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# Newton's "System of the World"

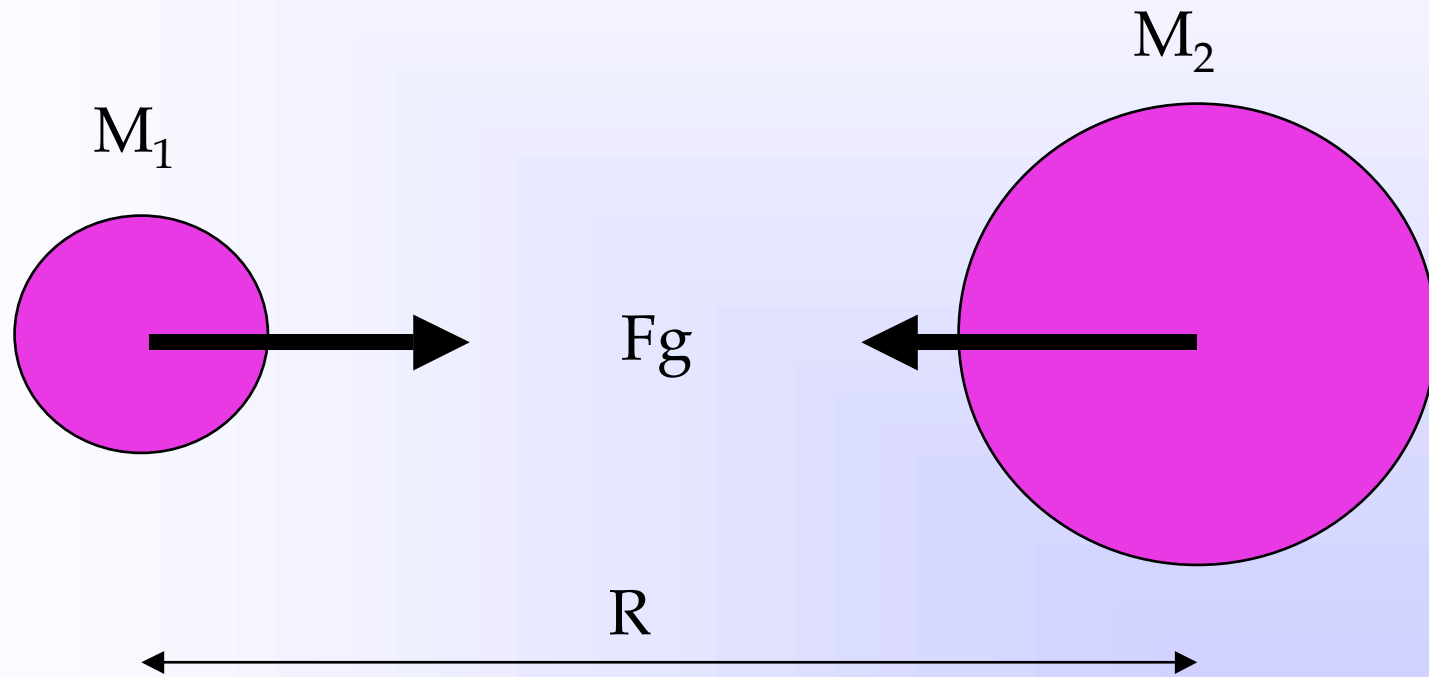
# Newton's grand synthesis

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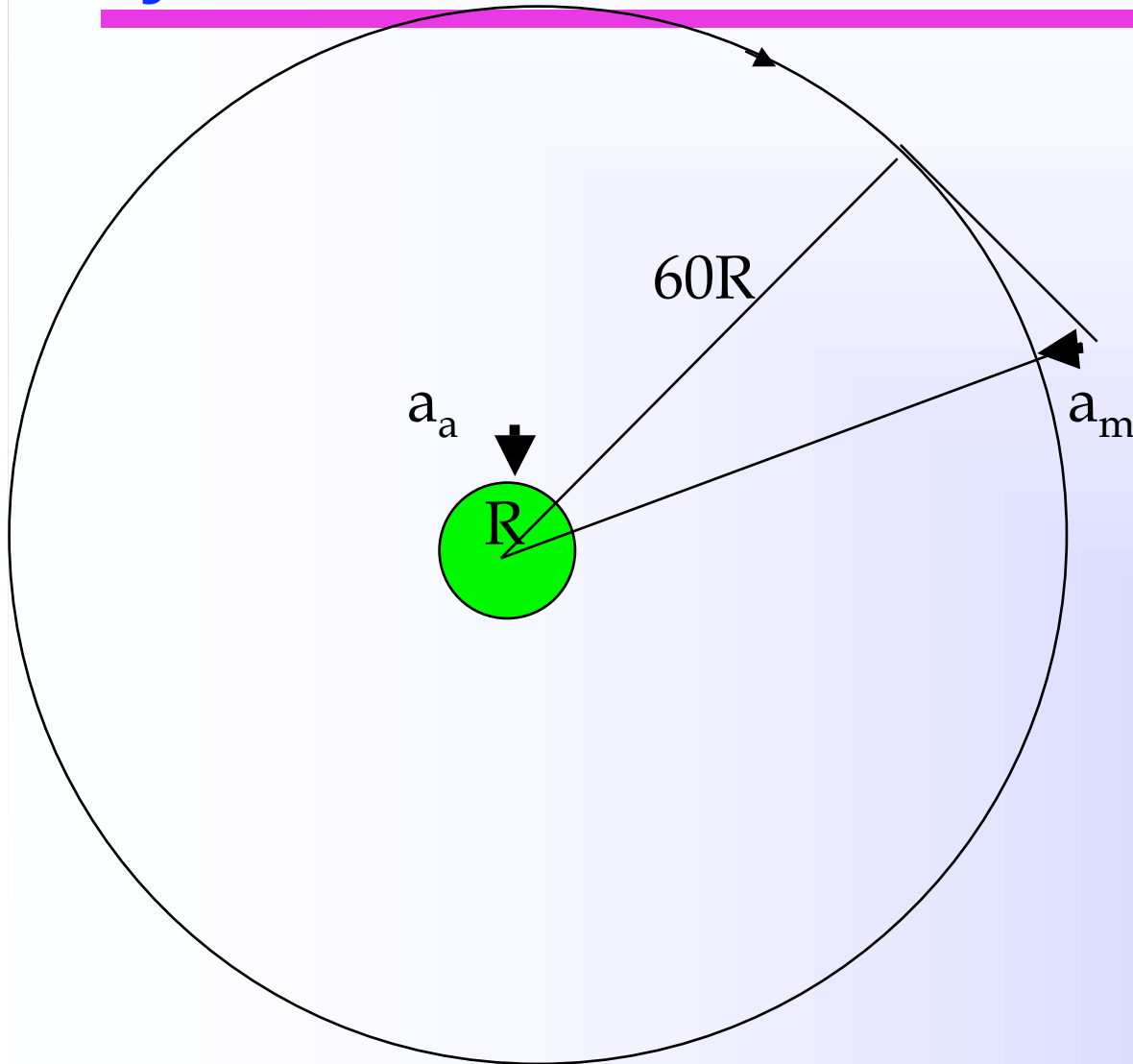
- Unified physics of heavens/earth with three laws of motion
  - Inertia
  - $F = ma$
  - Action = reaction
- Universal gravitation:  $F_g = GM_1M_2/R^2$
- “Hypothesis non fingo” on causes
- God as the “cosmic mechanic”
- Newton's mathematical philosophy vs. Descartes' **mechanical** philosophy

# Universal gravitation

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# Newton's "Moon Test" implies yes



$$F = ma = GmM_e/R^2$$

Thus, near Earth,  
 $a = GM_e/R^2$

For apple:

$$a_a = GM_e/R^2$$

For moon:

$$a_m = GM_e/(60R)^2$$

or  $a_a/a_m = 3600$   
(confirmed by  
observations!)

# Newton's "System of the World"

## ■ Explains Kepler's Laws

### ➤ Law of elliptical orbits

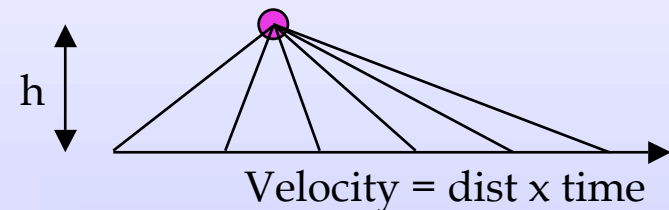
- Inverse square, central forces produce conic section orbits

