

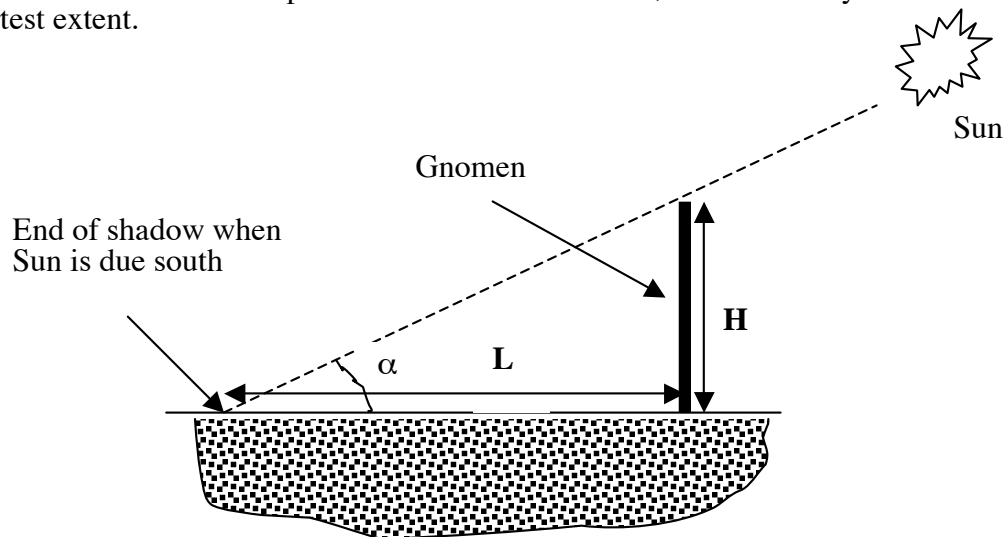
## Astronomy 4 Term Project

### Measuring the Solar Altitude and Determining the Geographical Latitude of Hanover

**Introduction:** The gnomon, or vertical stake which casts a shadow as the Sun moves across the sky, is considered to have been the first astronomical instrument, used already by pre-literate societies. With this instrument, you can measure the altitude of celestial objects (i.e., their angle above the horizon). Greek astronomers like Ptolemy used the gnomon to determine the length of the year with considerable accuracy. And from measurements of the Sun's altitude and knowledge of the Sun's position in the sky on the day in question, you can compute your geographical latitude (i.e., angular distance from the earth's equator to your zenith, with the apex of the angle being the center of the earth).

**Task:** Your goal for this project is to make a series of measurements, at about three-day intervals, of the Sun's maximal altitude (around noon), and from these data to compute the geographical latitude of Hanover. To do this, you will need: i) a gnomon and some type of plumb bob; ii) a measuring tape to determine the length of the Sun's shadow and the height of your gnomon (available for signing out in Room 200 in Wilder Laboratory); iii) an almanac giving the Sun's declination for the dates of your observations; and iv) a protractor or some means of computing tangents.

**Procedures:** First you will measure the maximal solar altitude about every third day for the remainder of the term. You will need to observe the shadow cast by the gnomon when the Sun reaches its highest point in the sky on a given day, i.e., when the shadow is the shortest. At this instant, the Sun is on the meridian for Hanover, i.e., the great circle on the celestial sphere that passes through both poles and Hanover's zenith point). This happens around noon (or more precisely, half way between sunrise and sunset), when the Sun is due south. Since we are currently on daylight saving time, and since the Sun's true motion does not follow exactly the regular advance of clocks, you may want to watch the Sun's shadow for up to 30 minutes around noon, to make sure you catch it at its shortest extent.



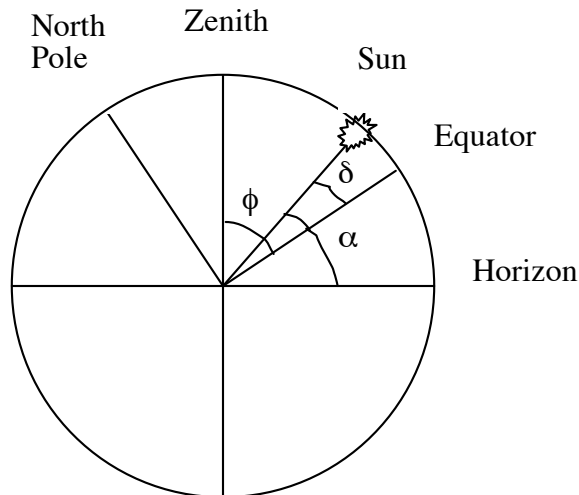
To find the solar altitude (denoted by the Greek letter alpha,  $\alpha$ ), you can make a scale drawing on a piece of paper (for example, by drawing each distance one-tenth the actual size) of the gnomon and the shadow, and then measure the angle  $\alpha$  with a protractor. Or you can use the formula:

$$\alpha = \tan^{-1} (H/L),$$

where **H** is the height of the gnomon, **L** the length of the Sun's shadow, and  $\tan^{-1}$  means the angle whose tangent is **H/L**. This value can be looked up in a table of trigonometric functions, or computed with a scientific pocket calculator. For example, if your gnomon is 1.2 m tall and the shadow's length is 0.87 m, the ratio of **H/L** is 1.38. The angle whose tangent is 1.38 is  $54^\circ$ , which would be the solar altitude for that observation.

In making this measurement, it is important that your gnomon be exactly vertical. Since early astronomers didn't have access to carpenter's levels, in which bubbles indicate positions of the vertical and horizontal, use a plumb bob to align your gnomon. For best results, you will want to use as tall a gnomon as possible, so that the uncertainties in your measurement of length will be small compared with the lengths being measured.

To compute the geographical latitude of Hanover (or wherever you have placed your gnomon), imagine the earth compressed to a point in the center of the following diagram of the celestial sphere. This diagram shows the relationship between your measured solar altitudes ( $\alpha$ ) and your geographical latitude (designated by the Greek letter phi or  $\phi$ ). The outer circle marks the meridian through your zenith.



From the diagram, you can see that  $\phi = 90^\circ - (\alpha - \delta) = 90^\circ + \delta - \alpha$ .

To compute  $\phi$ , you therefore must not only to measure  $\alpha$  but also to know the value of  $\delta$  for the date of your observation. The declination ( $\delta$ ), or angular distance of the Sun from the equator, varies throughout the year. At the equinoxes, the solar  $\delta = 0^\circ$ . At the solstices, the solar declination reaches a maximum of about  $23.5^\circ$  on June 21, and minimum of about  $-23.5^\circ$  on December 21. Do you see why? Daily solar declinations can be found in *The astronomical almanac 2005* (Washington, D.C., 2005) [Kresge Ref QB8. U12 2005].

**Results:** i) Write a brief description of the construction and site of your gnomon, and of your procedures for measuring the solar altitudes. ii) Present your data, listing the date, time, shadow length, solar altitude, and solar declination for each measurement. For each observation, compute the geographical latitude. iii) If your computed latitudes vary widely and you did not move your gnomon from Hanover to Montreal, discuss possible sources of error in your observations. iv) Using the above diagram, prove that the geographical latitude ( $\phi$ ) equals the altitude of the North Pole, i.e., that by measuring the height of the North Star you will have measured your geographical latitude.