reducing the noise on the display is to use the average function of the oscilloscope. To do this, press the DISPLAY button on the scope. Select the average function by pressing the button under the word "Average" at the bottom of the scope display screen. The number of samples to be averaged (8, 64 or 256) can be also be selected. To return the oscilloscope display to its original display mode, press the DISPLAY button and select "Normal" by pressing the button under the word "Normal" at the bottom of the display screen. [*Note: When the oscilloscope is in the average mode the display will be slow to respond to any changes in the signal -- such as when you change the frequency of the sine wave. You may find this slow response annoying. You may find it more convenient to leave the scope in the normal mode until you are ready to make measurement on the displayed signal. At that point, switch to the average mode, make your measurement and then return to the normal display mode.*]

3. Using the fine tuning knob on the oscillator, find the actual resonant frequency of the circuit. The frequency which gives the largest peak-to-peak voltage is the resonant frequency.

   The auto store feature of the oscilloscope can be used to make this measurement. Press the AUTO-STORE button in the upper right hand corner of the oscilloscope's control panel. Use the fine tuning knob on the oscillator to sweep the frequency back and forth across the resonant frequency. As you do so observe the oscilloscope display. The oscilloscope will capture each of the frequencies you tune through and display them on the screen without erasing the previously displayed trace. A lighter colored envelope of traces will appear behind the current trace on the screen. Once the largest peak-to-peak voltage becomes evident, adjust the frequency of the current trace until its peak matches the peak of the previously stored traces. Read the resonant frequeny from the frequency counter.

To return the display to its normal mode, press AUTO-STORE to turn off the auto-store feature and then press ERASE to erase the previously stored traces.

4. Set the frequency to approximately 10 kHz below the resonant frequency. Press AUTO-SCALE to get a reasonable display of the signal. Measure the peak-to-peak voltage of the signal. If the signal is noisy, you may want to put the scope into average mode before making this measurement.

   Note: The oscilloscope has a peak-to-peak voltage measurment option. To use it, press the VOLTAGE button at the top middle of the control panel of the oscilloscope. Choose the "V p-p" option by pressing the appropriate button at the bottom of the screen display. The peak-to-peak voltage will be displayed directly below the grid in the display. If you are using the average mode, you should wait until the scope has averaged the signal before selecting the peak-to-peak voltage option.

   Increase the frequency by 2 kHz and repeat this measurement. Repeat this process until you are 10 kHz above the resonant frequency. Finally, make a peak-to-peak measurement at the resonant frequency.

   Plot this data and compute the Q of the circuit. Check that the loading by the drive and pick up circuits has a negligible effect by varying the number of turns of coupling wire and if necessary extrapolate f_r to zero loading. If necessary, correct the resonant frequency for the finite Q. What are you measuring?

5. The inductance computed using equation (2) is the inductance of the coil only. It does not include any inductance between the two straight wires connecting the coil to the capacitor. The inductance of a pair of parallel wires of length d, diameter a, and separation between centers b, is

$$L = \frac{\mu_o d\left[1 + 4\ln\left(b/a\right)\right]}{4\pi} .$$

   Measure the length, diameter and separation of the wires connecting the inductor to the capacitor. Check to see if the correction to L/μ_o is significant.

6. An inductor has a “self-resonance” at a wavelength approximately twice the total length of wire in the coil. This self-resonance arises because the inductor is actually an LC circuit all by itself since the windings have some capacitance which appears in parallel with the inductance. Compute the approximate value of this self resonance. Disconnect the inductor from the capacitor and find the resonant frequency of the inductor by itself. How big is this effect? How should you correct for it?

7. Using your measurements and the results you obtained for questions 2 and 3, compute L/μ_o and C_C/ε_o, and hence c. Estimate the errors in your measurement due to: the uncertainty in f_r, the uncertainty in the dimensions of the capacitor and inductor, and those due to the uncertainty of the various corrections you have chosen to include. How accurately do these corrections need to be determined?

B. Propagation Method

1. The apparatus for this part of the experiment is shown on the next page. Check to see that the apparatus is actually setup as shown in this diagram. The 12 V DC power supply provides the DC current to power the diode laser. This DC current is fed through the 10 dB amplifier box (but is not amplified) and out to the Simpson multimeter where its value is measured. From the Simpson multimeter, the DC diode current travels to the bias tee where the output from the HP 8601 Sweeper/Oscillator is added to the DC current -- thus modulating the diode current at the frequency set on the sweeper/oscillator. [The output from the sweeper/oscillator is also fed into Channel 1 of the oscilloscope from the directional coupler.] The modulated DC current is then sent to the diode laser. The light from the laser travels down the optical bench and is reflected back towards the laser by the retroreflector. The path of the laser light is adjusted so that the returning light passes just above the laser, through the lens and is focused onto the photodetector. The signal from the photodetector is fed through the 28 and 10 bB amplifers and then into Channel 2 of the oscilloscope.