### ➤ Law of equal areas

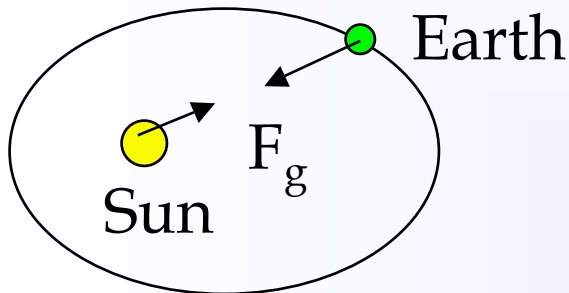
- Holds for inertial motion
- Holds for motion under central forces

### ➤ Harmonic Law ( $R^3/P^2 = \text{constant}$ )

- Holds for Jupiter's satellites, or Sun's planets



# Weighing the Sun



By Newton's 3d Law, the Earth and Sun attract each other; hence both are being accelerated by gravity

For sun:

$$a_s = GM_e/R^2$$

For earth:

$$a_e = GM_s/R^2$$

Total acceleration:

$$a = a_s + a_e = (M_e + M_s)G/R^2$$

Central acceleration:

$$a = V^2/R, \text{ where}$$

$$V = \text{Distance/Time} = 2\pi R/P$$

Let total acceleration = central acceleration

$$\text{Hence: } (M_e + M_s)G/R^2 = (4\pi^2 R)/P^2$$

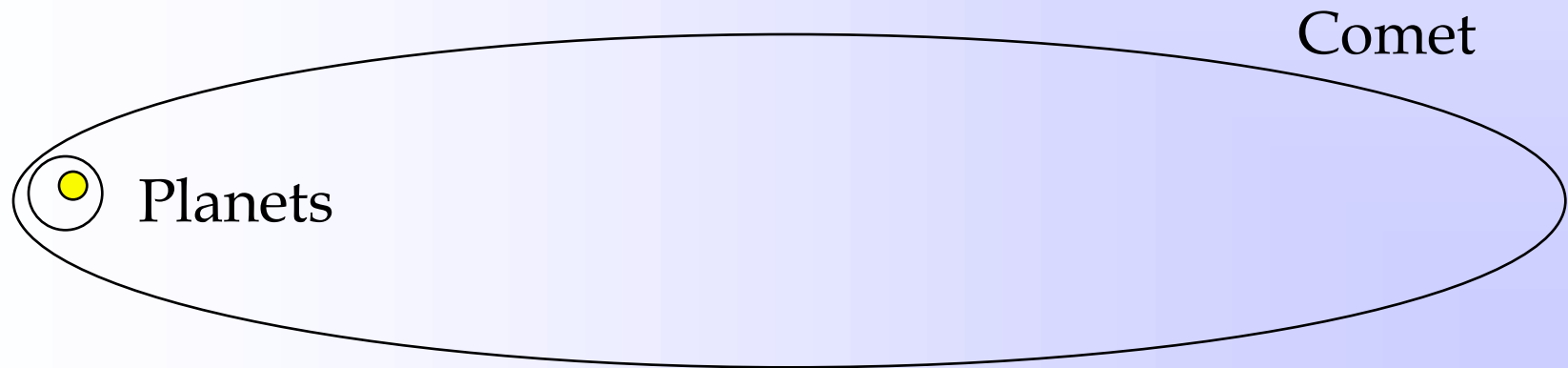
$$\text{Assume } M_e \ll M_s, \text{ so that: } M_s = (4\pi^2/G) (R^3/P^2) = 2 \times 10^{30} \text{ kgs}$$

This is cool (and yields Kepler's third law!)

