Effect of Surface Roughness on Phonon Thermal Conductance in the Quantum Limit

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We have investigated phonon energy transport and the reduction in the thermal conductance in the quantum limit due to phonon scattering by surface roughness using full 3-dimensional elasticity theory for an elastic beam with a rectangular cross-section. For $T < 0.4K$ and a thickness to width ratio as large as 0.38, a comparison between thin plate theory and full numerical results for the mode dispersion relations using the xyz-algorithm suggests that a thin plate model gives a good approximation for the thermal conductance, as well as providing more intuitive expressions for the scattering coefficients. Our analysis also suggests that, as in the scalar model, phonons propagating in higher modes are localized near their onset frequencies. At low frequencies we find scattering coefficients proportional to ω^2 for the compression and torsion modes as in a simple scalar model for the waves used previously, but $\omega^{3/2}$ for both in-plane and out-of-plane bending modes. The scattering of the out-of-plane bending mode is much smaller than that of the other three modes. The phonon scattering yields a decrease in thermal conductance below the universal value at low temperature as observed in the experiment by Schwab et al. [Nature **404**, 974 (2000)].