2. <u>System setup procedure.</u> This is a delicate operation. Please follow these instructions carefully. Do NOT touch any adjustments on the optical bench or associated equipment. (Sliding the retroreflector back and forth on the optical bench is OK.) The optical alignments are difficult to do and have already been done for you. Mucking about with any of the optical components may destroy the alignment and then the person responsible for aligning the equipment will be a very, very unhappy camper!!!

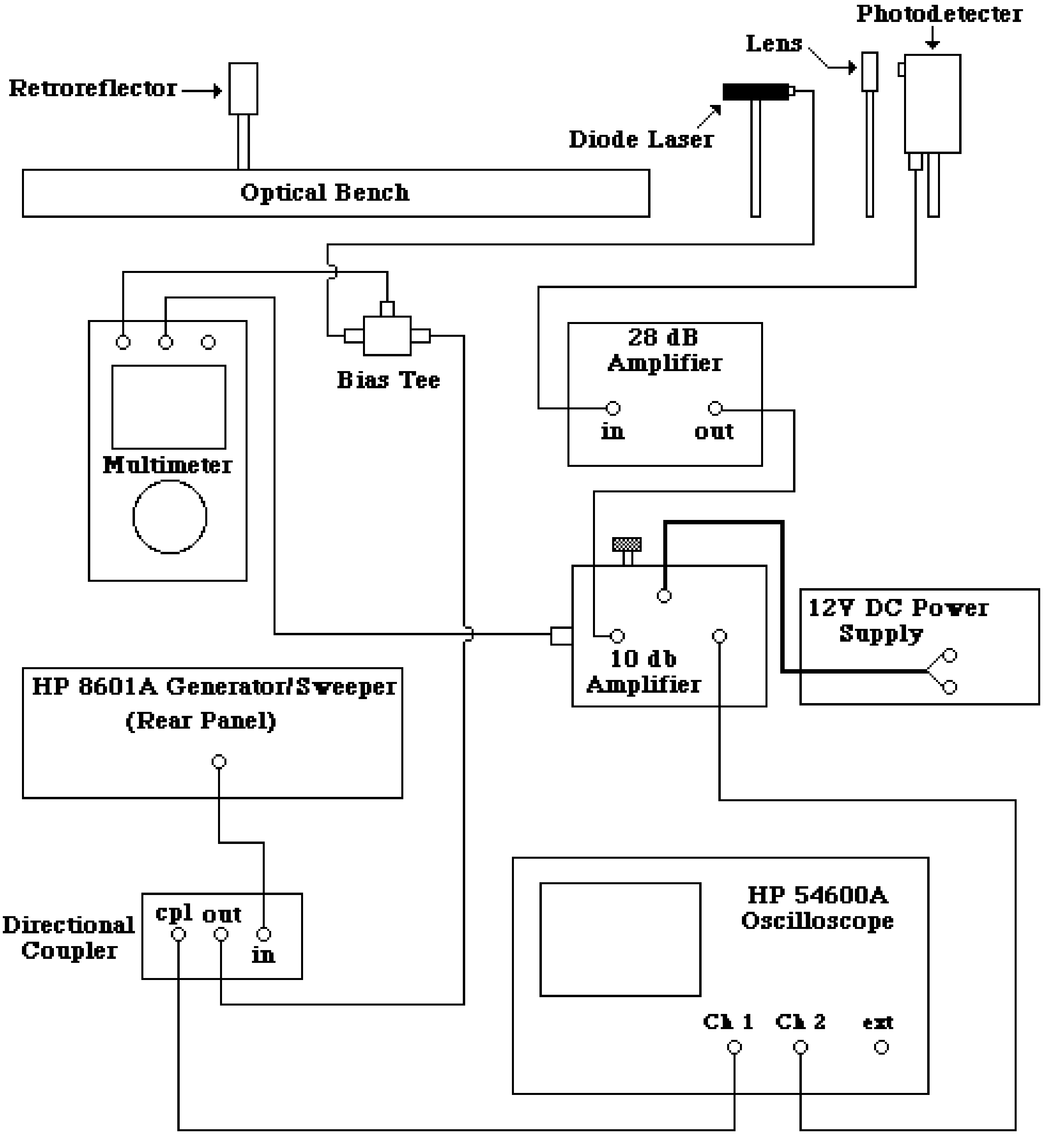
Photodetecter
Lens
Retroreflector
Diode Laser
Optical Bench
Multimeter
Bias Tee
28 dB Amplifier
in
out
10 db Amplifier
12V DC Power Supply
HP 8601A Generator/Sweeper (Rear Panel)
Directional Coupler
cpl
out
in
HP 54600A Oscilloscope
Ch 1
Ch 2
ext

Figure 2

a. Turn on the HP 8601 Sweeper/Oscillator. The power switch is on the lower left.

