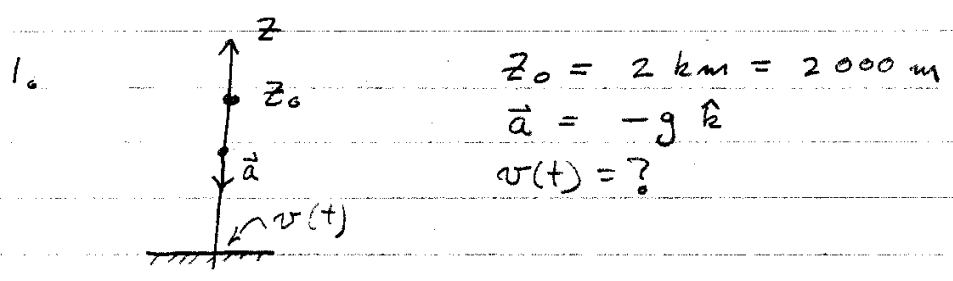


Physics 3 Quiz 3

14Jul03

1. Suppose a 1 gram raindrop were to form at a height of 2 km above the ground. If there were no air friction, at what speed would this raindrop hit your head?
2. Some rowdy fraternity brothers build a trebuchet that is capable of launching very substantial water balloons with a speed of 30 m/s. Since these gentlemen have studied physics they know that they will get maximum range if they launch their balloons at an elevation angle of 45 degrees. So they set up their trebuchet to do so. In an assault on a rival house at a range of 35 m, how much time elapses from launch to impact?
3. The sling of the trebuchet extends to 3 m high when the balloon is released. How high up the wall of the target building is the point of impact?
4. How much bodily harm will a water balloon do at 30 m/s? Answer: more than you want to think about.

Quiz 3 14 Jul 03



The raindrop accelerates freely straight down under the influence of gravity. So this is a one-dimensional problem on the z -axis. The equations of motion are

$$v(t) = v_{0z} + a_{0z} t$$

for velocity, and

$$z(t) = z_0 + v_{0z} t + \frac{1}{2} a t^2$$

for position.

Since this is all one dimensional, I'll simplify the notation:

$$v = v_0 + at$$

$$z = z_0 + v_0 t + \frac{1}{2} a t^2$$

Now note that $v_0 = 0$ because the raindrop started with no velocity. The value of z at the ground is $z = 0$

(2)

Also, $a = -g$, the downward acceleration due to gravity. Thus

$$v = -gt$$

$$0 = z_0 - \frac{1}{2}gt^2$$

v is what we want. To use the first equation, I need t . Get it by solving the second.

$$t = \sqrt{\frac{2z_0}{g}} \quad (= 20.2 \text{ sec})$$

Now substitute this expression for t into the first equation

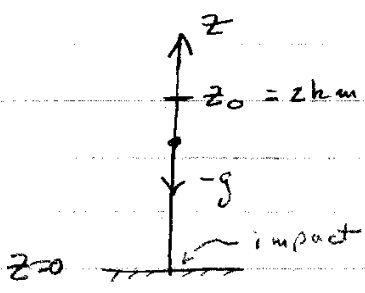
$$v = -gt = -g\sqrt{\frac{2z_0}{g}}$$

$$= -\sqrt{2gz_0} = -198 \text{ m/s}$$

The minus sign means the velocity is directed in the negative direction along the z -axis. It is moving down.

If I were taking the Quiz, my answer would look like this:

1.



$$z_0 = 2 \text{ km}$$

$$v_0 = 0$$

$$v = ? \text{ at } z = 0$$

$$v = v_0 - gt$$

$$z = z_0 + v_0 t - \frac{1}{2} g t^2$$

$$z = 0 \text{ at impact ; } v_0 = 0$$

$$0 = z_0 - \frac{1}{2} g t^2$$

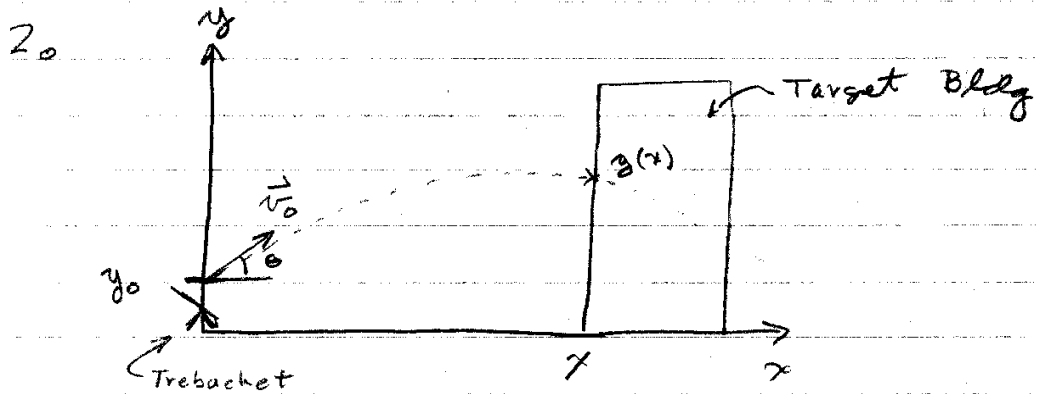
$$t = \sqrt{\frac{2z_0}{g}}$$

$$v = -gt = -g \sqrt{\frac{2z_0}{g}}$$

$$= -\sqrt{2gz_0}$$

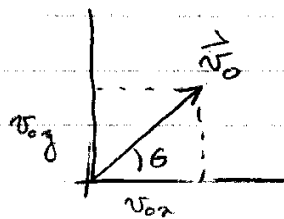
$$= -\sqrt{2 \times 9.8 \text{ m/s}^2 \times 2000 \text{ m}} = -198 \text{ m/s}$$

(4)



$$\begin{aligned} v_0 &= 30 \text{ m/s} & x &= 35 \text{ m} \\ \theta &= 45^\circ & t &= ? \text{ when } x = 35 \text{ m} \\ y_0 &= 3 \text{ m} \end{aligned}$$

Resolve \vec{v}_0 into components



$$v_{0x} = v_0 \cos \theta$$

$$v_{0y} = v_0 \sin \theta$$

The time to the wall depends only on the x -component of velocity

$$x = x_0 + v_{0x} t$$

$$x_0 = 0$$

$$t = \frac{x}{v_{0x}}$$

$$= \frac{x}{v_0 \cos \theta} = \frac{35 \text{ m}}{30 \text{ m/s} \cos 45^\circ} = 1.65 \text{ sec}$$

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3. Need $y(x)$. But that's the same as $y(t)$. So

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

We have $t = \frac{x}{v_{0x}}$, so

$$y = y_0 + \frac{v_{0y}}{v_{0x}} x - \frac{1}{2} g \left(\frac{x}{v_{0x}} \right)^2$$

$$= y_0 + \frac{v_0 \sin \theta}{v_0 \cos \theta} x - \frac{g}{2 v_0^2 \cos^2 \theta} x^2$$

$$= 3 \text{ m} + \frac{\sin 45^\circ}{\cos 45^\circ} 35 \text{ m} - \frac{9.8 \text{ m/s}^2}{2 \times (30 \text{ m/s})^2 (\cos 45^\circ)^2} (35 \text{ m})^2$$

$$= 3 \text{ m} + 35 \text{ m} - 13.3 \text{ m} = 25 \text{ m}$$