# Comets explained

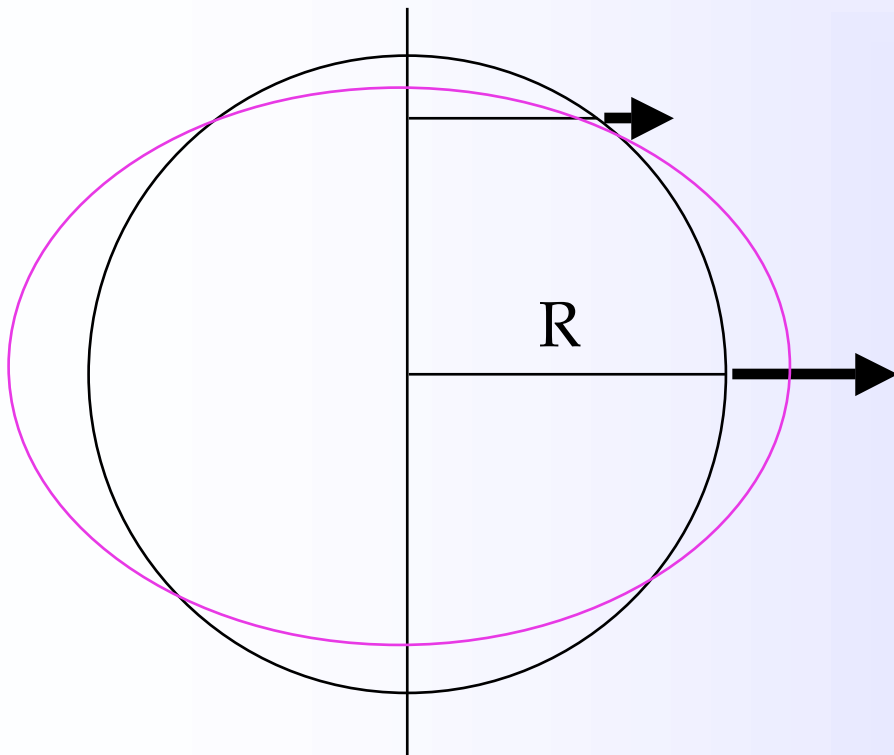
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- Halley's comet, 1682
  - Halley speculates that same comet appeared in 1531, 1607, 1682
  - Newton computes orbital parameters, predicts return in Dec 1758
  - Confirmed!



# Oblate shape of Earth

- What force required to hold spinning earth in spherical shape?



$$a = v^2/R$$

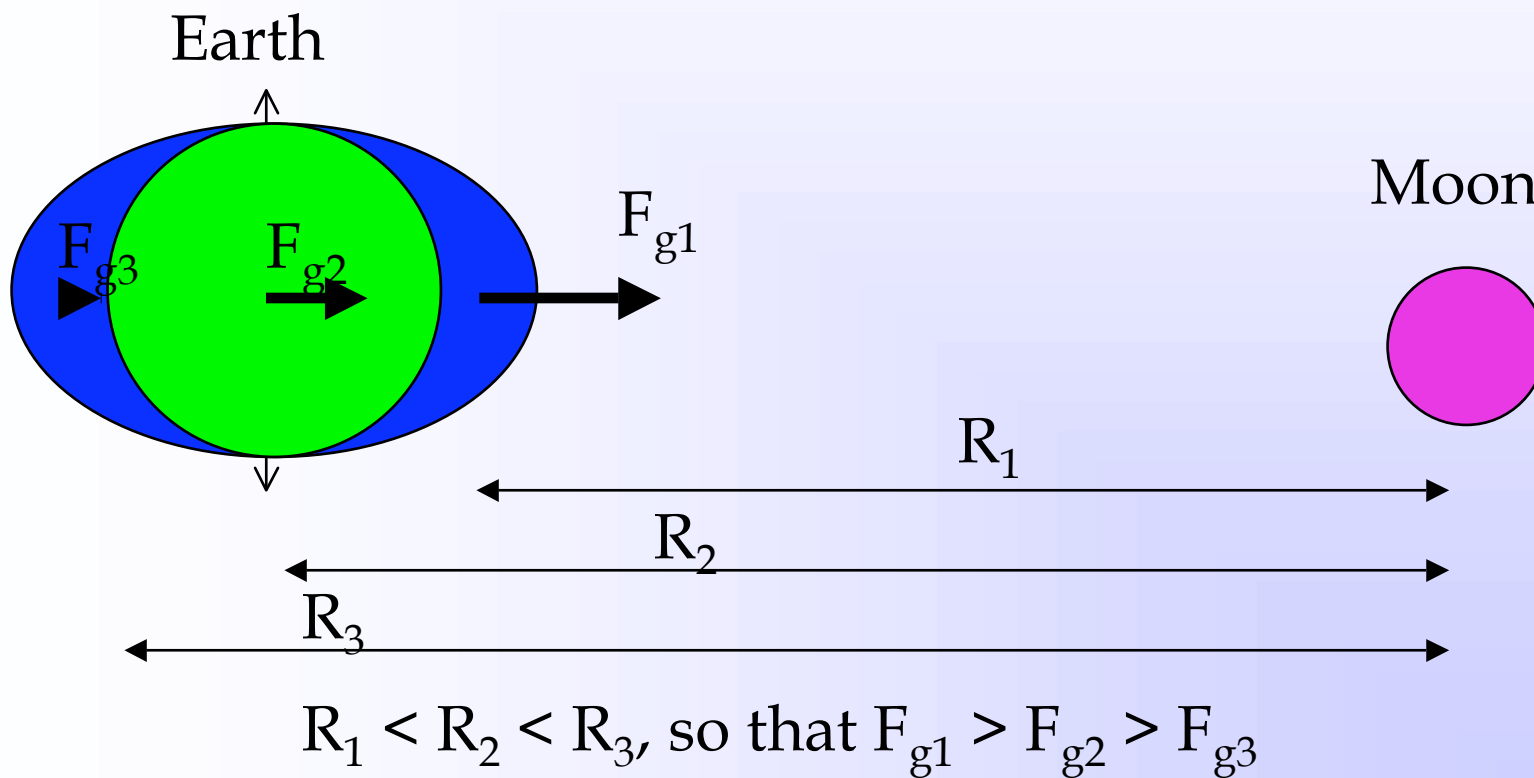
(a greatest at equator)