   There is a large knob on the left that controls the frequency of the output. A small display above the knob gives the frequency setting in MHz. Set the frequency to 95 MHz.

   To the right of the frequency display is a four position toggle switch with positions labels "CW/ SYM/VIDEO/FULL". Set this switch to the "CW" position.

   Below and to the left of the above four position switch is a RANGE toggle switch with two settings. Set this switch to 110.

   On the right of the control panel is knob labeled OUTPUT LEVEL that controls the signal amplitude. Around the outside of the knob are blue numbers representing the maximum output power range in dB (decibels with respect to a reference level of 1 mW) and black numbers representing the maximum output in volts (across 50 Ω). Set the OUTPUT LEVEL to a maximum output of +20 dB. The blue +20 dB labels should be at the top of the dial under the black dot.

   The red center part of the OUTPUT LEVEL knob controls the actual power level which you can monitor on the little meter above the knob. Turn up the power level until it reads 2.5 V on the black middle scale on the meter.

   Below the OUTPUT LEVEL knob is a three position toggle switch labeled "1 KHz Mod". Set this switch to the "Off" position.

b. Turn on the oscilloscope.

   1. Press the SETUP button on the control panel. Select "Default Setup" from the menu at the bottom of the display screen.
   2. Press the MAIN/DELAYED button on the control panel. Set the "Time Ref" option to "Left".
   3. Press AUTO-SCALE.

   A small part of the RF output from the sweeper/oscillator is picked off by a directional coupler and fed into Channel 1 of the the oscilloscope. You should now see this signal on the oscilloscope display screen.

c. The knob on the rear of the 10 db amplifier box controls the current to the diode laser. Turning the knob so that the top of the knob moves to the right turns the current down. Set this knob to the full off position.

d. The 12 V DC power supply has two switches on the right. The bottom switch is the main power switch for the supply. Turning this lower switch on turns on the power to the supply, but does not turn on the output jacks and therefore will not turn on the laser. To turn on the power to the laser you must also turn on the upper switch. Turn on both switches.

e. Set the Simpson meter to the DC current 100 mA full scale range. **Slowly** turn up the DC current to the laser using the knob on the back of the 10 dB amplifier. Monitor the current on the Simpson meter. Do not allow the current to move above 55 mA. Currents above 55 mA will damage the diode laser.

   [Note: The current output is limited by hardware inside the box so that turning the current all the way up will give you a maximum current of 50 mA. Therefore, there should be no way you can turn up the current to levels that will damage the laser. However, hardware can fail. Thus, you should still monitor the current to make sure it does not accidently exceed the 55 mA value.]

   Put your hand or a sheet of paper in front of the laser diode to verify that it is operating. **DO NOT LOOK INTO THE LASER!** While this laser is not powerful enough to cause serious injury, why take risks with the only eyes you've got? Looking into laser beams is an *incredibly* bad habit to develop. Many research lasers are powerful enough to completely blind you right off, no second chances.

f. Turn on the photodetector. The switch is on the bottom rear of the photodetector box. Press AUTO-SCALE on the oscilloscope. If all is working well you should now see two signals on the scope display. The top signal is the modulated signal sent to the diode laser. The bottom signal is the signal received from the photodetector.

g. Position the retroreflector at the 150 cm mark on the optical bench. Block the laser beam. Do you still have a signal? If so, why? Adjust the frequency of the RF drive to minimize this spurious signal, bearing in mind that you would like to keep the frequency high.

h. Using the vertical position controls on the oscilloscope, position the channel 1 display in the middle of the screen and position the channel 2 display to overlap the channel 1 signal.

Slowly move the retroreflector to the end of the optical bench. The phase difference between the two signals should change as you move the retroreflector down the optical bench. There should be a usable signal on channel 2 when the retroreflector is at its farthest position from the photodetector. If any of these conditions are not met, the optical system may be out of alignment. Do NOT attempt to align the system yourself. See your lab TA or the course instructor.

