Key Concepts for the Lecture of 16Jul03

Using Newton's Laws

- In the study of Kinematics, we developed an ability to understand and interpret the *equations of motion*. We have studied the special case of constant acceleration (which includes the special case of constant velocity) extensively. Given an initial position, an initial velocity, and an acceleration for a body, we can compute its later velocity and position at any time. We can predict the time evolution of the system. It is Newton's second law that gives us the acceleration to use in the equations of motion. Newton's second law states that a particle accelerates under the influence of a force. Just as your intuition suggests, the acceleration is proportional to the magnitude of the force, and inversely proportional to the mass (inertia) of the object being accelerated:

$$\vec{\boldsymbol{F}} = m\,\vec{\boldsymbol{a}}$$

- To analyze a system in which the components undergo linear motion, use the following general procedure:

  1. Analyze the system identifying each body (or particle or object) capable of motion. Take note of their masses and any geometrical constraints like hanging from a rope or sliding on a plane.

  2. Examine each body to identify all its interactions and the forces acting on it. Note that Newton's third law requires that every force arise from an interaction of two bodies, and that the forces associated with each interaction act on the two bodies in precisely opposite directions with precisely equal magnitudes. There will be a law of physics which governs each interaction and which will allow you to compute each force.

  3. Draw a free-body diagram for each body in the system. Carefully identify each force acting on the body, and draw it in the correct direction--and with the correct magnitude if you know it. Label each force clearly. Again, note that each force is the result of an *interaction between two bodies*. The body you are working on is always the *pushee*; you should be able to identify the *pusher* for each force.

  4. For each body, add up the forces. *You must do this addition vectorially.* The resultant of this sum is the total force (or net force or resultant force) on the body. It is this force that causes the acceleration described in Newton's second law.

  5. Use Newton's second law to write down the equation of motion for each body separately. If you have done all the steps above correctly, this basically a clerical task. The equation of motion for each body in the system is

$$\Sigma\,\vec{\boldsymbol{F}} = m\,\vec{\boldsymbol{a}}$$

6. If you perform step 5 for each body in the system, you will have a system of *n* equations in *n* unknowns. There will be three equations for each body--one each for the x, y, and z components of force and acceleration--and there will be one set of these three equations for each body. Clearly this can be a complex algebraic system, but its solution is the complete kinematics of the system.

- In general, the unknowns in the system of equations of motion will be components of acceleration and components of unknown forces of interaction between pairs of particles.

- There are several important tricks to simplify the derivation of the equations of motion.

1. In many systems, the individual bodies may only move in one dimension. In these cases, you can simplify the summation of forces by separately determining the component of force along the axis of motion. Then you can characterize each force with a signed scalar value (the scalar magnitude of the force), and you can characterize the acceleration with a signed scalar value. You are free to choose the direction along the axis of motion that you will treat as positive. Be sure to do this consistently!

2. Examine the geometry of any constraints on the system to see if you can equate any forces or accelerations. For example, a massless, inextensible rope in tension will exert the same tension force on the two bodies it connects. Tension is always a pull. (You can't push a rope.)

3. An object resting or sliding on a surface can only move in a direction parallel to the surface. Whatever the force is that presses the body to the surface is exactly canceled by the "normal force" of the surface pushing back on the body. It is safe to leave these forces out of the free-body diagram and the equations of motion. But don't neglect to include any component of applied force *parallel* to the surface.

4. The judicious choice of coordinate system can simplify things greatly. Notice that you can use a different coordinate system for each object in the system. You just have to be careful how you account for the forces of interaction between pairs of objects so that they are represented correctly in the coordinate system appropriate to each body.

5. It is often possible to group objects together and treat them as a single object with mass equal to the sum of the masses of the objects. *Be very careful using this technique!* You must be able to justify the combination of masses, or risk obscuring interactions that contribute to the dynamics. (Think of the dumbwaiter problem.)

- The procedure described here has tremendous generality. Although it often results in intractable sets of equations of motions, you can use it very effectively to analyze systems that involve straight-line motion of the components at constant acceleration.