$$F = ma = mv^2/R = 4m\pi^2R/P^2,$$

so force is greatest at equator and bulges out earth there

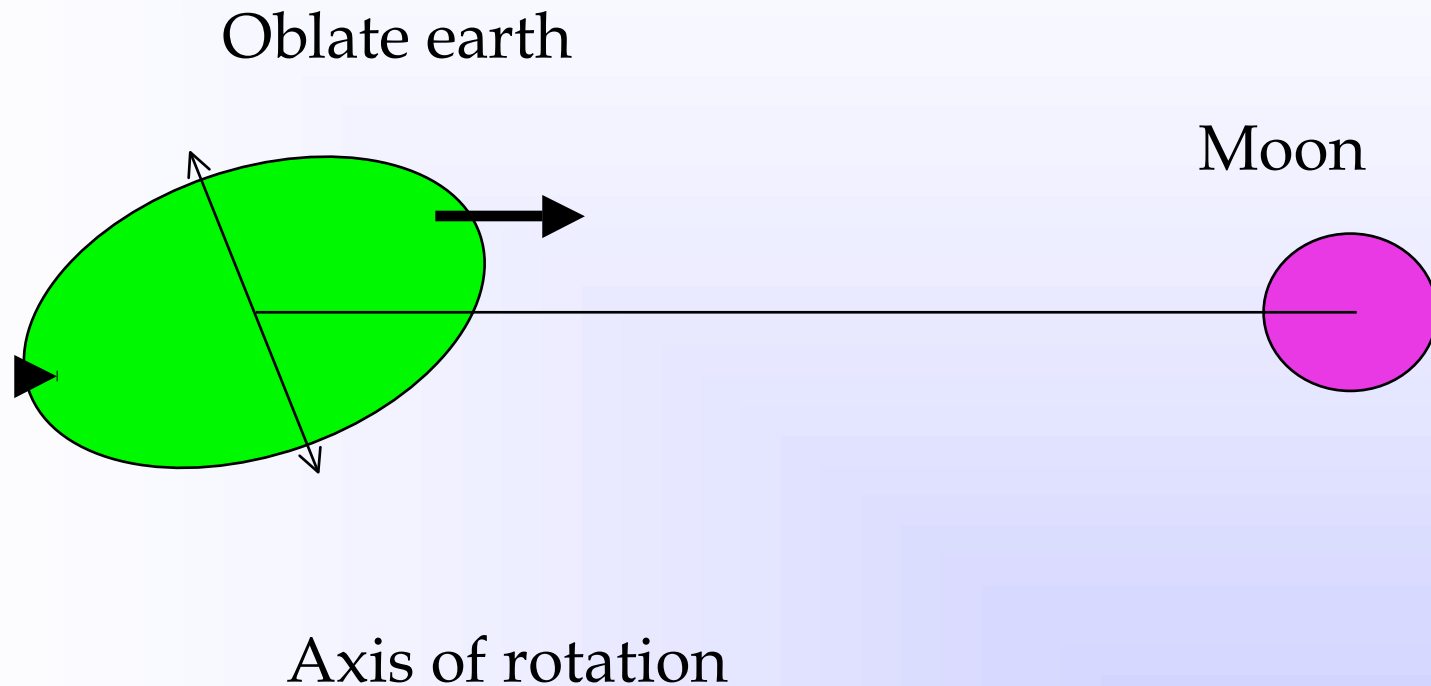
Confirmed by French in 1730s

# Two tides daily



# Precession of Earth

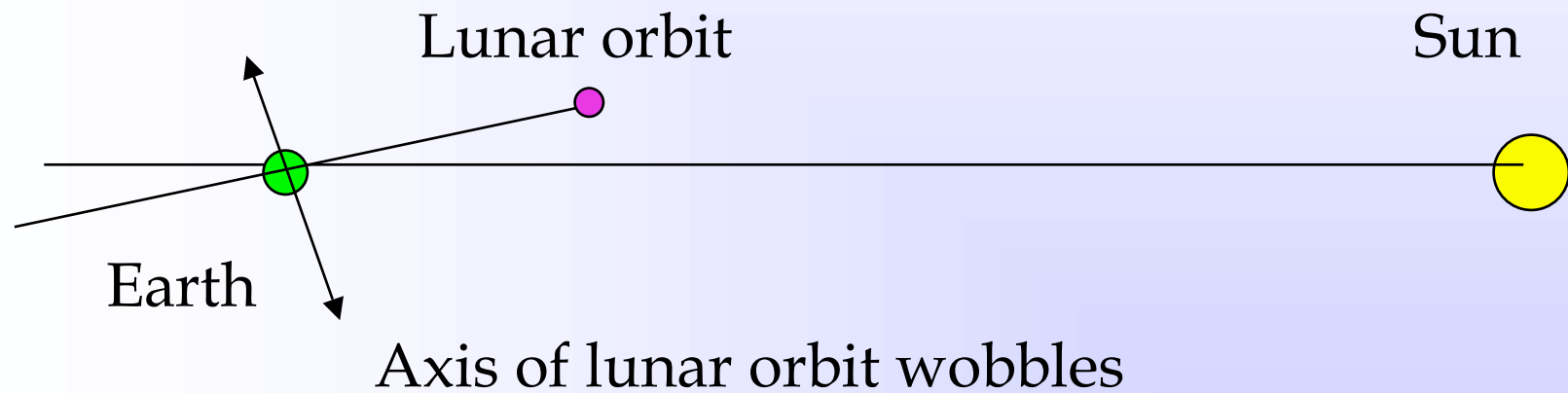
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Explains precession of stars on celestial sphere!

# Precession of lunar orbit

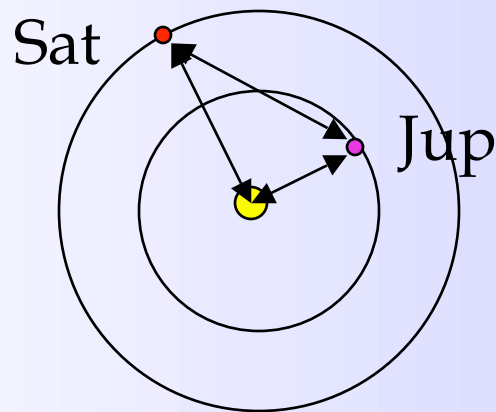
- Explains 19-year period of lunar nodes



# Perturbation of Jupiter, Saturn

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- 3-body problem cannot be solved exactly
  - Newton's approx. techniques explain slight deviation of Jup/Saturn from Kepler's laws
  - Perturbations require God's periodic intervention if solar system is to remain stable, says Newton



# Newtonian cosmology in 18c

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- Positional astronomy at national observatories
  - Greenwich, Paris, also Danzig, Nuremberg
  - Problem of longitude at sea
    - Accurate clocks on ships?
    - Accurate time-keeping with moon?
    - Board of Longitude's 20,000 pound prize
- Laplace, *Celestial Mechanics*, 1799
  - State-employed mathematician, friend of Napoleon
  - Stable solar system without God
  - Problems with Mercury (88" / century)
  - Nebular hypothesis (spinning gas cloud + gravity)
    - Explains rotational directions and planes of planets & Sun
    - First new cosmogony since Plato