Key Concepts for the Lecture of 7Jul03

Kinematic Overview

- The position of a particle is given by the coordinates of the particle in the chosen coordinate system. Position is a vector whose components are the coordinates of the particle:

$$\vec{\boldsymbol{r}} = x\,\hat{\boldsymbol{i}} + y\,\hat{\boldsymbol{j}} + z\,\hat{\boldsymbol{k}}$$

- Velocity is the rate of change of position:

$$\vec{\boldsymbol{v}} \equiv \frac{d\vec{\boldsymbol{r}}}{dt}$$

- Since the components of the vector velocity are independent, each "scalar component" of velocity obeys its own equation of motion:

$$v_x(t) \equiv \frac{dx(t)}{dt}$$

- If the velocity is known, then the position is

$$x(t) = \int v_x(t)\,dt$$

- For the *special case* of constant velocity, the position is

$$x(t) = x_0 + v_{x0}\,t$$

$$where \quad v(t) = v_{x0} = const$$

$$and \quad x_0 = x(0)$$

  The x_0 term arises from evaluating the constant of integration. In this case, x_0 is the initial position. As time increases, the position increases linearly from the initial value. Its rate of increase is the velocity.

- In the more general case, velocity can change as a function of time. Acceleration is the rate of change of velocity:

$$\vec{\boldsymbol{a}}(t) \equiv \frac{d\,\vec{\boldsymbol{v}}(t)}{dt}$$

- Since the components of the vector acceleration are independent, each "scalar component" of acceleration obeys its own equation of motion:

$$a_x(t) \equiv \frac{dv_x(t)}{dt}$$

- If the acceleration is known, then the velocity is

$$v_x(t) = \int a_x(t)\, dt$$

- For the *special case* of constant acceleration,

$$v_x(t) = v_{x0} + a_{x0}\, t$$

$$where \quad a_{x0}(t) = a_{x0} = const$$

$$and \quad v_{x0} = v_x(0)$$

The v_{x0} term arises from evaluating the constant of integration. In this case, v_{x0} is the initial velocity. As time increases, the velocity increases linearly from the initial value. Its rate of increase is the acceleration.

- Still for the *special case* of constant acceleration a_{x0}, initial velocity v_{x0}, and initial position x_0, the position is

$$x(t) = x_0 + v_{x0}\, t + \frac{1}{2}\, a_{x0}\, t^2$$