2. Before starting the data taking, some comments on the use of the oscilloscope in this part of the lab are in order. Before continuing, move the retroreflector to the 150 cm mark on the optical bench.

   a. As you can see the signals are quite noisy. The noise can be minimized when taking data by using the averaging option of the oscilloscope. The procedure for using this option is the same as used in part I of this experiment. Turn on the averaging option now.

   b. The oscilloscope has an option for measuring time base quantities. Press the TIME button at the top of the oscilloscope control panel. The time measurement options will appear at the bottom of the display screen. Select the "Freq" option. The frequency of the signal will appear under the display grid. The signal whose frequency was measured is indicated under the "Source" option. The period of the signal can also be measured by selecting the "Period" option.

   c. Time intervals can also be measured using the cursor function of the scope. Press the CURSOR button at the top of the oscilloscope control panel. [Note: The time cursors will also automatically appear when you press the TIME button. However, they will not become active until you press the CURSOR button.] Two vertical cursors should appear on the screen. To the right and below the CURSOR button is a position control knob for the cursors. Turn the knob. One of the cursurs should move. In the cursor menu at the bottom of the screen the active curser will be highlighted - either t1 or t2. If cursor t1 is active, select cursor t2 or vica versa. Turn the same cursor position knob. The second cursor should now move on the screen.

      Make cursor t1 the active cursor. Position it on one of the the crests of the signal displayed on channel 1. Make t2 the active cursor. Position cursor t2 on one of the the crests of the signal displayed on channel 2. Note that the time interval between the two cursors is displayed below the display grid along with the time value of t1, the time value of t2 and the $1/\Delta t$.

   d. Using the techniques discussed in b and c above, you can now measure the phase difference between the two signals. How? Try it.

You now know enough about the scope to make the measurements required in this part of the lab.

3. To make a direct measurement of a speed, you need to know the distance traveled and the time it took to travel that distance. The object of this part of the lab is to make a direct measurement of the speed of light. What measurements will you need to make? How can you measure the distance traveled by the light and the time it took to travel that distance using the apparatus at hand? Make the necessary measurements.

   Note: This portion of the lab procedure is purposely vague. It is up to you to figure out what measurements need to be made. Be sure to explain in your lab notebook what measurements you made and how they can be used to make a direct measurement of the speed of light.

4. Calculate c from your measurements.

5. Shut-down procedure. When you are done taking data, please complete the following steps to shut down the apparatus.
   a. Turn off the photodetector. The battery inside is expensive to replace.
   b. Turn the DC laser current all the way down.
   c. Turn off both switches on the 12 V DC power supply.
   d. Turn down the RF power level and shut off the RF generator.
   e. Turn off the oscilloscope.

Lab Report

Your lab report should be a record of what you did in the lab. It should include a short explanation of what you were trying to do, any deviations from the stated procedure, all raw data, and any conclusions your can draw from the data. Also include the following answers to the following questions:

1. (a) Show from equations (1), (2) and (3) that $(\varepsilon_0 \mu_0)^{-1/2}$ does indeed have the dimensions of a velocity.

   (b) To get a velocity one needs a length and a time. What experimentally determined quantities serve these functions in the electrical part of the experiment? (Ignore end and other corrections to the simple formulae).

2. The simple calculation that leads to Equation (1) neglects the fact that the electric field is not perfectly uniform near the ends of the capacitor. An approximate end correction for this effect is to assume the field *is* perfectly uniform, but that the length of the capacitor is increased by an amount $(b - a)/2\pi$ at each end. (see J.Clerk Maxwell, *Treatise on Electricity and Magnetism*, 3rd ed, 1891, art. 202; republished by Dover Press, 1954). Compute this to obtain a corrected value for C_c/ε_o.

3. (a) Show that if end corrections are neglected the inductance of the coil is given by Eq. (2).

   (b) Using the formula for the field along the central axis inside a coil carrying current *I*:

$$\frac{B}{I} = \frac{\mu_o N}{2l}\left(\cos\left(\theta_1\right) - \cos\left(\theta_2\right)\right) \qquad (4)$$

   where θ_1 and θ_2 are the angles subtended at the field point by the ends of the coil (see E.M. Purcell, *Electricity and Magnetism*, section 6.5. The derivation is in cgs units, but replacing $4\pi/c$ by $\mu_o$will convert the formula to MKS units.). The end corrections reduce the inductance of the coil by a factor $(1 - r / l)$, where terms of order $(r / l)^2$ are neglected and the field is assumed to be uniform over cross-section of the coil.

4. What corrections did you include when computing c for this part of the experiement. Which did you ignore? In both cases, why?

5. Estimate and discuss the random and systematic errors associated with the direct measurement of the speed of light made in part II of the experiment.

6. Was Maxwell right? Do the two values for c from the different measurement methods agree within experimental error? If not, what might have gone wrong?