Determination of the symmetry classes of elastic crystalline bodies

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The method of determining the symmetry class to which a crystalline medium belongs is proposed. It is assumed that the matrix $\tilde{\mathbf{C}}'$ representing the tensor of elastic constants *C*' is given in some arbitrary (laboratory) coordinate system (LCS). Assuming that waves propagate along a longitudinal normal (LN) (i.e. that waves are pure longitudinal and pure transverse) one may direct the axes of the coordinate system along the polarization vectors. In such coordinate system, without loss of generality, one may assume that the elastic constants C_{45}, C_{35} and C_{34} vanish [1]. So, we are looking for all such Euler rotations of LCS (enumerated by q and described by matrices $\tilde{\mathbf{A}}_q$) transforming $\tilde{\mathbf{C}}'$ to $\tilde{\mathbf{C}}_q^{//} = \tilde{\mathbf{A}}_q \tilde{\mathbf{C}}^{/} \tilde{\mathbf{A}}_q^{-1}$ with $\left(\tilde{\mathbf{C}}_q^{//}\right)_{34} = 0$ and $\left(\tilde{\mathbf{C}}_q^{//}\right)_{45} = \left(\tilde{\mathbf{C}}_q^{//}\right)_{35} = 0$ [1,2]. Generally, this problem can be solved only numerically. Then performing rotations around each of new *z*-axes (which are directed along the polarization vectors of the longitudinal waves) by the angles π, 2π/3, π/2, π/6, and looking to the form of transformed tensor of elastic constants, we check if these axes are the symmetry axes. If the answer is positive, the axis is one of acoustic axes, i.e. pure transverse modes propagating along it are degenerate. If all axes are only the longitudinal normals the medium is triclinic. In the opposite case the established set of symmetry axes allows one to determine the class of symmetry to which the medium belongs. Thus, it is possible to determine the crystalline coordinate systems (CCS) in which the matrices representing the tensor of elastic constants have simple and standardized forms. Knowing LCS and CCS one may find the Euler angles characterizing the orientation of the elastic body with respect to LCS. We designed a suitable program of numerical calculations performing the described procedure.

The accuracy of calculations is determined mainly by errors of values of elastic constants. The elements of matrices representing tensors of elastic constants in LCS can be measured for example by immersion technique [4].

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